ELEMENTS OF LOGIC

DEDUCTIVE AND INDUCTIVE

VOL. I

BY

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To

My Revered Father,

Kavirāj Umā Charan Sen,

Çikitsā-Viśārad

THE ELEMENTS OF LOGIC

PREFACE

The present work is primarily intended as a text-book for College students, preparing for University Examinations. It embraces a full course of Logic, both Deductive and Inductive, and in these pages I have tried to explain the problems, principles and rules of Logic in a way best suited to the understanding of those to whom the subject is new.

The work consists of four Books, treating successively of Introductory topics, Deduction, Induction, and Fallacies. In Book I, I have attempted to explain the Psychological and Metaphysical problems which have a bearing on Logic. Such problems, no doubt, present a good deal of difficulty to beginners ; still I think it is on the whole desirable that they should be taught at the very outset to view Logic as a propædeutic to Philosophy ; and from my long experience as a teacher, I know that it is quite possible to make these introductory topics intelligible and interesting to them. Book II treats of the Traditional doctrine of Syllogism in three parts—the first dealing with Terms, the second with Propositions, and the third with Inferences. Book III treats of the problems of Induction. Its object is to 'show the grounds on which our scientific knowledge rests, the methods by which it has been built up, and the defects from which it must be free.' Here I have drawn illustrations from everyday experience and from well known phenomena of Nature, so that students may easily understand the application of the principles and rules. In Book IV, I have given an exhaustive classification of Fallacies—Deductive

and Inductive—and explained them by means of suitable examples.

Each Book contains a large number of questions for solution together with directions for answering them. I hav-- also supplied in abundance arguments gleaned from various works on Logic, as well as from the examination papers of different Universities. A few Sections and certain Notes, which treat of controversial matters or embody difficult discussions, have been printed in smaller type. These are intended for comparatively advanced students and may be omitted by beginners. Moreover, I have added a thorough and complete marginal analysis to enable my readers to follow, remember, and revise the course.

As the idea of writing this work on Logic grew out of the lectures delivered to my pupils for nearly two decades, it is but natural that I should draw upon various standard books on the subject. It will also be noticed that I have freely made use of quotations from different authorities on the subject with a view to interesting the young learners in the original works.

I must here avail myself of the opportunity of recording my indebtedness to Rai Krishna Chandra Bhattacharyya Bahadur, M.A., P.R.S. (late Principal, Hooghly College and late Professor of Philosophy, Calcutta University), who has not only gone through a considerable portion of the manuscript, but also encouraged me to bring out this work; as well as to my esteemed colleague, Prof. Mohini Mohan Mukherjee, M.A., to Profs. Abaninath Basu, M.A., B.L., and Jyotish Chandra Banerjee, M.A., pupils and colleagues of mine, and to Mr. Sasibhusan

Barik, B.A., another brilliant student of mine, all of whom have rendered ungrudging service in the preparation of this work.

K. S.

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Book I

INTRODUCTION

CHAPTER I

PLACE OF LOGIC IN THE SCHEME OF KNOWLEDGE

§ 1. **What is Logic?** The world of things (facts and phenomena) comprises two great divisions, *viz.*, the world of matter and the world of mind. A little examination will show that a thing must be either matter or mind. *Matter* is that which has *extension*, or which occupies space. Thus the table before me, yonder tree, the star in the sky are all examples of matter, for they all occupy certain portions of space. These are extended or space-filling things. But take any living sentient creature, *e.g.*, man. He has not only a body—an extended material organism, but also a mind manifesting itself in the powers of thinking, feeling and willing. *Mind*, therefore, is that which has *consciousness*, or which can think, feel and will. Thus I work out a mathematical problem (thinking), I am pleased or displeased with a thing (feeling), I move my hand to pluck a fruit (willing),—these are the workings

The two broad divisions of things—Matter and Mind.

Matter is known by extension.

Mind is known by consciousness or the functions of Thinking, Feeling and Willing.

and manifestations of mind. (The world of matter is called the External world or the Objective world, and the world of mind is called the Internal world or the Subjective world.) In knowing a thing, the mind is the subject because it knows, and the thing is the object because it is known; hence 'Subjective' means internal or pertaining to mind, and 'Objective' means external, real or pertaining to the world of objects.

Meanings of 'Subjective' and 'Objective.'

The distinguishing characteristic of man consists in his attempt to know, and the systematic knowledge of the different departments of the world gives rise to different Sciences.

Now, we know that men are rational, that rationality is the characteristic which distinguishes man from all other species of animals. Rationality, again, manifests itself in the attempt to understand, in the curiosity to know the facts and phenomena of this world. And for convenience of study men have divided the world into several ideal departments—the department of plants, the department of stars, the department of minerals, etc. The systematic and connected study of each particular department gives rise to a special Science. Thus we have the Sciences of Botany, Astronomy, Mineralogy, etc. Corresponding to the two great divisions of things, matter and mind, we have two great divisions of Sciences, material or physical and mental. *Material sciences* deal with things and relations of things belonging to the external world of *Nature*, while *mental sciences* deal with facts and phenomena of the internal world of *Mind*.

A science is either material or mental, according as it deals with matter or with mind.

Again, a science may be either theoretical or practical. A science is *theoretical* or speculative when it simply studies facts as they actually are, or as they actually happen, and tells us about their nature, working and development. Thus Botany is a theoretical science, because it tells us simply about the nature, structure and development of plants,

A science is either theoretical or practical, according as it deals with things as they are, or

without ever entering into the question of their practical utility. A science is *practical* when it studies things as they ought to be, or as we wish them to be. A practical science is also called an *Art*. It has an end in view, and lays down rules for guidance of conduct. Thus Surgery is an art, because it has an end in view, *viz.*, healing of diseases, and lays down rules of operation for its attainment.

with things as they ought to be.

Mind, as indicated above, manifests itself in the three functions, *viz.*, thinking, feeling and willing, which form the subject-matter of Psychology. Psychology is a theoretical study of mind—it is a theoretical science which conveys to us the knowledge of the nature, origin and development of thinking, feeling and willing as they actually take place in a normal human mind. Psychology, again, gives rise to *three practical sciences*, corresponding to the three functions of mind. Thus:—

While Psychology is a theoretical mental science,

The Psychology of Thinking gives rise to **Logic** —a practical science—which studies how to regulate *thinking* for the attainment of truth.

The Psychology of Feeling gives rise to **Æsthetics** —a practical science—which studies how to regulate *feeling* for the production and realisation of the beautiful.

The Psychology of Willing gives rise to **Ethics** —a practical science—which studies how to regulate *willing* or action for the attainment of right conduct.

Logic, Æsthetics, and Ethics are practical mental sciences, respectively rising out of the Psychology of Thinking, of Feeling, and of Willing.

Thus Truth, Beauty and Duty are respectively the ends of Logic, Æsthetics and Ethics, which inquire into the nature and conditions of their respective ends, and lay down rules for the attainment of these ends.

From what has been said above it is clear that

(1) Logic is a *science* ;

(2) Logic is a *mental science*, dealing with facts of mind, as opposed to a material science dealing with facts of the external world ;

(3) Logic is a *practical mental science* (rising out of the Psychology of Thinking), which regulates our thinking, *i.e.*, lays down rules for the guidance of thinking, in order to attain truth.

Provisional definition of Logic.

Logic may thus be provisionally defined as the *Practical Science which regulates Thinking in order to attain Truth.*

CHAPTER II

General Subject-matter and Definition

§ 1. **What is Thought? Thought in General and Logical Thought.** We have just seen that Logic is the practical science which regulates thinking in order to attain truth. Hence, a clear understanding of *what Logic is* requires us to know the meanings of such words as 'Thought', 'Truth', and 'Science'. As Logic owes its origin to the Psychology of Thinking, the first thing we have to do is to understand clearly what thinking or thought is.

Thinking, Feeling and Willing—explained and illustrated

We have noticed above that mind has three fundamental functions, *viz.*, thinking (otherwise known as knowing, cognition or intellection), feeling (affection or emotion) and willing (desire, volition, conation or action). Suppose there is an orange before me on the table ; it produces certain sensations in me, viz., sensations of shape, size, colour, etc. These sensations (passive affections of the mind by an external object) are interpreted by the thinking activity of mind, and thus a knowledge of the particular orange in all its relations is obtained. This is *Thought* or knowing activity of mind. Next, the thought of the orange as a fruit very sweet to the taste may please me. This is *Feeling* or emotion. (Elementary feeling or sensation precedes thought, while emotion or feeling proper follows it.) Lastly, I may desire to taste the orange, or put forth effort to throw it out of the window. This is *Willing* or volitional activity of mind. Any state of mind is analysable into the above three functions.

It must not be supposed, however, that mind is wholly engaged for a time with one function, then abandons it and enters upon another, but rather all three are present in varying degrees in each and every state of consciousness. One may, however, predominate in intensity over the other two on a particular occasion, and give its colouring to the whole. Thus in solving a problem or in reading something from memory, the intellectual or thinking aspect is predominant, in feeling angry with an enemy, the feeling or emotional side is predominant, in lifting a weight, the volitional or active aspect is predominant. But in each case the remaining two are also present, only in a less marked degree.

analysed and the several constituent factors explained and illustrated.

As Logic is concerned with *Thought*, let us more closely analyse its nature. **Thought** includes several factors, *viz.*, Sensation, Perception, Memory, Imagination, Conception, Judgment and Reasoning. *Sensation* is the mind's susceptibility of being affected by the qualities of an external thing. The particular orange before me produces certain sensations of shape, colour, etc., in my mind. *Perception* is the intellectual activity by means of which mind interprets the raw materials supplied by sensations, and arrives at a clear knowledge of the particular orange. *Memory* and *Imagination* are the intellectual activities by which the knowledge and the idea acquired by perception are preserved, reproduced and modified when required. Thus even when the particular orange is removed, the idea continues to linger in my memory, and I can reproduce and represent it in my mind when required. Further, I can imagine an orange of a larger shape or of sweeter taste than any actually experienced by me. *Conception* is the intellectual process of forming a general idea, called Concept, on an examination of a number of particular objects. Thus, I can frame the general notion 'Orange', which represents the whole class of such fruits, and not this or that particular orange.

The result of the process of conception is called a Concept, General Idea or Notion. A Concept when expressed in language is called a Term. (For formation of Concept, *vide* Ch. V, § 1.) *Judgment* is the intellectual process of establishing a relation of agreement or disagreement between two ideas. By Judgment we affirm or deny one idea of another. Thus, when I say 'The orange is sweet', I affirm 'sweetness' of 'orange'. Here a relation of agreement is established between the two ideas 'orange' and 'sweetness'. The result or product of the process of judgment is also called a Judgment, and a Judgment when expressed in language is called a Proposition. *Reasoning* or Inference is the intellectual process of passing from one or more given judgments to some other new judgment. Reasoning is thus a process of passing from the known to the unknown, from the given to the new, from the observed to the unobserved, from the present to the past, distant and future. Thus, if I pass from the two given judgments 'All oranges are sweet' and 'This is an orange' to another new judgment 'This is sweet', I am said to reason or infer. Or, if on examining many oranges and finding them to be sweet, I pass over to the judgment 'All oranges are sweet', I am said to reason or infer. An Inference or Reasoning when expressed in language is called an Argument.

Psychological Thought means any intellectual activity; and

Now, 'thought', in the *wide* sense or the sense in which it is used in Psychology, stands for all the intellectual activities explained above. And there are two well marked divisions of intellectual activities :—

(1) Activities of Acquisition, *viz.*, Perception and Reasoning. Perception is the immediate

knowledge of particular things present before our senses, and reasoning or inference is the mediate knowledge of things past, distant and future. (For the distinction between immediate and mediate knowledge, *vide* § 6.)

(2) Activities of Conservation, *viz.*, Memory and Imagination, by which we preserve, reproduce and modify the ideas and knowledge acquired by perception and inference.

'Logical thought' means Reasoning or inferential thought together with its adjuncts—Conception and Judgment.

In Logic, however, the word 'Thought' is used in a *narrow sense.* **Logical Thought**, or Thought which falls within the province of Logic, is that which is concerned with the *acquisition* of knowledge. And even within the sphere of acquisition, the range of Logical thought is limited to the second form, *viz.*, Reasoning or Inferential thought. Perception is excluded from the sphere of Logic, for perception cannot be regulated. Logic teaches us to infer correctly, but cannot teach us to perceive; we perceive according to the natural capacity of our senses (*vide* § 6). Thus, when we say that Logic is the 'Science of Thought', we mean that it is the 'Science of Reasoning'. And as Reasoning directly involves the processes of Conception and Judgment (see § 2), Logical thought is restricted to mean sometimes the processes of Conception, Judgment and Reasoning, and sometimes the products of these processes, *viz.*, Concepts, Judgments and Reasonings.

Reasoning is the mental process of passing from the 'known',

§ 2. Reasoning in General. We have seen above that 'Thought' in Logic means Reasoning, and Logic is the science which teaches us to think, *i.e.*, to reason, correctly. Let us explain here the nature of Reasoning more fully. **Reasoning** is the mental process by means of which we extend our

knowledge beyond what is actually perceived to things past, distant and future. Reasoning, therefore, in its *wider* sense, includes the following processes: —

'given' and 'present' to the 'unknown', 'new', and 'past, distant and future.' Reasoning, in the wider sense includes conception and judgment,

(1) *Concepton.* This is the process of forming concepts or general ideas on an examination of a number of individual objects (see Ch. V, § 1).

(2) *Judgment.* This is the process of comparing two such concepts so as to recognize a certain relation between them.

(3) *Inference.* This is the process of passing to a new judgment as following from one or more such judgments already known

while in the narrower sense, is identical with inference. The known truths from which reasoning starts are called premises or data, while the new truth arrived at by reasoning is called conclusion.

Reasoning, in its *narrower* sense, is identical with the process of inference, *i.e.*, the process by which we pass from one or more given and known truths to some other truth not already known. The new truth arrived at by the process is called the *conclusion*, and the known truths with which the process starts are called the *data* or *premises*. The conclusion may be drawn from a single premise (Immediate Inference) or from more (Mediate Inference). The conclusion may be more general than the premises (Inductive Inference), or may not be so (Deductive Inference). In all correct inference the conclusions must follow consistently and necessarily from the premises, but they may or may not agree with actual things. (*Vide* Formal and Material Truth, § 3.)

Distinction between Deductive and Inductive Reasoning.

Deductive and Inductive Reasoning. Reasoning falls under two great divisions, Deductive and Inductive. In Deductive Reasoning we start with one or more given premises and arrive at a conclusion which can, in no case, be more general than

the premises. Deductive Reasoning may be either immediate or mediate. It is *immediate* when the conclusion follows directly from a single premise; *e.g.*, 'All philosophers are men', therefore, 'Some men are philosophers'. It is *mediate* when the conclusion follows from more than one premise; *e.g.*, 'All philosophers are men', 'Socrates is a philosopher'; therefore, 'Socrates is a man'. In Inductive Reasoning we start with particular facts of experience and arrive at truths of general application beyond the range of our experience. For example, on finding that many individual men—John, James, Socrates, etc.—are mortal, we conclude 'All men are mortal'. Inductive Reasoning is usually *mediate*. (Formal and Material Reasoning are respectively identical with Deductive and Inductive Reasoning. *Vide* § 3.)

What is Truth?

§ 3. Truth : Division of Logic into Formal and Material. We have seen before that the end of Logic is the attainment of Truth. Let us now explain the nature of Truth and also indicate the division of Logic according to the different aspects of it. **Truth** means the agreement of thoughts with themselves or with actually existing things. Suppose I try to think a table which is white and not-white at the same time, or which is round as well as square;—here there is no consistency of my ideas with themselves, the different ideas conflict with each other, and I am said to be in error. When our thoughts or ideas are free from such inconsistency and inner contradiction, they are said to be *formally true*. Thus **Formal Truth** means self-consistency or agreement of thoughts with themselves. Again, it may be that my thoughts are free from inconsistency, but they may or may not agree

Distinction between Formal and Material Truth.

with reality or facts. When our ideas, besides being self-consistent, also agree with actual facts, they are said to be *materially true*. Thus the judgment 'Men are mortal' is not only free from self-contradiction, but is also in agreement with fact—hence it is materially true ; but the judgment 'Men are quadrupeds' is materially false, for, though free from inconsistency, it is not in accordance with fact. Thus **Material Truth** means agreement of thoughts or ideas with facts or actually existing things. Hence we must distinguish between the two aspects of Truth—between formal validity and material reality, between possibility and actuality, between self-consistency and consistency with facts.

It is clear from what has been said above that nothing can be true, in the fullest sense of the term, unless it is true materially as well as formally. Formal truth may or may not rise to material truth, but the latter necessarily includes the former. What is possible may or may not be actual, but what is actual is necessarily possible. Thus, what is materially true must be formally true, but what is materially false may or may not be formally false, for though something may not actually exist, the thought of it may at least be self-consistent. Again, what is formally true may or may not be materially true, but what is formally false must be materially false, for what is self-contradictory can never actually exist.

Having thus understood the nature of truth in its two aspects—formal and material, let us now explain the distinction between Form and Matter of Thought, between Formal and Material Reasoning, and between Formal and Material Logic.

Distinction between Form and Matter as applied to common things.

Form and Matter of Thought. The distinction between form and matter may be understood in connection with common things. Medals struck from the same die have exactly the same *form*, but they may be of various *matter*, as gold, silver, brass, etc. In pottery, rough earth as given to the potter is the matter, while the form (the relation existing among the different parts of a pot) is what is communicated to the matter by the exercise of his art. This distinction between form and matter has been introduced into the province of thought. By

Distinction between Form and Matter of Thought.

Form of Thought we mean the way in which the mind thinks, and by **Matter of Thought** we mean the things thought about. *Matter* signifies all that is given *to*, *form* signifies all that is given *by*, thought. The Laws of Thought determining thinking constitute the die, *i.e.*, the *form*, in which we put the various kinds of objects, *i.e.*, the *matter*, about which we think.

Thought may differ either in form, or in matter, or in both.

Note. It is easy to see that just as ordinary things may differ from one another either in form, or in matter, or in both, so thought may also differ either in form, or in matter, or in both. An iron ball and a golden ball from the same die are different in matter, but the form is the same. Again, two golden balls from two different dies are different in form, but the matter is the same. Similarly, the two judgments 'No men are perfect' and 'All men are imperfect' agree in matter, because the two judgments carry one and the same sense, while they differ in form, because one judgment is affirmative and the other is negative. Again, the two judgments 'All men are mortal' and 'All men are imperfect' agree in form, because both the judgments are universal affirmative; while they differ in matter, for one expresses a relation between 'man' and 'mortality', and the other, between 'man' and 'imperfection'.

Distinction between Formal and Material Reasoning.

Formal and Material Reasoning. **Formal Reasoning** means reasoning carried on according to the Fundamental Laws of Thought or Formal Laws

of consistency, without any regard to its agreement with facts. **Material Reasoning** means reasoning carried on with reference to real things and relations of things. By formal reasoning we arrive at formal truth, *i.e.*, truth free from inner contradiction. By material reasoning we arrive at material truth, *i.e.*, truth in agreement with facts. Again, in a piece of reasoning there are premises and a conclusion. Reasoning is *formal* if the conclusion necessarily and consistently follows from the premises, no matter whether it agrees with fact or not. Reasoning is *material* when the conclusion, besides being consistent with the premises, also agrees with reality or fact. For example, let us reason thus:—

All men are quadrupeds,
James is a man;
∴ James is a quadruped.

Here the form of reasoning is quite correct (for, as we shall see later on, the reasoning is in the perfect form *Barbara*), inasmuch as the conclusion follows consistently from the assumed premises. But the reasoning is materially false, for the premise (All men are quadrupeds) and also the conclusion (James is a quadruped) do not agree with facts. Again, let us reason thus:—

John is mortal,
James is mortal,
Socrates is mortal,
etc., etc., etc.,
∴ All men are mortal.

Here we have an example of material reasoning, for the conclusion 'All men are mortal' follows consistently from the premises 'John is mortal', 'James is mortal', etc.; and, moreover, it is in accordance

with fact or actual experience. It may be observed that in formal reasoning we simply accept the premises as true, while in material reasoning we proceed by examining the premises as real facts. The conclusion in material reasoning must accord with fact, and to ensure this, the premises must also be actual facts of experience.

Distinction between Formal and Material Logic. **The business of Formal Logic is to establish conclusions in consistency with the given premises, without any reference to actual facts;**

Formal and Material Logic. The end of Logic is the attainment of truth by means of reasoning or inference. Now truth as well as reasoning (explained above) is either formal or material. Hence Logic is called Formal or Material according as its aim is the attainment of formal or material truth by means of formal or material reasoning. Formal Logic does not question the material truth of the premises, but accepts them as true, and draws inferences which follow consistently from the given premises, without any reference to actual facts. It starts with premises assumed to be true and establishes a conclusion which does not violate the Fundamental Laws of Thought (*i.e.*, the Principles which determine the formal validity of all reasoning, *vide* Ch. VI), and is thus free from self-contradiction.

(1)	(2)
All men are mortal,	All men are quadrupeds,
James is a man ;	James is a man ;
∴ James is mortal.	∴ James is a quadruped.

In Formal Logic, argument (2) is as much valid as argument (1), although it is quite clear that argument (2) is materially false. **Material Logic,** on the other hand, is not satisfied with bare freedom from inconsistency, but insists on the agreement with things actually existing in nature. It starts with the criticism of the premises, looks to the consistency of

while that of Material Logic is to establish conclusions in agree-

the conclusion with the premises, and establishes a conclusion in accordance with the real state of things found in nature. Formal (or Deductive) Logic is also called Logic of Consistency, or Subjective or Abstract or Pure Logic; and Material (or Inductive) Logic is also called Logic of Truth, or Objective or Concrete or Applied Logic.

ment with actual facts.

Note. According to Formal Logicians (such as Hamilton, Mansel, Whately), Logic is essentially formal. Thus Hamilton defines Logic as the 'Science of the Formal Laws of Thought', and holds that Logic has only formal truth for its end. This view maintains that Logic inquires into the universal laws of correct thought and has nothing to do with the objects of thought. It is the business of the special sciences to inquire into the conditions of material truth. They further hold that Logic cannot deal with material objects, as they are always changing, and as it is difficult to ascertain the real nataure of things without having recourse to Metaphysics. Logic, therefore, deals only with ideas without reference to material things. But Material Logicians (such as Mill, Bain, Spencer and others) maintain that the end of Logic is material truth. Logic will be but a useless science if the conclusions established are not in accordance with facts. Again, why do we accept the rules of Logic as true? The answer is: because they are capable of being verified by appeal to facts. We may observe in this connection that Logic aims at truth, and nothing is true, in the proper sense of the term, unless it is materially as well as formally valid. Hence Logic cannot ignore the material aspect. It must, however, be pointed out that the primary object of Logic is to free thought from self-contradiction, and its secondary object is to secure agreement of thought with things. Thus Logic is in the first place formal, and in the second place material, and it has material truth for its ultimate end.

Different views regarding the nature of truth aimed at by Logic. While Formal Logicians restrict the scope of Logic to formal truth only, Material Logicians extend its scope to include material truth also.

§ **4. Parts of Logic.** Logic, we know, is the science of reasoning; and we also know that reasoning or argument is the process of passing to a new judgment or proposition by means of one or more given judgments or propositions; and that a judgment or propositon is a combination of two concepts or terms. Thus 'Men', 'Mortal', and 'James' are three terms; by means of these terms we frame two propositions, *viz.*, 'All men are mortal' and

Three parts of Logic—Terms, Propositions, and Reasonings or Inferences.

'James is a man'; next, by means of these two propositions we frame the following argument—

All men are mortal,
James is a man;
∴ James is mortal.

Hence, though reasoning or inference forms the principal subject-matter of Logic, yet reasoning cannot proceed without concept (term) and judgment (proposition). Reasoning is liable to be false, if the terms in it are indistinct and vague and if the propositions are false. Thus we see that Logic is no less concerned with the correct use of terms and propsitions. Hence it may be said that Logic falls under **three parts**, *viz.*, Terms, Propositions, and Reasonings or Inferences.

What is Method?

Note. To these three parts some Logicians add a fourth, *viz.*, Method, which consists in arranging our reasonings on any subject so as to render the whole subject easily intelligible Method has been defined in the Port Royal Logic as "The art of disposing well a series of many thoughts, either for discovering truth when we are ignorant of it, or proving it to others when it is already known." But it is doubtful whether Method should have a place in Logic.

The two broad divisions of Logic are Deduction and Induction.

Deductive and Inductive Logic. It has been said that Logic is the science of reasoning, and reasoning is either Deductive or Inductive. Hence Logic has been usually divided into two main parts, *viz.*, Deductive and Inductive; the one treats of Deductive reasoning or inference, the other of Inductive. The points of difference between Deductive and Inductive inference may be set forth as follows:—

Points of difference between

(1) In Induction we pass from particular facts observed to general truths, and, in all cases, the conclusions are more general than the premises; while in Deduction we pass from general truths to particular

facts, and, in no case, is the conclusion more general than the premises. The general truths or principles arrived at by Induction are applied to new cases by Deduction. Having observed many particular cases of human mortality, having experienced that John, James, Thomas, etc., are mortal, we conclude that all men are mortal. This is **Induction.** Next, when we meet with a new individual, say, Socrates, we apply the general truth 'All men are mortal' to his case in this way:—

Deduction and Induction. **In Induction the conclusion is always more general, while in Deduction it is never more general, than the premises.**

All men are mortal,
Socrates is a man ;
∴ Socrates is mortal.

This is **Deduction.**

(2) Deduction aims at formal truth, whereas Induction aims at material truth. In Deduction the premises are taken for granted, and we only see if the conclusion consistently follows from the premises, no matter whether it agrees with fact or not. Thus Deduction is concerned with formal consistency, and Deductive Logic has been identified with Formal Logic. Induction establishes a general truth, and in order to ensure the correctness of the conclusion, the particular facts must be carefully and correctly observed. In Induction the premises (*i.e.*, the facts) and the conclusion (*i.e.*, the generalization) must agree with facts. Thus Induction is concerned with material truth, and Inductive Logic has been identified with Material Logic.

Deduction aims at formal, while Induction aims at material truth.

Note 1. Some Logicians (*e.g.*, Whately) identify reasoning with Deduction, and Logic with Deductive Logic. According to them, Induction is not a separate department of Logic at all, but only a form of Deduction (*vide* Ch. XXI, § 4, 7). Others (*e.g.*, Mill) hold that reasoning is essentially Inductive, and Deduction is only a form of interpreting the general truth obtained through Induction by applying it to new cases. But we shall see later on (Ch. XXI, § 6) that neither Induction alone nor Deduction

Some Logicians maintain that Induction is no part of Logic at all, while others hold that

Deduction is no inference at all.

alone is a self-sufficient process of reasoning. The two together constitute the whole process of reasoning. Though one cannot be separated from the other, though one supplements the other, yet, for convenience, the two forms of inference are discussed separately with a view to understand clearly the nature, function and test of each.

What part of Logic is Deductive and what part Inductive?

Note 2. The terms 'Deductive' and 'Inductive' are used in connection with inference only. Deductive inferences (Immediate inference and Syllogism) are treated of in Deductive Logic, while Inductive inferences are treated of in Inductive Logic. Terms, Propositions and subsidiary processes of Definition, Division, etc., belong to Induction and Deduction alike. These are discussed in Deductive Logic only for the sake of convenience. Again, Inductive Logic not only treats of Inductive inference, but also of the auxiliary processes such as Hypothesis, Explanation, Classification and Naming

What parts of Logic are formal and what parts material?

Further, Deductive reasoning is formal, because it is not concerned with the actual truth or falsity of the premises and of the conclusion. Inductive reasoning is material, because it is concerned with the actual truth or falsity of the reasoning. The doctrines of Definition, Division, Classification, Denotation and Connotation of Terms, the Predicables and the validity of Terms are material As the doctrine of Propositions is concerned with the form of the expression of thought, it is mainly formal; but the Import of Propositions is material, because here reference to the reality of the judgment is necessary.

§ 5. Science and Art : Theoretical and Practical Science : Positive and Normative Science.

We have begun with the definition of Logic as the practical science which regulates thinking in order to attain truth, and we have also touched on the meaning of science—theoretical and practical. Let us here indicate more fully the nature of scientific knowledge, the distinction between Science and Art, and the bearing of this upon the character of Logical study.

A Science is an inquiry into the laws explaining the nature and behaviour of

Science. Science means knowledge systematized and organized. It is a systematic attempt to ascertain the laws explaining the nature and behaviour of a group of facts belonging to a particular department of the world. Thus Botany is a science, because it inquires into the laws

explaining the nature, working and development of plants or the vegetable department of the world. Similarly, Astronomy aims at discovering the laws of heavenly bodies; Physics aims at discovering the laws of such phenomena as heat, light, electricity and gravity; Biology inquires into the phenomena of life; Psychology discovers the laws of mental processes.

a particular group of phenomena.

Characteristics of Scientific Knowledge.

Now, Scientific knowledge is distinguished from ordinary unscientific knowledge by the following *characteristics* or marks:—(1) Speciality, (2) Generality, (3) Certainty, (4) Accuracy, and (5) System.

(1) Scientific knowledge is limited to the study of the phenomena belonging to a particular department of the world.

(1) Scientific knowledge is confined to some particular department of the world of matter or of mind. A particular group of facts forms the subject-matter of a particular branch of knowledge called a *Special* Science. (Thus Botany is concerned with plants, Physics with matter and energy, Biology with life, Astronomy with heavenly bodies, and Psychology with states of mind.

(2) It is always general.

(2) Scientific knowledge must always be *general*. The aim of Science is always to find out laws which govern the phenomena of a definite department of the world. It is true that Science takes particular facts into consideration, but it does so only to discover the general truths underlying these facts. The general truths which hold many isolated facts together are called Laws or Principles. For example, we have the Law of Gravitation in Astronomy, the Laws of Motion in Physics, the Law of Relativity in Psychology, and so on.

(3) It is always certain.

(3) Scientific knowledge must always be *certain*, while ordinary knowledge may be of a probable nature. Probable statements must be proved to be

true before they can be incorporated into Science. Science never accepts anything which has not been supported by reason. Fate or chance has no place in Science.

(4) It is accurate.

(4) Scientific knowledge must always be *accurate*. Vague, indefinite and random statements of popular knowledge have no place in Science. The statements of Science are determinate and exact. Science tries to express the relation of phenomena by exact quantitative measurement. Hence, no science is perfect unless it admits of mathematical treatment.

(5) It is methodical

(5) Scientific knowledge must always be *systematic* and *methodical*. Every Science proceeds according to a definite method or order in arranging its topics. Unsystematic and random grouping together of facts does not constitute scientific knowledge. Thus in every science, there is a particular method of studying the particular group of facts constituting its subject-matter. Generally, a Science begins by defining the terms used in it, stating the axioms and postulates taken for granted, and then proceeds to arrive at more complex laws from simpler ones.

Distinction between Science and Art.

Science and Art. The word 'Science' is sometimes used in a wide sense to mean both Theoretical and Practical Science, and it is sometimes used in a narrow sense to mean *Theoretical* or *Speculative Science* as distinguished from *Practical Science* or *Art*. "The distinction between science and art is, that a science is a body of principles and deductions, to explain some object-matter: an art is a body of precepts, with practical skill, for the completion of some work. A science teaches us to know, and an art to do; the former declares that something exists

A science teaches us

with the laws and causes which belong to its existence, the latter teaches how something must be produced" (Thomson, *Outlines of the Laws of Thought*, pp. 11-12). "The language of science is, This is, or This is not; This does or does not happen. The language of art is, Do this, Avoid that. Science takes cognizance of a phenomenon and endeavours to discover its *law*; art proposes to itself an *end*, and looks out for means to effect it" (Mill's *Essays on Political Economy*).

to know, and an art to do.

We may further ask: Does Science precede Art, or Art precede Science? We see that Science is concerned with knowledge—it teaches us *to know*, and Art is concerned with practice—it teaches us *to do*. And as the right *doing* of a thing requires the *knowledge* of the thing, it may be said that Art is based upon Science, and consequently Science precedes Art. But ordinarily men had begun *doing* a thing before Science imparted any systematic knowledge of it. So, properly speaking, Art in its crude state precedes Science. Thus Art, which is not based on scientific ground, is called by Bain empirical art or art proper. Hence a distinction is often drawn between Art and Practical Science. When an Art is based upon Science, it is called a Practical Science. Art is out and out practical, but Practical Science is both theoretical and practical. Art teaches us *to do*, but Practical Science teaches us *to know how to do*.

The question as to the relative priority of Science or Art.

Art proper precedes Science.

When Art is based on Science, it is called Practical Science.

It may be observed in this connection that no line of absolute distinction can be drawn between a Science and an Art, or between a Theoretical Science and a Practical Science. It must not be supposed that in Science there is no reference to practice, and in Art there is no reference to knowledge. The

The distinction between Science and Art is rather relative than absolute.

One is incomplete without reference to the other.

truth is that one necessarily supplements the other, for a science which does not ultimately serve some practical purpose is of no use, and an art which is not based on scientific knowledge is apt to be misleading. As Bain says, "The final end of all knowledge is Practice, or the guidance of conduct. There are numerous departments of practice, according to the needs of human being; and every one of these reposes upon knowledge more or less accurate" (*Deduction*, p. 28). We call a science theoretical, when it is principally concerned with conveying knowledge of a particular group of facts; and we call a science practical, when it is principally concerned with the application of such knowledge in practice. In one the theoretical and in the other the practical side is predominant.

Logic is both a Science and an Art.

Is Logic a Science or an Art? Some Logicians hold that Logic is a mere Science and thus ignore altogether its practical side; others maintain that Logic is a mere Art, and thus ignore its theoretical side. The correct view, however, seems to be that Logic is both a Science and an Art. Logic is a Science, inasmuch as it inquires into the laws and conditions of correct thought, and thus teaches us to know in what correct thinking consists. Again, Logic is an Art, inasmuch as it has an end in view, *viz.*, attainment of truth, and lays down rules for the guidance of thought, so as to secure correctness and avoid error. Thus Whately has defined Logic as 'The Science and Art of Reasoning', emphasizing thereby that Logic is both a Science and an Art, inasmuch as it teaches what the processes of correct reasoning are and how best to direct them so as to attain its end, *viz.*, *truth*. In short, Logic is to be regarded as a Practical Science, for it

is concerned with the inquiry into the general conditions of valid reasoning as well as with the application of these in practice for attaining truth and avoiding error.

A Positive Science is concerned with the actual, while a Normative Science with the ideal.

Positive and Normative Science. A Positive Science inquires into phenomena as they are actually found to happen in nature. It is concerned with facts as they actually *are*, without any reference to an ideal or what *ought to be*. Thus Botany is a positive science, because it inquires into the laws concerning plants as they *actually are* and *behave* in nature. A Normative science, on the other hand, defines and explains some norm or ideal, and judges the merits or demerits of things from the standpoint of the norm. It is concerned with what *ought to be*. Thus Logic is a Normative science, because it is concerned with Truth conceived as an ideal to be realized, and judges our thoughts to be correct or incorrect according as they realize or fail to realize the ideal.

Logic is both a Normative and a Practical Science.

Again, a distinction is sometimes drawn between a Normative Science and a Practical Science. It has been said that while the former only inquires into the nature of the ideal, the latter shows us the way how to attain the end or ideal. It is quite obvious that Logic is both a Normative and a Practical Science, for it not only sets up Truth as the norm or standard of thinking, but also lays down rules for its attainment.

§ 6. Knowledge : Immediate and Mediate. As the purpose of Logic is to guide our thought in the attainment of true knowledge, let us explain here the nature and kinds of knowledge with a view to determining what kind of knowledge forms the proper subject-matter of logical inquiry.

Knowledge is the possession of true ideas accompanied by a belief that they are true.

What is Knowledge? Knowledge consists in the possession of a system of true ideas, *i.e.*, ideas corresponding to things and relations of things, accompanied by a belief in their truth. Knowledge thus implies:—(1) a system of ideas in the mind (subjective side), (2) a system of things and relations of things (objective side), (3) correspondence between the two (truth), and (4) a belief in their correspondence. If any of the above four factors be wanting, there can be no knowledge in the true sense of the term If 'ideas' be absent, there cannot be any knowledge, for 'things' must be known by the knowing mind in terms of ideas. If 'things' or 'facts' be absent, how can we speak of correspondence, and how can there be truth without correspondence? The idea of the 'centaur' is not a true one, for there is no fact to correspond to. Further, absence of 'belief' makes knowledge impossible, for in a dream I have 'ideas' which may also correspond to 'facts', but yet these ideas fail to constitute knowledge, for when I awake I lose all 'belief' in the correspondence. Moreover, mere 'belief', without 'correspondence', gives rise to what is called *error*.

Mere belief without correspondence gives rise to error.

Distinction between Immediate and Mediate knowledge. Immediate knowledge is perceptual,

Immediate and Mediate Knowledge. Knowledge is either Immediate or Mediate. By *Immediate Knowledge* we mean knowledge gained by direct perception, without any inference or reasoning from premises. Perception is either internal or external. *Internal perception* (or Introspection, or Self-consciousness) gives us knowledge of the states of our own mind (the internal world), as when we say 'I am hungry', 'I am pleased'. *External perception* gives us knowledge of external objects, *e.g.*, horse, table, tree, etc., through the exercise of our sense-

organs, the eye, ear, etc., as when we say 'This horse is red', 'That is the sound of the bell', 'Here is a round table'.

(1) Immediate or perceptual knowledge can reveal only what is *present* to us and directly affects us, it cannot reveal what is past, distant and future. Hence (2) it further follows that perception can only give us the knowledge of *singular facts,* and not of general truths. By immediate knowledge or perception I can know 'This piece of gold is heavy', but I can never know 'All metals are heavy'. It must be observed, however, that immediate knowledge as gained by perception supplies us with ultimate and elementary premises, which enable us to proceed to complex ideas and universal truths.

while Mediate knowledge is inferential.

By *Mediate Knowledge* we mean inferential knowledge, *i.e.*, knowledge gained indirectly *through the medium of* premises. Thus I find tears in the eyes of a man and I infer that he is sorrowful. Here my knowledge of the tears is immediate, but the knowledge that the man is sorrowful is mediate, for I know it not directly, but indirectly through the medium of tears in his eyes. Mediate knowledge is either Inductive or Deductive. Thus I find that individual men, A, B, C, etc., are mortal ; from these particular facts of experience I infer 'All men are mortal'. Here we have an example of mediate knowledge gained by Induction. Now, if I find any new man 'X', I may, from my previous knowledge, come to know that 'X' is also mortal. (The premises are—'All men are mortal' and 'X is a man'.) Here we have an example of mediate knowledge gained by Deduction.

Can we know general

Note. It may be observed here that immediate knowledge will give only singular propositions, but mediate knowledge will give both general and singular ones—

propositions immediately

general ones are given by Induction, and singular ones by Deduction from general ones previously known. It is contended by one school of philosophers (the *Intuitive* or *A priori* School) that there is an important class of general propositions (axioms), which are *immediately* known without the help of inference. These are called Intuitive, Axiomatic or *Necessary Truths*, gained by an immediate act of *Intuition*. The nature of Intuition and Necessary Truths will be explained in Ch. XXII, §§ 1 and 2.

Is Logic concerned with Immediate knowledge?

Place of Immediate Knowledge in Logic. According to Mill, Bain and most English writers, Logic is concerned with mediate or inferential knowledge; they altogether exclude immediate knowledge from the sphere of Logic. For, they say, "What is perceived is certain once for all, and no logical rules can make it more so." So Bain urges, "We cannot escape from them (facts of present consciousness), we cannot be more or less convinced of them by any method of procedure" (*Deduction*, p. 32). According to Ueberweg, on the other hand, both immediate and mediate knowledge come within the province of Logic. He defines Logic as "the Science of the Regulative Laws of Human Knowledge," and holds that "material or real truth cannot be made the end of Logic unless the truths known by Perception are brought within the jurisdiction of Logic. Logic in short cannot realise its end—the attainment of truth—unless it treats of all the processes of knowing and lays down conditions to which they must conform in order that they may attain their end, namely truth. The form of Perception like the form of Reasoning *should* be treated in Logic" (*vide* Dr. Ray, *Inductive Logic*, p. 123).

Mill's view

Ueberweg's view

Again, it has sometimes been held that Logic is directly and primarily concerned with mediate knowledge and only indirectly or secondarily concerned with immediate knowledge, for mediate Inductive knowledge is ultimately based upon facts of percep-

tion. It is impossible to say that all metals have weight unless we actually observe that some particular cases of metal, *viz.*, gold, etc., have weight. Our Inductive conclusions must be wrong if our perceptions of particular cases are wrong. Hence some general rules of methodical perception (Observation and Experiment) are laid down in Inductive Logic.

It may be observed, however, that Ueberweg's view is too wide. Logic teaches us to reason or infer correctly and not to perceive correctly, for perception cannot be regulated Perception depends, for its correctness, not on logical rules, but on the natural capacity of the senses and on long practice and training. No amount of logical rules can make a person a good observer. Again, Logic is viewed as the Science of Proof (*cf.* Mill's and C. Read's definitions of Logic) ; mediate knowledge being concerned with past, distant and future things, requires proof to establish itself, but immediate knowledge requires no proof, for we readily believe what we are directly conscious of, and no logical rules can make our immediate knowledge more certain than it is in itself. Thus Mill very truly remarks: "No Science is required for the purpose of establishing such truths (truths immediately known) ; no rules of art can render our knowledge of them more certain than it is in itself. There is no logic for this portion of our knowledge" (*System of Logic*, Intro. § 4). Hence, properly speaking, Logic is concerned only with mediate or inferential knowledge, and immediate or perceptual knowledge falls outside the scope of Logic.

Ueberweg's view seems to be too wide,

while Mill's view seems to be correct.

Note. It has sometimes been said that though Mill excludes perception from the sphere of Logic, he actually includes it in Logic under the heads of Observation and Experiment. But it may be said in reply that Observation and Experiment are not treated of as parts of Logic, but

According to Mill, Observation and Experiment are

no parts of Logic, but only means of applying the Logic to facts.

only as a means of applying Logic to actual propositions (*vide* Bain, *Logic*, Intro. 52). Again, it has been urged that 'we never perceive the existence of a thing without perceiving what it is, *i.e.*, recognizing it, or referring it to some kind or class already known more or less, and this involves more or less inference. Thus perception is really always mixed up with inference, and therefore liable to error; and is therefore a fit subject for logical treatment.'

Perception involves an element of inference.

But it might be said in reply that the element of inference involved in the perceptual process admits of logical treatment. We protest against the possibility and necessity of bringing perception *per se* under logical test.

The conditions of a correct definition of Logic

§ **Definition of Logic.** From what has been said above concerning the nature of Logic, we are now in a position to attempt a correct definition of it, and to examine the more common definitions that have been proposed. Our work begins with the definition of Logic as 'the practical science which regulates thinking for the attainment of truth'. Let us see what *the conditions of a correct definition of Logic* are, and how far these conditions are fulfilled by our provisional definition.

A correct definition of Logic must imply: (1) Logic is a Practical Science.

(1) In the first place, a correct definition of Logic must state the double (theoretical and practical) aspect of Logical study. Logic, we have seen above, is both a science and an art, or an art based on science, *i.e.*, a practical science. In our definition this condition is fulfilled by the use of the words 'Practical Science' and 'Regulate'.

(2) Thought, as used in Logic, means reasoning or inferential thought.

(2) In the next place, while attempting to define Logic correctly, we have to indicate the meaning in which the term 'Thinking' is used. In Logic, 'Thinking' is used in a narrow sense to mean reasoning or inferential thought (thus excluding perception or immedate knowledge from the sphere of Logic); and reasoning, again, has been taken to mean both the process and product of inference itself and of its adjuncts, conception and judgment. Our provisional definition may be thus objected to on the

ground that the use of the word 'Thinking' there was rather vague.

(3) Truth, as the end of Logic, is formal as well as material.

(3) Again, 'Truth' in our definition means material as well as formal truth, for we have seen that to be true or valid, thought must not only be self-consistent, but must also agree with fact.

(4) Logic is also concerned with certain auxiliary processes.

(4) Further, correct reasoning will depend on several auxiliary processes (or processes 'subservient to reasoning'), *viz.*, Conception and Judgment on the one hand, and Definition, Division, Naming, etc., on the other. We have already indicated that we use the word 'Thinking' in our definition to mean reasoning with its necessary adjuncts, conception and judgment. But our definition excludes the auxiliary processes of Definition, Division, Naming, etc.

(5) Logic is concerned both with the processes and the products of thought.

(5) Moreover, we must know that Logic is concerned both with the processes and the products of thought. It is, however, more concerned with the products than with the processes, as it inquires whether the products of thought are valid or invalid. And this condition is supplied in our definition by the use of the word 'Truth', which refers to an idea, a thought, a product of thinking. This will be more clearly indicated if we add 'and avoidance of error' to the definition.

(6) Logic is a Science both of Proof and of Discovery.

(6) Lastly, our definition must indicate that Logic is not only the Science of Evidence or Proof (as Mill holds), but also the Science of Discovery, for to prove one thing to be false is to show in what truth lies, and scientific discoveries would have been impossible without logical methods and rules to guide our thought (*vide* Ch. XXVI, § 3). Our definition has fulfilled this condition by the use of the words 'attainment of Truth', for truth is said to be attained when we pass from the known to the

unknown (*i.e.*, discover), and when our passage is warranted, and the result is proved to be true, by logical rules. This requisite will be better provided for if we add 'and avoidance of error' to our definition.

Our provisional definition supplemented.

Hence our provisional definition is to be modified and restated thus: *Logic is the Practical Science which regulates Reasoning and the auxiliary processes for the attainment of Truth and avoidance of error.*

Various definitions criticized.

Note 1. **Definitions of Logic criticized by Mill and Bain** Mill and Bain point out the defects of the more common definitions in the following manner :—

I. Logic is the art of reasoning.—*Aldrich.*

This definition is shown to be open to the following objections :-

Aldrich's definition.

(1) It is too narrow for it ignores the theoretical aspect of Logic. It makes Logic to be an art merely, without recognizing the fact that it is at the same time a science.

(2) It is vague--for it is not clear whether the word 'Reasoning' has been used to imply both Deduction and Induction, or Deduction alone. In fact, older Logicians identified reasoning with Deduction.

(3) Agian, it is inadequate--for it excludes the processes auxiliary to reasoning, *viz.*, Definition, Division, Naming, etc.

Whatley's definition.

II. Logic is the art and science of Reasoning. *Whately.*

This definition removes the first objection to Aldrich's definition, but is open to the last two objections.

Thomson's definition.

III. Logic is the science of the Laws of Thought.—*Thomson.*

This definition has been shown to be open to the following objections :—

(1) It is too narrow—for it ignores the practical aspect of Logic.

(2) The word 'Thought' is ambiguous. It may mean either all the processes of Thinking, *viz.*, Perception, Memory, etc., or only the processes of Conception, Judgment and Reasoning. In this respect the definition is too wide.

(3) Again, the expression 'Laws of Thought' is ambiguous, even though we understand what thought is. For, it may either mean the laws to which thought must conform in order to be self-consistent—thus making Logic purely *formal* or Deductive; or it may mean the laws to which thought must conform in order to agree with fact as well as with itself—thus making Logic *material* or

Inductive. In this connection we may also consider the following allied definitions :

Hamilton's definition.

(A) Logic is the science of the Formal Laws of Thought.—*Hamilton*.

(B) Logic is the science of the Laws of Thought as Thought.—*Do.* (*Reid's Works*).

These definitions are also open to the first two objections to Thomson's definition given above. Moreover, these definitions are too narrow, for the expressions 'Formal Laws of Thought' and 'Laws of Thought as Thought' mean the laws to which thought must conform in order to be consistent with itself—thus identifying Logic with Formal (Deductive) Logic, and excluding Material (Inductive) truth from the province of Logic.

Definition of the Port Royal Logic

IV. Logic is the science of the operations of the understanding in the pursuit of truth.—*Port Royal Logic.*

This definition is good in so far as it indicates the double (theoretical and practical) aspect of Logic. Moreover, it is right in making 'attainment of truth' the end of Logic, and the 'study of the operations of the understanding' the means of attaining it. But it is open to the following objections :—

(1) The term 'Truth' is ambiguous, for it may mean either perceptual or inferential truth, or both. Hence the definition is too wide, for Logic is concerned only with inferential truth.

(2) It ignores processes other than the operations of understanding, *e.g.*, the auxiliary processes of Definition, Division, Naming, etc.

Mill's definition.

Note 2. **Mill's Definition of Logic.** Having examined the above definitions, Mill amends the definition of the *Port Royal Logic* and defines Logic thus :—

Logic is the science of the operations of the understanding that are subservient to the estimation of evidence (both the process itself of advancing from known truths to unknown, and all other intellectual operations in so far as auxiliary to this). This definition is a satisfactory one, as it fulfils all the conditions of a correct definition.

Note 3. **Some other definitions tested.**

Jevons' definition.

V. Logic is the Science of Reasoning.—*Jevons.*

This definition ignores the practical aspect of Logic. Though Jevons admits the twofold character of Logic, he does not mention it in his definition, for according to him, Logic is more a science than an art. Again, the term 'Reasoning' is insufficient and vague, for in the restricted sense of the older Logicians it means *Deduction* only; even in its wide sense—implying Deduction and Induction—the term is too narrow, for Logic is also concerned with certain auxiliary processes, *viz.*, Definition, Division, Naming, etc.

Welton's and Ray's definitions.

VI. Logic is the science of the principles which regulate valid thought.—*Welton.*

III. Logic is the science of the regulative principles of thought.—*Dr. Ray.*

These two definitions are for the most part comprehensive. But they may be said to be open to two objections :—(1) The definitions are too wide—for the word 'Thought' may mean all the intellectual activities—Sensation, Perception, Memory, Imagination, etc. Moreover, it remains vague whether by valid thought we are to understand both 'correct precept and correct inference,' or either of these. Hence it is not clear whether the sphere of Logic shall be confined to inferential truth, or extended to include perceptual truth as well.

(2) These definitions overlook the fact that Logic has also to deal with certain auxiliary processes, *viz.*, Definition, Division, Naming, etc.

Meanings of 'Things', 'Thought', and 'Language'.

§ 8. Thought, Language and Things. *Things* mean objects which one can think about. These are either substances or attributes. A substance is a thing which can be thought to exist by itself, and may be either corporeal or incorporeal, *e.g.*, wood, mind. An attribute is a thing which is thought to depend for its existence upon a substance, *e.g.*, hardness (of wood).

Thoughts or *ideas* are mental representations of things. When an idea represents a singular individual thing, it is called a percept or idea, *e.g.*, the idea of 'John'. When an idea represents a class, it is called a concept or general idea or notion, *e.g.*, the idea of 'Man'.

Language means expression of ideas by means of signs. Language may be of different kinds—tactual, visual and vocal. But the most convenient form of expressing thought is supplied by means of articulate sounds or words.

The etymology of the term 'Logic' implies an almost

Thought and Language. The word 'Logic' is derived from the Greek word 'Logos', which means reason as well as speech, and the etymology of the term 'Logic' implies that Logic is concerned with the correct use of reason and speech for the attainment

of truth and avoidance of error. This ambiguous use of one and the same word 'Logos' to signify either reason or speech may be accounted for by the fact that reason and speech, *i.e.*, thought and language, are regarded as inseparably connected as two aspects of the same thing. And this tendency to identify thought and language has been carried by some Logicians so far as to define Logic as 'the science which treats, in a general way, of the origin, significance, and practical use of vocal signs or speech'. But this is certainly going too far. Though a very intimate relation between thought and language exists, yet the two cannot be treated as identical ; for, while thought is essentially the same everywhere, language admits of endless variety of forms. Logic is concerned with the invariable forms of thought, and not with the variable forms of expression. Again, there has been much discussion of the question whether thought is at all possible without language. Thought, however, is not altogether impossible without language ; for—

connection between thought and language.

An extreme view of Logic regarding it as wholly conversant about the use of language.

In fact thought is not altogether impossible without language.

(*a*) Animals and infants can think to a certain extent, though they have no language in the proper sense of the term.

(*b*) Language is said to give expression to our thought, hence it may be understood that thought exists before, and awaits, expression.

But though a rudimentary form of thinking is possible in the animal and the child without language, yet it is true that higher and more complex forms of thinkings are impossible without language.

Complex and abstract thinking, however, is not possible without language.

Note. **Functions of language in relation to thought.**

(1) "Language gives the power of analysing complex wholes" (*Welton, Manual of Logic, I, Ch. I,* § 2). An

Five main functions of

language in relation to thought.

impression is received as a whole, and with the help of language it is analysed into simpler factors.

(2) "Language makes possible the formation of Concepts" (*ibid.; also vide* Ch. V, § 1). Moreover, language gives fixity and definiteness to our ideas or concepts, which without language would soon become vague and uncertain. Thus, without language, conception, judgment and reasoning, or higher thought would have been impossible.

(3) "Language shortens the process of thinking" (*ibid.*).

(4) "Language is a direct means of communicating thought. It is this function of language which makes all social intercourse possible, and enables each person to profit by the knowledge acquired by others with whom he is brought in contact. Thus, mental development is facilitated and made definitely more speedy, and of greater breadth and richness, than would be possible if each mind were condemned to exist in isolation" (*ibid.*).

(5) "Language is a means of recording thought. This is the great use of writing and printing, and is evidently an extension of the function last described. By this means we can benefit by the experience and share in the knowledge and thoughts, not only of those few persons whom we may chance to meet, but of men of all times and all places who have given us, in their writings, a record of their intellectual work" (*ibid.*).

Again, Language without reference to thought is meaningless. Language which does not signify a thought is meaningless jargon. Hence we see that thought and language are inseparably connected, that neither has its proper significance without the other.

Thought has always reference to things thought of.

Thought and Things. Thought has always a reference to a subject which thinks and an object which is thought of. Our thought or idea is a mental representation of a thing. Thus thought, without a thing thought of, is a contradiction in terms. Again, things are of value to us so far as they can be apprehended by the mind, *i.e.*, so far as they can be known by us in terms of thought.

Logic regards thought to be the essence, and things and

Relative importance of Thought, Things and Language in Logic. We see that thought, things and language are closely connected, that one can be of little importance apart from the other two, that

one rises to its full significance only along with, and by means of, the rest. Thus, to be true, thought must agree with things ; things are of value in so far as they can be thought of ; thought, again, must be expressed in correct language ; and language must stand either for a thought or for a thing, otherwise it is meaningless. But all these are not equally fundamental. It is thought, the truth of which we seek in agreement with things ; again, it is thought, the expression of which we seek in correct language. Thus, thought is the *essence*, and things and language are the necessary *supplements*. Hence Logic is primarily concerned with consistency of thoughts with one another or subjective consistency ; and secondarily concerned with agreement of thoughts with things, or objective consistency, and with expressions of thoughts in correct language, or linguistic consistency. The latter two are only instrumental in developing and perfecting the first one.

language to be the necessary supplements. Logic is primarily concerned with subjective consistency, and secondarily concerned with objective and linguistic consistency.

§ **9. Different Schools of Logic.** The above consideration of the relation of thought, things and language leads to the important question of the different Schools of Logic holding different views with regard to the proper subject-matter of Logic. The question here is: With what is Logic chiefly concerned—language, thought, or things? The answer given is different according to the different Schools of Logic—Nominalistic or Linguistic School, Conceptualistic or Subjective or Formal School, and Objective or Realistic or Material School.

Three different Schools of Logic holding different views as to the proper subject-matter of Logic.

(1) **Nominalism.** According to Nominalistic Logicians (*e.g.*, Whately),* Logic is entirely con-

Nominalism holds that Logic is

* Whately, *Elements of Logic*, p. 37.

concerned only with language and linguistic consistency.

versant about *language*.) In the Logical proposition 'Man is mortal', the terms 'man' and 'mortal' may be considered subjectively as mental notions or ideas, objectively as real things existing outside the mind, and verbally as mere words or names. Nominalism holds that Logic is concerned only with *names* or words and not with the corresponding ideas or things, and a logical proposition is to be regarded as expressing a relation, not between two ideas nor between two things, but between two words or names. According to this view, Logic is defined as the *science of the correct use of language*. Logic is thus only a philological science—a Grammar of Reasoning.

Conceptualism holds that Logic is concerned only with ideas and subjective consistency.

(2) **Conceptualism.** (According to Conceptualistic Logicians (*e.g.*, Hamilton, Mansel), Logic is concerned only with *thoughts* or ideas, and not with words or things.) A Logical proposition, according to them, is to be regarded as a judgment expressing a relation between two ideas, without any reference to their expression in language or to their correspondence with actually existing things. This is the Subjective, Conceptualistic, Formal, or Psychological view of Logic. According to this view, Logic aims at *formal consistency*, *i.e.*, consistency of our ideas (of the subject and predicate) among themselves. Thus Logic is merely the *science of consistency*, and has nothing to do with things or relations of things.

Hamilton and Mensel's view.

Hamilton, accordingly, defines Logic as 'The science of the Formal Laws of Thought or the science of the Laws of the Form of Thought'. Mansel also defines Logic as 'the science of the Laws and Products of Pure or Formal thinking'.

(3) **Realism.** According to Realistic Logicians (*e.g.*, Mill, Bain, Spencer*), Logic is concerned directly with actually existing *things* and their mutual relations and correlations. A Logical proposition, according to them, expresses a relation, not between two ideas, but between two things which those ideas represent. This is the Objective, Realistic, or Material view of Logic. According to this view, Logic aims at *material truth, i.e.*, agreement of our ideas with facts, and not merely at formal consistency. Thus, Logic is the *science of the most universal relations of things.* It is not simply the science of the Laws of the Form of Thought, but the science of the Laws of the relations between things. Logic is thus regarded as the most general of all sciences, and things and relations of things, which constitute the subject-matter of Logic, are not merely thought-relations, but actual objective facts.

Realism holds that Logic is directly concerned with things and objective reality.

Criticism : the Comprehensive View. A little reflection will make it clear that none of the above three views, taken exclusively, will give us the correct conception of the subject-matter of Logic. In the first place, the Linguistic view cannot be maintained by itself. Language, to be significant, must have reference to thoughts or things. As language is merely the medium of expressing thoughts and things, it becomes meaningless jargon when considered apart from thoughts and things. Thus the Linguistic view must go hand in hand either with the Subjective or with the Objective view. This is the reason why Logicians like Whately and Mill, apparently holding the Linguistic view,

The Nominalistic view is not sufficient by itself;

it must be combined either with the subjective or with the objective view.

* *Vide* Herbert Spencer, *Principles of Psychology*, II, § 302.

have been compelled to fall back either on Conceptualism (*vide* Whately, *Elements of Logic*, p. 37) or on Realism (*vide* Mill, *System of Logic, I*, p. 21). In the next place, Logic aims at the attainment of truth, and truth is not only formal, but also material ; hence our thought, to be true in the proper sense of the term, must not only be free from inner contradiction, but must also agree with objective relations of things. Hence Conceptualism must be combined with Realism. Last of all, Logic never enters into the question of the ultimate nature of things, which is the subject-matter of Metaphysics. Like all special sciences, Logic accepts things as they are actually perceived by us or as they actually reveal themselves to our thought. Hence things divorced from thoughts can have no place in Logic. Thus Realsim must be combined with Conceptualism.

So also Conceptualism must be combined with Realism, and Realism must be supplemented with Conceptualism.

As ideas must be expressed in correct language and must represent facts, the true view will be a combination of Conceptualism, Realism and Nominalism.

The right view, therefore, seems to be that there is no necessary antagonism between these three views. In a certain sense, Logic is concerned with all the three—language, thought and things. Language is indispensable, because we cannot express our thought or talk about things without it. Even the results of thought could not be retained without the aid of language. Again, language is meaningless without the thought which it expresses ; and we do not talk without meaning something. Moreover, when we think, our endeavour is to get as near truth as possible ; hence we seek not merely formal consistency, but also material validity, *i.e.*, correspondence of our thoughts with things. And this is especially the case in Inductive Logic. Thus we see that words are only expressions of ideas, which, again, are only so many ways of looking at things, so that there is nothing to prevent these

apparently conflicting opinions from being harmonised into one *comprehensive view*. Hence we conclude that the true view will be a combination of Conceptualism, Realism and Nominalism, inasmuch as Logic is found to be concerned with self-consistency, material validity and correctness of language. It must, however, be admitted that *thought* constitutes the most important factor. It is from *thoughts* that Logic may be said to proceed to the consideration of *names* and *things*. "*Logic* treats of things so far as they are the objects of thought, and *it* treats of language so far as it is the embodiment of thought" (Jevons, *Elementary Lessons*, p. 11).* Hence we must hold that Logic deals primarily or directly with *thought*, and secondarily or indirectly with *language* in so far as it expresses thought, and with *things* in so far as they constitute the objects of thought.

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* The italics are our modifications.

CHAPTER III

Logic and the Sciences

As the principles of Logic are more or less perfectly employed in all the Sciences and Arts,

§ 1. **Relation of Logic to Sciences in general.** We have already explained the nature of science in Ch. II, § 5. For further description and classification of sciences, see Inductive Logic, Ch. XXIII, § 3. We are to discuss here the place and position of Logic among the Sciences. Logic is the most general and fundamental of all Sciences, inasmuch as its principles are more or less perfectly employed in every scientific study. A Science, as already explained, is a systematic or methodical treatment of a subject, *i.e.*, every Science has to *define* the terms it employs, to *classify* its subject-matter and to *infer* conclusions from given data. As Logic teaches us how to define, classify and infer correctly, it is the basis of all Sciences. In short, no scientific study is possible without reasoning, and Logic teaches us to reason correctly. Hence Logic has aptly been described as the *Science of Sciences* (*Scientia Scientiarum*). Moreover, if Logic is the Science of Sciences, it is also the *Art of Arts* (*Ars Artium*), for an art is only the practical aspect of a science, and the correctness of an art depends upon that of the science on which it is based. Thus Logic is the foundation of all Sciences—theoretical and practical.*

it has rightly been called the Science of Sciences and the Art of Arts.

Logic is the most

Again, if we classify sciences according to their *degree of generality*, Logic must be placed at the

* Read Jevons, *Elementary Lessons in Logic*, p. 6.

'learned' and 'not-learned' cannot be at the same time affirmed of John. But, as Welton points out, Mill's statement of the Principle of Contradiction, like that of the Principle of Identity, is rather a postulate referring to *expression* than a principle of thought. Sir W. Hamilton regards this principle as equally primary with that of Identity, and he calls it the Principle of *Non-contradiction*, since it enjoins the absence of contradiction as an indispensable condition of thought, and its force is not affirmative but negative a thing can *never have* two contradictory qualities at the same time—two contradictory terms cannot be affirmed of the same thing.

Some prefer to call it the Principle of non-contradiction

The Principle stated

§ 4. **The Principle, Law, or Axiom of Excluded Middle.** The simplest statement of the Principle is the formula '*A is either B or not-B*'. Other forms are: (1) "Everything must be or not be" (Jevons): (2) "A either is, or is not, B" (Welton): (3) "Either a given judgment must be true or its contradictory, there is no middle course" (Thomson). (4) "The double answer, yes and no, cannot be given to one and the same question understood in the same sense" (Ueberweg): (5) "Of contradictory attributions we can only affirm the one of a thing, and if one be explicity affirmed the other is denied" (Hamilton). This Principle means that two contradictory qualities cannot both be false of an individual thing at the same time and in the same sense; if one be false of an individual thing, the other must be true of it. If an individual thing be not one of the contradictories, it must necessarily be the other; if it be excluded from one class, say, that of 'good', it must necessarily be included in the other, *viz.*, that of 'not-good'; for there is no intermediate possibility between the two contra-

The Principle means that two contradictory qualities cannot both be false of an individual thing at the same time and in the same sense.

dictories—there is no middle course or third alternative—the middle course, is *excluded* (hence the name Excluded Middle). Mill states the Principle thus: "It is allowable to substitute for the denial of either of two contradictory propositions the assertion of the other." This statement means that the *denial* of the proposition 'John is learned', *viz.*, 'John is not (denial) learned', is the same as, and thus can be substituted for, the *assertion of its contradictory*, *viz.*, 'John is (assertion) not-learned (contradictory)'.

Mill's Statement.

Contradiction and Excluded Middle. The Principle of Contradiction and the Principle of Excluded Middle are not two independent laws. The Principle of Excluded Middle is, in fact, only the other half of the Principle of Contradiction. The Principle of Contradiction lays down that two contradictory qualities cannot both be true of an individual thing, one must be *false*; the Principle of Excluded Middle interprets it more fully by stating that two contradictory qualities cannot both be false of an individual thing, one must be *true*. Thus, they are two aspects or different interpretations of one and the same principle. For this reason some Logicians have combined them into one Principle under the name of the Principle of Contradiction. Ueberweg combines them into one Principle, *viz.*, the 'Principle of Contradictory Disjunction', which is thus stated: 'A is either B or is not-B', which means that A cannot be *both* B and not-B (Contradiction) and that it must be *one* or the *other* (Excluded Middle).

The Principle of Contradiction and the Principle of Excluded Middle are but two aspects of one and the same Principle.

Hence Ueberweg combines them into one.

The Principle of Excluded Middle holds good in the case of contradictory terms, and not in the case of contrary terms. (For distinction between the

The Principle of Excluded Middle

holds good in the case of contradictory terms and not in the case of contrary terms.

two, see Ch. VII, § 7.) The force of the Principle of Excluded Middle lies in the fact that two contradictory attributes, like 'white' and 'not-white', cannot both be false of a thing, one must be true. But this does not apply to contrary terms, for though two contrary terms, like 'white' and 'red', cannot both be true, yet both may be false of a thing. It may, however, be noted that the force of the Principle of Contradiction applies equally well to contrary and to contradictory terms. The force of the Principle of Contradiction lies in the fact that two contradictory attributes cannot both be true of a thing, one must be false. Now, neither two contradictory terms, *e.g.*, 'white' and 'not-white', nor two contrary terms, *e.g.*, 'white' and 'red', can both be true of the same subject. If any one in each pair be true of a thing, the other in the same pair must be false.

The force of the Principle of contradiction applies equally well to contrary as also to contradictory terms.

The Principles of Contradiction and Excluded Middle hold good when the subject is singular and not general.

The Principles of Contradiction and Excluded Middle hold good in the case of an individual thing, and not in the case of a class of things. In other words, the Laws are applicable when the subject (of which two contradictory qualities are either affirmed or denied) is a *singular* term (*i.e.*, one which signifies a single individual thing only), and not a *general* term (*i.e.*, one which signifies any one of a number of things). Let us take a general term such as 'man' and a pair of contradictory terms such as 'cruel' and 'not-cruel', and form two propositions 'Man is cruel' and 'Man is not-cruel'. If the term 'man' be taken to mean 'some men', then the two propositions 'Some men are cruel' and 'Some men are not-cruel' are both true, and thus, in this case, the Principle of Contradiction fails. Again, if the term 'man' be taken to mean 'all men', then the two

propositions 'All men are cruel' and 'All men are not-cruel' are both false, and thus, in this case, the Principle of Excluded Middle fails.

The three Fundamental Principles lie at the root of all logical processes.

§ 5. Remarks on the Three Fundamental Principles. The three Principles, as explained above, are indispensable for securing formal truth, *i.e.*, for bringing our thoughts into harmony with one another. Now, what is materially true must first be formally true; hence, it has been said that these Fundamental Principles of Thought are assumed in all our thought—formal as well as material. No valid thought is ever possible without conformity to these principles which lie at the root of all logical processes. They are called Principles of Formal Reasoning, because they are the universal and necessary conditions of all valid thinking, whatever the matter may be.

Attempts have been made to reduce the three Principles to one, *viz.*, that of Identity.

Attempts have sometimes been made to reduce the three Principles to one, *viz.*, that of Identity. It has been held that Identity, Contradiction and Excluded Middle are only different expressions of Identity or Consistency. We have already seen that the Principle of Excluded Middle is only the other half of the Principle of Contradiction. It is also obvious that the Principle of Contradiction involves the Principle of Identity. For the Principle of Contradiction is—'A cannot be both B and not-B', which may be expanded thus: If A is *identical* with B, it must be B, and nothing but B; and if A is *identical* with not-B, it must be not-B, and nothing but not-B. Thus we may say that the other two principles are only expansions of the Principle of Identity. If A is A (Identity), it is evidently not not-A (Contradiction), and A can never be excluded alike from both of these possibilities, A and not-A (Excluded Middle). Dr. Keynes expresses this close relation by the formulæ:[1] "I affirm what I affirm, and deny what I deny; if I make any affirmation, I thereby deny its contradictory; if I make any denial, I thereby affirm its contradictory." But perhaps it is going too far to attempt to reduce the Principle of Contradiction to that of Identity. Even though it be said that the Principle of Contradiction involves the Principles of Identity, it may, with equal cogency, be said that the Principle of Identity also involves the Principle of Contradiction. For, 'A is A' means A is A, and nothing but A, *i.e.*, 'A is A and not not-A', which means 'when A is A, it is not not-A', *i.e.*, 'A and not-A cannot both be true of A'. It seems to be more reasonable to regard Identity and Contradiction as two equally fundamental Principles, for they correspond to the equally fundamental and necessary mental processes of Assimilation and Discrimination. Assimilation may depend on Discrimination, and Discrimination may depend on Assimilation, but the one is as primordial, as the other. Hence Identity and Contradiction should be regarded as essentially different

But it seems plausible to regard Identity and Contradiction as two essentially distinct Principles.

principles, representing two radically distinct tendencies of the mind.

Instances of the practical application of these Principles.

We have already observed that these principles lie at the root of all logical processes. Here are a few instances of the *Application of the Principles*. The Definition of Terms is based on the Principle of Identity. The definition of 'man' as a rational animal implies that man is identical with rational animal. Division by Dichotomy is based upon the Laws of Contradiction and Excluded Middle. The Principle of Identity is the basis of Immediate Inference by Conversion, while the Principles of Contradiction and Excluded Middle are applied in Aristotle's scheme of Predicables, Immediate Inferences by Obversion, Contraposition, Inversion and Opposition. Again, the Principles of Identity, Contradiction and Excluded Middle form respectively the foundations of Affirmative, Negative, and Disjunctive Syllogisms.

The Principle of Sufficient Reason stated.

§ 6. **The Principle of Sufficient (or Determining) Reason.** This Principle as first formulated by the great German Philosopher, Leibnitz, stands thus: "No fact can be found to be real, no proposition true, without a sufficient reason why it is in this way rather than in another", or, "Whatever exists, or is true, must have a sufficient reason why the thing or proposition should be as it is and not otherwise." This Principle has explicitly a twofold significance: (1) In the first place, it ensures the formal validity of our reasoning, inasmuch as it requires us to conform to the Principles of Identity, Contradiction and Excluded Middle. Every proposition must have a reason why it is so, and not otherwise, *i.e.*, every judgment, and consequently every concept constituting the judgment, must be free from self-contradiction, and every inference must be supported by the premises, *i.e.*, must follow consistently from the premises. Thus the Principle serves as the Principle of Consistency, and lies at the root of all Formal or Deductive reasoning. (2) In the second place, it ensures the material validity of our reasoning, inasmuch as it requires us

Twofold significance of the Principle. It forms the ground of both Deduction and Induction.

to find a reason or cause for every fact, for every occurrence of an event. The facts and events of this world are connected with one another as cause and effect, and a fact or occurrence is said to be explained and understood by means of its cause, which gives rise to it. Thus the Principle serves as the Principle of Causation and lies at the basis of all Inductive or Material reasoning. Hence we see that the Principle of Sufficient Reason is connected alike with, and forms the ground of, both Deduction and Induction.

Subordinate Axioms and Postulates.

§ 7. **Other Axioms and Postulates of Logic.** Besides the Fundamental Principles explained above, we mention below some subordinate Axioms and Postulates, as well as the Principles which regulate Inductive reasoning and which we leave for consideration in the Inductive Portion.

Dictum de omni et nullo.

(1) Aristotle's *Dictum de omni et nullo* (statement concerning All and None), which may be rendered thus: Whatever is affirmed or denied of a class distributively, may be affirmed or denied of everything contained in that class. For the exposition of this dictum and its substitutes—the canons of the Syllogism—*vide* Ch. XVII, § 5.

The Principle of Similarity.

(2) *The Principle of Similarity.*

This is only an aspect of the Principle of Identity, and signifies that different things, though different in some respects, may yet be *identical* in some other respect or respects. Thus while Identity may be described as perfect similarity, *i.e.*, unity in essence, Similarity may be described as imperfect identity, *i.e.*, unity in some respect or respects. The Principle of Similarity is the ground of all Inference—Deductive, Inductive and Analogical (*vide* Ch. XV, § 1).

The Principle of the Uniformity of Nature.

(3) ***The Principle of the Uniformity of Nature,*** and

The Law of Causation.

(4) ***The Law of Causation,*** and

The Principle of Ground and Consequent.

(5) ***The Principle of Ground and Consequent*** (which is only an aspect of the Principle of Sufficient Reason and another way of expressing the Law of Causation), are regarded as the Grounds of Material Truth or Induction, and will be explained in the Inductive Portion (Ch. XXV).

Hamilton's Postulate.

(6) *Hamilton's Postulate*: "Logic postulates to be allowed to state explicitly in language all that is implicitly contained in thought." It means that it is permissible to change the form of terms, propositions and arguments, in order to render their meaning more clear, provided the meaning remains the same. If the identity of thought be preserved, it is immaterial what verbal changes are intro-

duced. We shall subsequently see that in describing logical characters of terms and propositions and in testing arguments, we are allowed to make any necessary verbal change, so long as the meaning is left unchanged. This postulate is no doubt a corollary of the Principle of Identity.

Venn's enumeration of the Postulates of Logic

(7) *Venn's Postulates of Logic*: According to Venn and others, Logic assumes the following postulates:—

(*a*) A duality of things—There exist two separate substances, the mind which knows and the matter which is known.

(*b*) The possibility of knowing things.

(*c*) That mind and matter are subject to uniform laws.

(*d*) That things are as we actually perceive them. That the world appears essentially the same to all observers, present, past and future.

(*e*) That there is a distinction between truth and falsity, and there is some test whereby we may distinguish the one from the other.

(*f*) That the language used is to be used always in the same sense.

§ 8. **Exercises.**

1. When Logic is defined as 'the Science of the Laws of Thought,' explain what is meant by the words 'Science,' 'Law' and 'Thought.'

2. Logic has been defined as 'the Science of the Formal Laws of Thought'. Explain what is meant by 'Thought' and 'Formal Laws of Thought'. Do you consider this definition adequate? If not, substitute what you consider to be a better one, giving your reasons.

Give and explain the definition of Logic which appears to you to be the most satisfactory.

State and criticize the definitions of Logic given by Whately, Hamilton, Thomson, Jevons and Welton.

State and explain J. S. Mill's definition of Logic.

6. Explain the definition of Logic as the science of the necessary laws or forms of thought.

7. Clearly indicate the place of Logic in the scheme of knowledge.

8. What is Thought? In what sense is the term used in Logic?

9. What is Reasoning? Explain and illustrate the main forms of it.

Explain the distinction between the Form of Thought and the Matter of Thought, also the distinction between Formal and Material truth.

11. Distinguish between the Form and the Matter of Reasoning and between Formal and Material Logic, stating the end aimed at by each. Which department of Logic is essentially Formal, and which Material, and why?

12. What are the three different parts into which Logic is divided? Show that all three necessarily belong to it.

13. Distinguish Deductive from Inductive Logic. Are Logic and Deductive Logic identical?

14. Explain fully what you mean by a Science. In what sense is Logic a Science? In what relation does it stand to other Sciences?

15. What is a Science? What is an Art? What is the relation between the two? Discuss the relative priority of each. Illustrate your meaning by examples. Do you consider Logic to be a Science, or an Art, or both? State your reason fully.

Distinguish between (*a*) Science and Art; (*b*) Theoretical Science and Practical Science; (*c*) Positive Science and Normative Science; (*d*) Practical Science and Normative Science. Also indicate under which of these Logic falls.

17. What is Knowledge? What is Truth? Explain the difference between Immediate and Mediate Knowledge, giving examples. How is it possible to advance from the former to the latter? When you go out in the morning, and find the roads in a muddy condition, you believe at once that there has been rain during the night; explain what is *immediate* and what is *mediate* in your knowledge of this subject, giving your reasons.

18. Fully discuss the question of the place of Immediate Knowledge in Logic.

19. Point out the relation between Thought, Language, and Things.

20. What are the functions of Language in relation to Thought? How far is it correct to say that Logic is concerned with Language?

21. Point out the relative importance of Thought, Things, and Language in Logic.

22. What different views have been held with regard to the subject-matter of Logic? Which of these views would you accept, and why?

23. Give a brief account of the different Schools of Logic. To which School do you belong?

24. Is Logic more or less general than the other Sciences? Give reasons. In what sense is Logic the Science of Sciences?

25. Contrast the Province and Method of Logic with those of Psychology.

26. Explain the relation of Logic to (*a*) Metaphysics, (*b*) Grammar, and (*c*) Rhetoric.

27. Clearly indicate the Province or Scope of Logic.

28. What is the use of studying Logic? Can we say that the study of Logic is useful, when persons who have never studied it reason correctly? Give reasons for your answer.

29. What are the mental processes concerned in Logic? Distinguish between Term and Concept, Proposition and Judgment, Argument and Inference.

30. What is a Concept? How is a Concept formed? What is Abstraction?

31. Distinguish Realism, Nominalism and Conceptualism, and point out their bearing on Logical Doctrine.

32. Examine the different views that have been held with regard to the nature and existence of Concepts.

33. What are the requisites of a correct definition of Logic Show that the definition you adopt satisfies the requisite conditions.

34. What do you mean by (*a*) the word 'Principle', and (*b*) the phrase 'Principles of Logic?' State what you take to be Fundamental Principles of Logic, and explain their meaning clearly.

35. Explain the expressions 'Principles of Formal Reasoning' and 'Fundamental Laws of Thought.' Explain the general character of the Principles.

36. State and explain the Laws of Identity, Contradiction and Excluded Middle, and explain their significance.

37. State and explain the Principle of Sufficient Reason, and discuss its Logical place and value.

38. Are the Laws of Identity, Contradiction and Excluded Middle all equally fundamental and independent? Discuss.

39. Indicate the relation between the Principle of Contradiction and the Principle of Excluded Middle. Show whether each applies equally (*a*) when the subject is singular (*b*) when it is general.

40. Show that the Fundamental Principles are assumed in all thought. Illustrate the application of these Principles.

41. State the Principles of Identity and Contradiction from different standpoints; exhibit their relation to each other, and show how the Principle of Similarity is but an aspect of the Principle of Identity.

42. Discuss the Province of Logic in relation to Truth and Knowledge.

Book II

DEDUCTION

Part I

TERMS

CHAPTER VII

Import of Terms

§ 1. **Three Parts of Logic.** We have already seen (Ch. II, § 4) that though reasoning or inference forms the principal subject-matter of Logic, yet reasoning (or argument) cannot proceed without concept (or its verbal expression—term) and judgment (or its verbal expression—proposition). Thus we see that the reasoning or argument—'All men are mortal', 'John is a man', therefore 'John is mortal'—consists of three judgments or propositions. Each of these constituent judgments or propositions, again, is a combination of two concepts or terms. Thus the proposition 'All men are mortal' is made up of two terms, *viz.*, 'men' and 'mortal', joined together by the bond 'are', logically called the 'copula'. Hence the study of Inference requires a preliminary study of the nature of its constituent parts, the Propositions and Terms. Thus Logic is divided into three main Parts—the first (Part I) dealing with *Terms*, the second (Part II) with *Propositions*, and the third (Part III) with *Reasonings* or *Inferences*. We begin with the study of Terms, because a Proposition being composed of Terms, cannot be properly understood unless we first understand the nature of its constituent elements, the Terms.

Logic has been divided into three parts—Terms, Propositions, and Inferences.

§ 2. Import of Terms. Let us here explain the meaning of a 'Term' as distinguished from a 'Word' and a 'Name'.

A thing is an object of thought; it is either a substance or an attribute

Things: Ideas: Concepts. As already explained (Ch. II, § 8), a *Thing* means an object of thought. It may be either a substance or an attribute, either real or imaginary; thus 'God', 'centaur', 'whiteness', 'anger' are all examples of things.

An idea is a mental representation of a thing and is either singular or general.

Ideas are mental representations of things. Ideas may be either of individual things (singular ideas), or of classes of things (general ideas). A general idea is called a *Concept* or Notion. Thus a distinction may be drawn between a concept and an idea. While a concept represents a class, an idea represents either a class or an individual. Thus we speak of the concept, notion or idea of man, dog, etc.; but of the idea, and not the concept, of this individual man or of that particular dog. A concept or idea expressed in language is called a *term, e.g.*, man, wisdom, Socrates (see below). We have already explained the meaning of *Concept,* its formation and the different views as to its nature and existence (Ch. V).

'Term' has been used in two senses:

Terms and Words. A Term is a word or combination of words signifying some idea and capable of standing by itself as the subject or the predicate of a proposition. In a proposition, the subject is that of which something is said, and the predicate is that which is said of the subject. In the proposition 'Man is wise', the words 'man' and 'wise' are *terms,* and 'man' is the subject-term and 'wise' is the predicate-term. Terms are so called, because they form the two extremes (from Latin *terminus*, an end or extremity) or stand at the two ends of a proposition. The word 'Term' has

been used in two different senses: (1) In a wide sense, it means a word or combination of words which *may be* used as the subject or predicate of a proposition, *e.g.*, man, round, table, the King of England. We may use any one of these expressions as the subject or predicate of a proposition; for instance, 'The table is round,' 'The King of England is a man'. (2) In a narrow sense, it means a word or combination of words which *is actually* used as the subject or predicate of a proposition. In this sense, the words 'man', 'round', 'table' are not terms so long as they do not form parts of a proposition.

(1) In a wide sense, it means a word or combination of words which *may be* used as subject or predicate of a proposition.

(2) In a narrow sense, it means a word or combination of words which *is actually* used as so

Terms are now to be distinguished from *words*. In Logic words are divided into three classes:—

(1) *Categorematic* (from Gr. *Kategorema*, a predicate), (2) *Syncategorematic* (from Gr. *Syn*, with, and *Kategorema*), and (3) *Acategorematic* (from Gr. *A*, non, and *Kategorema*). A word or combination of words that is capable of being used by itself as the subject or predicate of a proposition is called *Categorematic*, *e.g.*, nouns, pronouns, adjectives, participles and certain parts of verbs. A word or combination of words which must be joined with other words in order to form a term is called *Syncategorematic*, *e.g.*, prepositions, articles, conjunctions, adverbs, and adjectives used as such. When a word cannot at all form part of a proposition, it is called *Acategorematic*, *e.g.*, interjections, nouns in the vocative case, and verbs in the imperative and optative moods. Hence we see that words are wider than terms. All terms are words, but all words are not terms; only categorematic words are terms. Again, a term may be constituted of more than one word, since the subject and predicate are sometimes

Words are divided into categorematic, Syncategorematic, and Acategorematic.

Categorematic words are terms—hence words are wider than terms.

formed of a combination of words. Thus, a proposition may contain many words, but there cannot be more than two terms in it.

Note. The division of words into categorematic, syncategorematic and acategorematic is really a division into Terms and Non-terms, and it is as absurd to speak of syncategorematic term as it is tautologous to speak of categorematic terms; for the very definition of term implies that a term must be categorematic. There is, however, one sense in which every word may become categorematic, *i.e.*, may be used as the term of a proposition. This sense is technically known as the *Suppositio Materialis* of a word, *i.e.*, speaking of a word itself as a thing. Thus we can say —'*Very* is an adverb,' '*Rich* is an adjective.' Here 'very' and 'rich' mean 'the word very' and 'the word rich.'

'Name' defined. A *Term* in the wide sense is equivalent to a *name*. In Logic, however, the word *Term* is used in its narrow sense, as given above.

Terms and Names. Mill defines a *name* as "a word (or set of words) serving the double purpose of a mark to recall to ourselves the likeness of a former thought, and a sign to make it known unto others." Thus it may be said that a concept or idea when expressed in language is a name or term in the wide sense of the word. In other words, a *term* in the wide sense is equivalent to a name, standing for any object of thought, real or imaginary, substantive or attributive, mental or material. But the word '*term*', as used in Logic, is restricted to its narrow sense and means a *name which actually stands either as the subject or as the predicate of a logical proposition.*

Denotation of a term indicates the individuals to which it applies, while Connotation implies the common and essential attributes

§ 3. Denotation and Connotation of Terms. By the **Denotation** of a term we mean the number of things to which the term is applicable in the same sense; by the **Connotation** of a term we mean the essential attributes which it implies, or on account of which it is so called. Most terms, and especially general terms, have a twofold meaning, *viz.*, meaning in *denotation* and in *connotation*. Thus the term 'man' means, in the first instance, all individual men

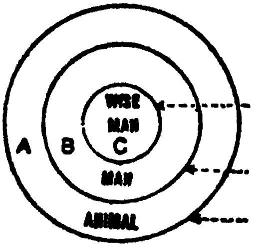
—John, James, Thomas, Brown, etc., to each of which the name 'man' applies in the same sense. In other words, the denotation of the term 'man' indicates *all the individual human beings*—past, present and future—to each of which the term 'man' is applicable. In the second instance, the term 'man' implies the attributes of animality and rationality, which are the *essential attributes possessed in common by all men* without exception; and the name 'man' is applicable to a particular class of animals, because of its possessing the essential attributes of animality and rationality, which are not found in others. Thus the denotation (Lat. *de*, down, *notare*, to mark) of a term consists of the individual or individuals to which the term applies. It is otherwise known as the *Extension, Extent, Breadth, Domain, Sphere, Scope, Compass*, or *Application* of a term. Again, the connotation (Lat. *con*, with, *notare*, to mark) of a term consists of the essential attribute or attributes possessed by the individuals denoted by the term. It is otherwise known as the *Intension, Intent, Depth, Force, Content, Comprehension*, or *Implication* of a term. possessed by those individuals.

Note. There are some difficulties involved in the idea of connotation. (1) We may ask: What attributes form the connotation of a term? (On this point and on the distinction between comprehension, intension and connotation, *vide* Keynes, *Formal Logic*, p. 20; Alfred Sidgwick, *Elementary Logic*, p. 100.) If it be said—the essential attributes, we may further ask: How are we to determine the essential attributes? Does it include all the attributes, whether known or unknown, of an object, or only those that are known at the time? (2) Again, we have seen above that the connotation consists of the essential attributes separated from the rest by the mind's power of abstraction; but it is difficult to get agreement among different minds as to purely abstract attributes. Different persons might frame different ideas as to the connotation of a term. (3) Moreover, are we to regard connotation *subjectively*, *i.e.*, depending on the qualities suggested and known at the time to an individual subject; or *objectively*, i.e., depending on the Difficulties involved in the idea of Connotation.

qualities actually existing in objects, whether such qualities are known or unknown; or *conventionally*, *i.e.*, depending on the essential qualities determined by scientific research and fixed by general agreement? It is generally maintained that *Logic is concerned with the scientific or conventional connotation* (*i.e.*, the essential attributes determined by scientific research or fixed by general agreement), and not with the subjective and objective forms (called by Keynes 'subjective intension' and 'comprehension' respectively) (4) Further, there is some difficulty with regard to Proper names. Do they connote a group of attributes, or are they merely unmeaning marks? For discussion, *vide* Ch VIII, § 10.

***Relation between Denotation and Connotation.* The Rule of inverse variation.**

In a series of general terms that can be arranged in order of subordination, the relation between their denotation and connotation is such that an increase or decrease in either will cause a corresponding decrease or increase in the other. In other words, *the denotation and the connotation vary inversely* in the case of several terms that can be arranged in a series as more general and less general.



Animality, Rationality and Wisdom.

Animality and Rationality,

Animality.

Take the terms 'animal', 'man' and 'wise man'; and let their denotation or extent be represented by the circles A, B and C respectively. The circle A contains within it the circle B, which, again, contains within it the circle C. This means that the denotation or extent of 'animal' is greater than that of 'man', and the denotation of 'man' is greater than that of 'wise man'; for 'man' is only a section of 'animal', and 'wise man' is only a section of 'man'. Again, 'animal' has for its connotation only the attribute

of animality, 'man' has for its connotation the attributes of animality and rationality, and 'wise man' has for its connotation the attributes of animality, rationality and wisdom ; for only rational animals are men, and only wise and rational animals are wise men. Now it may easily be understood that as we proceed from the group 'wise man' to the group 'man', and from the group 'man' to the group 'animal', *the denotation gradually increases while the connotation decreases.* Inversely, as we proceed from the group 'animal' to the group 'man', and from the group 'man' to the group 'wise man', *the connotation gradually increases while the denotation decreases.* Similarly, it may be shown that *when the denotation or connotation decreases, the connotation or denotation increases.* Of the three groups considered here, the group 'wise man' has the least denotation, but the highest connotation ; while the group 'animal' has the highest denotation, but the least connotation.

Is the rule always true? Yes, in the case of terms that can be arranged in a series of varying generality; no, in the case of a single term.

Thus the above rule of inverse variation is always true in the case of terms that can be arranged as higher and lower, and implies that the denotation and connotation of a term are invariably fixed. The denotation of a term is invariable, for it refers to all individuals, past, present and future, belonging to the class in question. The connotation of a term is also invariable, for in Logic, it means *conventional* or *scientific* connotation, *i.e.*, the essential attributes determined by scientfiic research, and accepted by general agreement as forming the meaning of a term. Consequently it follows that the rule does not apply to the denotation and connotation of a single term. If the rule is applied to *the same term,* it fails in two ways, inasmuch as it

assumes that the denotation and connotation of a term are not absolutely fixed, but are liable to increase or decrease by nature's course or by accident or by advance of knowledge. Thus (*a*) when the denotation of a term is increased or decreased by such individuals as may happen to possess *all* the attributes connoted by it, then the connotation will remain unaffected. For example, a new country inhabited by men may be discovered, thereby increasing the denotation of the term 'man', or an already existing country inhabited by men may be destroyed, thereby decreasing its denotation, but in both cases its connotation remains unaltered. (*b*) Again, when by scientific research, the connotation of a term is increased or decreased by such an attribute as may be possessed by *all* the individuals denoted by it, then the denotation of the term will remain unaffected. For example, a new attribute common to all metals may be discovered, or an attribute hitherto mistaken to form a part of the connotation of metals may be subtracted, but the denotation in either case remains unaltered. But as has been truly remarked, "the rule was never meant to apply to what happens to a single term through increasing knowledge or increasing number of individuals" (Mellone, *Text-Book of Logic*, p. 25).

'Inverse variation' must not be understood in any strict mathematical sense.

It may also be observed here that the general rule of inverse variation must not be regarded as a precise mathematical law of variation; as there is *no mathematically exact proportion* between the increase and decrease of the denotation and the connotation of a term, for a slight increase or decrease in one may be accompanied by a considerable decrease or increase in the other. (*Vide* Welton, *Manual of Logic, I*, p. 62.)

4. Relation of Terms considered in Denotation and Connotation. Take a number of terms, *e.g.*, 'Indian', 'Man', and 'Animal'; and let their denotations be represented (as in the adjoining

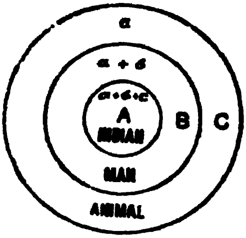


figure) by the circles A, B and C; and let their connotations be indicated by the small letters. Now, if a number of terms be so related that the denotation of one (say, 'Indian'—A) is included in that of another (say, 'Man'—B), then the term of wider denotation ('Man'—B) is called **Genus** in relation to the term of narrower denotation ('Indian'—A), which is called **Species**. The terms 'genus' and 'species' are entirely relative, so that a term which is a genus in relation to one term may be a species in relation to another. Thus the term 'man' is a species in relation to 'animal', but a genus in relation to 'Indian'. Again, the term 'Indian', though a species in relation to 'man', is a genus in relaiton to 'Bengalee'. Thus, *as regards denotation, species is contained in genus.* 'Genus' and 'Species'.

Again, the connotation of the term 'animal' is animality (*a*), the connotation of the term 'man' is animality and rationality (*a* and *b*). Thus the connotation of 'animal' is contained in that of 'man'; similarly, the connotation of 'man' is contained in that of 'Indian'. But 'animal' is a genus in relation to the species 'man', which, again, is a genus in

relation to the species 'Indian'. Hence, *as regards connotation, genus is contained in species.*

'Co-ordinate Species.' 'Super-ordinate and Sub-ordinate Terms'.

When two or more species are included in the same genus, they are called **Co-ordinate or Cognate Species** in relation to one another. The genus in relation to its species is called **Super-ordinate,** and the species in relation to its genus is called **Subordinate.** The highest genus which can no longer be regarded as a species is known as **Summum genus**; and the lowest species, which can no longer be divided into species but only distributed into individuals, is called **Infima species.** All the intermediate genera and species between the summum genus and the infima species are called **Subalterns.** The nearest genus and species to a term are called **Proximate genus and species.** The above relations are best illustrated by the following table, known as the **Tree of Porphyry** (commentator on Aristotle) or the **Ramean Tree** (from Ramus, the Scholastic Logician).

'Summum genus.'

'Infima Species.'

'Subalterns.' 'Proximate genus and species.'

The Tree of Porphyry.

Substance

Corporeal = Body | Incorporeal

Animate = Living being | Inanimate

Sensible = Animal | Insensible

Rational = Man | Irrational = Brute or lower animals

Socrates | Plato | and others (Individuals)

Here the term 'substance' is the summum genus, and the term 'man' is the infima species. The intermediate terms, *viz.*, 'body', 'living being' and 'animal' are subaltern genera and species in relation to one another. The terms 'animal' and 'man' are proximate genus and species in relation to each other. The terms 'man' and 'brute' are co-ordinate or cognate species coming under the same genus 'animal'. Again 'living being' is super-ordinate genus to 'animal', but subordinate species to 'body'. 'Socrates', 'Plato' and others are individuals or varieties. Thus we can select any particular term in the *Tree* and fully determine its relation to the rest. Take, for example, 'animal'. It is proximate and subordinate species to 'living being', co-ordinate species to 'insensible', super-ordinate genus to 'man' and 'brute'.

We may also note in this connection the meanings of Differentia, Proprium and Accidens, which will be frequently employed in the subsequent chapters. **Differentia** (or Difference or Distinguishing quality) is that attribute (or group of attributes) which distinguishes one species from other species coming under the same genus, as well as from the genus itself. Thus, *rationality* is the differentia of 'man', because it distinguishes 'man' from its co-ordinate species 'bird', 'beast', etc.,—which are irrational, as well as from its proximate genus 'animal'. Differentia is, therefore, the *excess of the connotation of a species over that of its proximate genus.* *'Differentia.'*

Proprium or Property. This is an attribute which is no part of the connotation of a term, but which follows from it either *causally* (*i.e.*, as an effect from a cause) or *deductively* (*i.e.*, as a conclusion *'Proprium.'*

from premises). A property is, therefore, common to every individual of a class, but is no part of the connotation. Thus, 'equality of three angles' is no part of the connotation of 'equilateral triangle,' since its connotation is 'equality of three sides'. But as 'equality of three angles' follows deductively from the connotation 'equality of three sides', it is a property of 'equilateral triangle'. Similarly, 'the power of arguing' is no part of the connotation of 'man,' but follows causally from 'rationality,' and is therefore a property of 'man'. If a property follows from the connotation of the genus, it is called a *Generic* property; if it follows from the differentia, it is called a *Specific* property. Thus 'feeling pleasure and pain' and 'having appetite' are the generic properties of 'man', since they follow from 'animality,' the connotation of the genus 'animal'; while 'making inference' and 'being virtuous' are specific properties of 'man', since they follow from 'rationality', the differentia of 'man'.

Generic and Specific

Accidens.

Accidens or Accident. This is an attribute which is possessed by a term, but which neither forms part of, nor follows from, its connotation. An accident gives, therefore, some new information about a term. Thus the birth-place of a man, the blackness of the crow, the whiteness of milk are examples of accident. An accident may be either *Separable* or *Inseparable*. And the distinction, again, is applicable in the case of an *individual* as well as a *class*. If an accident always belongs to an individual or to all the members of a class, it is called its inseparable accident. Thus, the height of a man is always found in him, and so is an inseparable accident with regard to an individual; the whiteness of milk is an inseparable accident with

Separable and Inseparable

regard to a class, for all milk is, in fact, white, though whiteness is neither a part of, nor deducible from, the connotation of milk. Similarly, the birth-place, parentage, colour, size, etc., of a particular person are his inseparable accidents, while the blackness of the crow, the whiteness of milk, etc., are inseparable accidents of the respective classes. If, on the other hand, an accident is sometimes present and sometimes absent in an individual, or if it belongs only to some individuals of a class but not to the whole class, it is called its separable accident. Thus the particular posture of a man is a separable accident with regard to the individual, for the man may next moment change his present posture for some other. And wisdom is a separable accident with regard to the class 'man', for all men are not wise, only some are. Similarly, the dress and habitation of a particular person are his separable accidents, while blindness, fair complexion, etc., are separable accidents of the class 'man'.

Proprium and Accidens compared.

From what has been said above it is clear that both 'property' and 'accident' agree in this, that they form no part of the connotation ; but they differ in this, that while property follows from the connotation, an accident does not so follow. Again, an accident may be removed from the class or the individual without affecting its essential nature, while to remove the property or the differentia would be to destroy, or at least to change, the nature of the individual or the class. As connotation consists of the genus and the differentia, the differentia conveys no new knowledge about the term beyond what we know by analysing its connotation ; but a property and, more particularly, an accident always give some new information, and thus add to the know-

ledge obtained by simply unfolding the connotation of the term.

A term is said to be *distributed* when it is used in its entire extent

§ 5. Distribution of Terms. A term is said to be *distributed* when it is employed in its whole extent or denotation, *i.e.*, when it is applied to all the individuals denoted by it. For example, in the propositions 'All men are mortal' and 'No men are perfect', the term 'man' is distributed, for it is used to denote all individual human beings without a single exception. Again, a term is said to be *undistributed* when it is employed in its partial extent or denotation and not in its entire extent, *i.e.*, when it is applied only to a part of its denotation, and not to the whole. For example, in the propositions 'Some men are good', 'Some men are not honest' and 'Most men are unhappy', the term 'man' is undistributed, for it is used to represent only a part of its denotation. We shall subsequently see that singular terms are always distributed, for they are always taken in their entire extent. General terms are distributed when they are qualified by 'all', 'the whole of', and the like. A general term is undistributed when it is qualified by 'some', 'a few', 'most' and the like words conveying the sense of 'some', which, in Logic, means 'at least one, though not all'.

It is said to be *undistributed* when it is used in its partial extent

Universe of Discourse means the more or less limited sphere within

§ 6. Universe of Discourse.* As already explained, the denotation of a general term embraces the whole range of objects to which the term can be correctly applied, and thus the denotation of the term will be identical with its universe of existence

* *Vide* Keynes, *Formal Logic*, p. 183; Boole, *Laws of Thought*, p. 166; Venn, *Empirical Logic*, p. 180; Welton, *Manual of Logic, I*, p. 59; Coffey, *Science of Logic, I*, p. 54.

i.e., with the realm of objects that existed in the past, that exist at present, and that will exist in future). But in our ordinary speech and intercourse we often employ terms not to signify the whole realm of objects to which the terms are correctly applicable, but only a more or less limited portion of this realm. This more or less limited sphere, within which the speaker intends his statements to apply for the time being, is called the *Universe of Discourse* or *Suppositio*. Thus we often use general terms with the intention that they are to be understood not in their entire extent, but only in their limited application, *i.e.*, application as limited by time, place and other circumstances. For example, when we say 'Everybody is talking about it', we do not mean all the inhabitants of the world, but all inhabitants within certain limits of space. It must be remembered, however, that the universe of discourse may be limited to a portion only of the actual universe of things (as in the above example), or it may be identical with the actual universe. Thus when we say 'Man is mortal', our universe of discourse is the same as the actual universe of men. Moreover, as Dr. Keynes says, "it must be clearly understood that the universe of discourse is by no means necessarily identical with the region of what we ordinarily call 'fact'; it may be the universe of dreams, or of imagination, or of some particular realm of imagination, *e.g.*, modern fiction, or fairy land, or the world of the Homeric poems" (*Formal Logic*, p. 183).

which the speaker intends his statement to apply at any particular time. It has always a reference to the context of the speaker's assertion.

The importance of the universe of discourse consists in the fact that Logic demands, in the interest of truth, that one term should be used in one unvaried meaning throughout a given argument, and

Importance of Universe of Discourse.

that, in order to determine the precise meaning of a term and to avoid ambiguous use of it, it must be understood in connection with the universe of discourse, *i.e.*, with reference to the context of our assertion.

Place of Universe of Discourse.

Although it seems proper to explain Distribution of Terms and Universe of Discourse in connection with the denotation of Terms, it must be remembered that Distribution of Terms and Universe of Discourse are attributes not of terms in themselves, but of terms as used in a proposition. For this reason some writers have discussed these topics under Propositions.

Terms are said to be opposed to each other when they imply attributes that cannot co-exist in the same subject.

Three kinds of opposition

§ 7. **Opposition in Terms.** We have seen above that terms may be *connected* with one another in respect of denotation, or that there may be a relation of inclusion among them; but they may also be *opposed* to one another in that respect, or there may be a relation of exclusion among them. Thus two terms are said to be opposed to, or incompatible with, each other, when they imply attributes that cannot co-exist in the same subject. This incompatibility or opposition of terms may be of three different kinds, *viz.*, (1) Contradiction, (2) Contrariety and (3) Simple repugnance.

***Contradictory* terms are mutually exclusive, and taken together, they cover the whole universe of thought and existence, leaving no**

Two terms are said to be opposed by way of *Contradiction*, when they are mutually exclusive and admit of no medium or third alternative. Thus the pairs of *positive* and *negative* terms, such as 'black' and 'not-black', 'good' and 'not-good', 'man' and 'not-man' are called *contradictories*. 'Black' and 'not-black' are called contradictory terms, because they exclude each other, and, taken together, they cover the whole universe of thought and existence,

leaving no room for a third alternative. Between 'black' and 'not-black' there cannot be a mean which is neither 'black' nor 'not-black', for 'not-black' includes in it all possible colours other than 'black', and 'black' and 'not-black' together exhaust the whole universe or range of colour. Two contradictory terms, therefore, are so related to each other, that if one be true of an object, the other must be false of it ; and if one be false of an object, the other must be true of it. Thus, if a thing is 'black', it cannot be 'not-black'; again, if a thing is not 'black', it must be 'not-black'; and if it is not 'not-black', it must be 'black'. Hence two contradictory terms can neither be affirmed nor denied of one and the same thing, *i.e.*, they cannot both be true or false of the same thing.

room for a third alternative.

Two terms are said to be opposed by way of *Contrariety*, when they are most opposed to each other within the same universe and are thus mutually exclusive, but admit of a third alternative or medium between them. Thus the pairs 'black' and 'white', 'good' and 'bad', 'rich' and 'poor' are called *contraries*. 'Black' and 'white' are called contrary terms, because although they exclude each other, yet, taken together, they do not cover the whole universe, and there is always room left for a third alternative. There is a mean possible between 'black' and 'white', *e.g.*, yellow, red, green, etc. Two contrary terms, therefore, are so related to each other, that if one be true of an object, the other must be false of it ; but if one be false of an object, the other is not necessarily true of it. Thus, if a thing is 'black', it cannot be 'white' at the same time, but if it is not 'black', it does not follow that it is necessarily 'white', for it may be yellow, red,

***Contrary* terms are mutually exclusive, but admit of a third alternative.**

green or of any other colour than black. Hence two contrary terms cannot both be affirmed of a thing but they may both be denied of it, *i.e.*, they cannot both be true, but they may both be false, of the same thing

Points of difference between Contradictory and Contrary terms

We may now summarize the *points of difference* between Contradictory and Contrary terms thus —

(1) Two contradictory terms, taken together, cover the whole universe, while two contrary terms do not. It follows from this that -

(2) Between two contradictory terms there is no medium, but there is always a mean between two contrary terms. Whence it follows, again, that—

(3) Two contradictory terms cannot both be true or false of an object, while two contrary terms cannot both be true of an object, but they may both be false.

***Simply repugnant* terms are also mutually exclusive, they are neither contradictories nor contraries.**

Terms are said to be opposed by way of *Simple repugnance*, when "without being collectively exhaustive like contradictories, or in extreme opposition like contraries, they nevertheless imply attributes which are mutually exclusive, and cannot, therefore, be simultaneously affirmed of the same object. Thus 'red', 'blue', 'green' are incompatible in this sense; so also 'made of wood', 'of delf', 'of iron', etc. Such objects of thought are also called *disparate*" (Coffey, *Science of Logic*, I, p. 60).

Material and Formal contradiction.

Note. Contradiction is of two kinds. When contradiction is discovered from an examination of the connotation and denotation of the terms, *i.e.*, from a knowledge of the matter signified by the terms, it is called *Material contradiction*. For example, 'male' and 'female', 'matter' and 'spirit', 'British' and 'foreign' are *material contradictories*. On the other hand, when contradiction is understood by the very form of the terms, and consists in prefixing 'not' or 'non' to the positive form, we have what is called

Formal or *Logical contradiction*. For example, 'good' and 'not-good', 'man' and 'not-man', 'black' and 'not-black' are *formal contradictories*. (*Vide* Welton, *Manual of Logic*, *I*, p. 65; Coffey, *Science of Logic*, *I*, p. 64.)

A pair of contradictories exhaust the whole universe of discourse, and not the whole universe of thought and existence.

It may also be noted here that according to some Logicians, a pair of contradictory terms do not exhaust the whole universe, but only the whole universe of discourse. Thus 'black', and 'not-black', it is said, cover the whole universe of colour, and not the whole universe of thought and existence, for 'not-black' cannot properly be said to mean everything in nature except black. Is it not simply absurd to say that mental things and qualities such as attention, courage, truth, justice, etc., are 'not-black'? (See Bain, *Deduction*, Bk. I, § 12.)

CHAPTER VIII

VARIOUS DIVISIONS OR CLASSES OF TERMS AND THEIR SIGNIFICANCE

Table of Division of Terms

§ 1. Table of Various Divisions of Terms. Terms may be divided, according to different principles, into various classes, which are given below in a tabular form :—

Term.	1	Simple or single-worded, *e.g.*, man, tree. Composite or many-worded, *e.g.*, man of business, Emperor of India.
	2.	Univocal, *e.g.*, town, street. Equivocal, *e.g.*, horse, foot.
	3	Singular, *e.g.*, John, the sun. General, *e.g.*, man, book.
	4.	Collective, *e.g.*, fleet, library. Non-collective, *e.g.*, man, the moon.
	5.	Definite, *e.g.*, this book, John. Indefinite, *e.g.*, a book, a man.
	6.	Concrete, *e.g.*, man, book. Abstract, *e.g.*, sweetness, redness.
	7.	Positive, *e.g.*, man, good. Negative, *e.g.*, not-man, not-good. Privative, *e.g.*, blind, lame
	8.	Absolute, *e.g.*, man, book. Relative, *e.g.*, father, pupil.
	9.	Connotative, *e.g.*, man, the earth. Non-connotative, *e.g.*, milkwhiteness, John.

Each of the above groups is exhaustive and independent, and every term must come under one or other of the members of each. As a particular triangle must be either equilateral, isosceles or scalene from one standpoint, and obtuse-angled, right-angled or acute-angled from a different standpoint, so one and the same term, *e.g.*, 'horse', may be brought under one or other of the members of each group. Thus we describe the logical characters of 'horse' by saying that it is simple, equivocal, general, non-collective, definite, concrete, positive, absolute and connotative.

A Simple term consists of a

§ 2. Simple and Composite Terms. A *Simple* or single-worded term consists of a single word, *e.g.*,

man, boy, student. A *Composite* or complex or many-worded term consits of a combination of words, *e.g.*, man of business, the present Vice-Chancellor of the Calcutta University.

single word, while a Composite term, of a combination of words.

§ 3. Univocal and Equivocal Terms. A term is *Univocal* when it has one definite meaning, *e.g.*, hydrogen, town, pice. A term is *Equivocal* when it has two or more meanings, *e.g.*, arm, civil, foot, sound ; when a term has more than one meaning, it is really equivalent to two or more terms, according to the *number of meanings*. The term 'civil' has several meanings—it is opposed to natural, to military, to ecclesiastical, to discourteous, etc. The precise meaning of a term is to be determined by reference to the universe of discourse (*vide* Ch. VII, § 7).

A Univocal term has one definite meaning, while an Equivocal term has more than one meaning.

Note. Equivocal terms have been divided into three classes; and one of these again, is divided into three sub-classes, with reference to the origin of each (*vide* Jevons, *Elementary Lessons*, pp. 30-36).

Different classes of equivocal terms.

Terms

- Univocal
- Equivocal
  - Terms equivocal in sound only, *e.g.*, air, heir; hare, and hair.
  - Terms equivocal in spelling only, as 'lead,' the metal, and 'lead,' as in following the lead of a person.
  - Terms equivocal both in sound and spelling.
    - Those arising from the accidental confusion of different words derived from different languages or from different roots of the same language; as, for example, 'mean' which means (*i*) medium (from *Lat. medius*), or (*ii*) base (from A. S. *gemoene*).
    - Those arising from the transfer of meaning by the association of ideas, *e.g.*, 'church' which may mean clergy.
    - Those arising from the transfer of meaning to analogous objects, *e.g.*, when we say 'this flower is sweet.'

A Singular term denotes a single thing only; it is of two kinds, viz.,

§ 4. Singular and General Terms. A *Singular* or individual term is one which can be applied in the same sense to only one specified individual thing, *e.g.*, John, the present Vice-Chancellor of the Calcutta University, circularity, this man, that book, the horse that I rode yesterday, the highest mountain in Asia. There are two descriptions of singular terms:—

Proper names,

(1) *Proper names*—which are *meaningless* arbitrary verbal marks assigned to individual objects Keynes defines a proper name as a "singular name given merely to distinguish an individual person or thing from others, its application after it has been once given being independent of any special attribute that the individual may possess" (*Studies and Exercises in Formal Logic*, p. 3). Thus, John, Socrates, Homer are examples of proper names. The same proper name may stand for different objects, but in different senses. The difficulty of ascertaining what the name stands for may be removed by reference to the context or the universe of discourse.

and Significant Individual names.

(2) *Significant Individual names*—which are general names limited, by other words, in their application to individual objects. These are not meaningless like proper names, but are *significant, i.e.*, they give us information about the attributes of the class to which the individual member named belongs, as well as about the attributes which distinguish this individual member from every other member of the class: *e.g.*, the present Vice-Chancellor of the Calcutta University, the author of the Iliad.

A General term, denotes a

A *General* term, on the other hand, is one which can be applied, in the same sense, to any one of an

indefinite number of things composing a class, *e.g.*, 'man', 'Vice-Chancellor of the Calcutta University', 'colour', 'virtue'. Thus the term 'Vice-Chancellor of the Calcutta University' is applicable in the same sense to each of the past, present and future Vice-Chancellors of the Calcutta University. Every general term is significant and implies that all objects of a class agree in certain common attribute or attributes, by vitrue of which the term is applicable to each of them in the same sense.

whole class of things.

Note 1. "A good test for deciding whether a term is general or singular is to see whether it will take 'all' or 'some' before it; for if it is a class name, statements may be made about *all* or *some* members of the class" (Coffey, Vol. I, p. 45). Sometimes one and the same term may be treated as both singular and general, according to the meaning attached to it. Thus the term 'God' is singular to a Monotheist, but general to a Polytheist. 'Space' is singular if space as a whole is meant, while general if a particular portion of space is meant. Names of materials (also called Substantial Terms)--earth, stone, gold, salt, mercury, water, flame—are singular, according to Bain; for, he says, they each denote the entire collection of one species of material. "But," as Keynes points out, "when we predicate anything of these terms, it is generally of *any portion* (or of some particular portion) of the material in question, and not of the entire collection of it *considered as one aggregate;* thus, if we say, 'Water is composed of oxygen and hydrogen,'—we mean any and every particle of water, and the name has all the distinctive characters of the general name. Again, we can distinguish *this* water from *that water,* and we can say 'Some water is not fit to drink;' but the word 'some' cannot be attached to a really singular term"—*Formal Logic,* p 12. (*Vide* Welton, *Manual of Logic, I,* p. 51.)

Sometimes one and the same term may be treated as both singular and general. Names of materials are singular, according to Bain; but general, according to Keynes

Note 2. It has sometimes been held that a proper name becomes general when different individuals bearing the *same* name form a separate class by virtue of this common feature alone, for example, when we say 'All Victorias are honoured in their names,' we treat the proper name 'Victoria' as a general term, as it implies all persons having the name 'Victoria' in common. But it seems proper to treat such a name as singular and not as general, for, as Mill rightly remarks, the name is not here affirmed of the different individuals in the same sense. There is, however, one case in which proper names may become general: "By

When proper names may become general.

antonomasia proper names may become general terms, as if we say '*A Johnson' would not have written such a book*, *i.e.*, any man of his genius for elaborate eloquence" (C. Read, *Logic*, p. 4).

A Collective term is applied to a group of similar things regarded as one whole, and not to the things taken one by one.

§ . Collective and Non-collective Terms: Collective and Distributive uses of Terms. A Collective term is one which can be applied to a group of similar things taken together and regarded as one whole. Thus the terms 'fleet', 'library', 'nation', 'regiment' are collective terms. Each of these terms is the name of *all* of a number of things taken together, and not of *each* of them taken separately. The term 'fleet', for example, implies a large collection of ships, and is applicable to all of them collectively, and not to any one of them separately. There is no distinctive opposition either between *collective* term and general term, or between *collective* term and any term in ordinary use. Hence, for the sake of an exhaustive division, every term must be either *collective* or *non-collective*.

The distinction between Collective and Distributive uses of terms.

There is, however, a real logical antithesis between *the collective and the distributive use* of a term in a proposition. A term is said to be used *collectively* when the assertion applies to the group as a whole; while it is said to be used *distributively* when the assertion applies to each member of the group individually. Thus in the proposition 'All the angles of a triangle are equal to two right angles', the word *all* is used collectively, for 'all the angles' means 'all the angles taken together'; while in the proposition 'All the angles of a triangle are less than two right angles', the word *all* is used distributively, for here 'all the angles' means 'any angle taken separately'. This distinction is of great importance, and the neglect of it may give rise to serious

mistakes in reasoning, for what is true of a term collectively may not be true of it distributively.*

A Collective term should be distinguished from a General term. While a general term is applicable to *each* of a number of things, a collective term is applicable only to *all* of a number of things *taken together*. For example, the general term 'man' means *each* individual man, while the collective term 'library' means a *collection of books, i.e.*, a number of books taken together, and not any one of them taken distributively or separately. Thus we can say 'John is a man', but we cannot say 'Chambers's dictionary is a library'. Again, a collective term may be either singular or general. Thus the term 'library' is general, being applicable in the same sense to any one of the various libraries (*i.e.*, collections of books) in the world. But the term 'the Imperial Library of Calcutta' is singular collective, being applicable to one particular collection of books alone. Similarly, the terms 'nation', 'fleet', 'regiment', 'senate', 'the first-year class', 'the army' are general collective; while the terms 'the English nation', 'the Baltic fleet', 'the 29th regiment of foot', 'the Senate of the Calcutta University in the year 1918', 'the first-year class of the Asutosh College of 1919', 'the French army' are singular collective.

A Collective term is to be distinguished from a General term.

A Collective term may be either singular or general.

Note. The division of terms into singular, general and collective, as made by some Logicians (Dr. Ray and others), is irrelevant and illogical. In the first place, the division of terms into singular and general is exhaustive, since every term must be either the name of only one thing or of more than one. In the next place, there can be no opposition between collective terms, on the one hand, and either singular or general terms, on the other; for, as we have seen above, collective terms may be either singular or general (*vide* Welton, *Manual of Logic, I*, p. 50).

Dr. Ray's division of terms into singular, general and collective appears to be illogical.

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* See Fallacies of Division and Composition discussed in Vol. II, Ch. XXXV. *Vide* Keynes, *Formal Logic*, p. 14.

The range or extent of application of a Definite term is precise, while that of an Indefinite term is uncertain

§ 6. Definite and Indefinite Terms. A *definite* term is one which precisely marks out an individual or a number of individuals. Thus 'this man', 'Jonn', 'the sun', 'the present leader of the House of Commons', 'all students', 'no students' are all definite terms, as the range of application in these cases is precisely determined. Similarly, all proper names, general terms in entire extent, and specialized individual (*i.e.*, singular) terms are all definite terms. An *Indefinite* term, on the other hand, is one which does not precisely mark out an individual or a number of individuals. In other words, the range or extent of application of such terms is uncertain. Thus general terms so limited in application as to denote unspecialized individuals and general terms in partial extent are indefinite. For example, 'a certain student', 'a man', 'some students', 'a leader of the House of Commons' are all indefinite terms.

A Concrete term is the name of an object, while an Abstract term is the name of an attribute or a group of attributes considered apart from the object in which it exists.

§ 7. Concrete and Abstract Terms. A *Concrete* term is one which indicates a substance (*i.e.* a person or object) or a class of substances—either mental or material. For example, 'man', 'the chair', 'animal', 'white house', 'red flower', 'sweet taste', 'warm feeling' are all concrete terms, because each of them indicates a definite substance having certain attributes. An *Abstract* term, on the other hand, is one which indicates an attribute or a collection of attributes apart from the substance in which it exists. For example, 'humanity', 'animality', 'wisdom', 'colour', 'virtue', circularity', redness' are abstract terms because each of them signifies an attribute or a collection of attributes apart from the substance in which it exists. The attributes implied by the abstract terms have no separate determinate existence of themselves, but are *thought of* as separate

from, or independent of, the substances in which they inhere—they are *abstracted* (or separated in thought) from the things of which they are the attributes.

Are all adjectives concrete (as Mill, Jevons, Welton and others think) or abstract (as Martineau, Fowler and others think)?

Note 1.—Welton agrees with Mill and Jevons in holding that adjectives are not abstract terms, but that *all adjectives are concrete and general.* For example, when we say 'Snow is *white*,' we mean that snow possesses the colour 'white,' and 'white' means 'white things,' and thus it is concrete and general. Others (*e.g.*, Martineau, Fowler) hold that *all adjectives are general and abstract*, for, according to them, adjectives are not names of things, but *attributives, i.e.*, words expressing only attributes apart from any objects possessing them. Thus 'white' would mean the attribute of 'whiteness,' and not 'white things.' But the above seem to represent two extreme views on the subject The right view seems to be that adjectives are sometimes *concrete and general* (*i.e.*, when they denote things and connote attributes, as 'white' meaning white things), and sometimes *abstract and general* (*i.e.*, when they signify attributes and imply attributes of those attributes, *e.g.*, '*great* strength,' '*high* magnanimity').

Are abstract terms singular or general?

Note 2. There is a diversity of opinion among Logicians with regard to the question *whether abstract terms are singular or general.* Jevons contends that they are singular, because the quality which each signifies is always one and the same. Others maintain that they are general, because the attribute signified by an abstract term may be possessed by each of a number of individual things. Some, again, maintain that the distinction of singular and general is not applicable to abstract terms. As Keynes says, "A still more satisfactory solution, however, is to consider these names as neither general nor individual, and to place them in a class apart" (*Formal Logic*, p. 12). But the generally accepted view is that of Mill, who holds that some abstract terms are evidently general, while others are singular. Mill says: "Some of them are certainly general. I mean those which are names not of one single and definite attribute, but of a class of attributes. Such is the word *colour*, which is a name common to whiteness, redness, etc. Such is even the word whiteness, in respect of the different shades of whiteness to which it is applied in common: ....But when only one attribute, neither variable in degree nor in kind, is designated by the name; as visibleness; tangibleness; equality; squareness; milkwhiteness; then the name can hardly be considered general; for though it denotes an attribute of many different objects, the attribute itself is always conceived as one, not many" (*System of Logic*, Bk. I, Ch. II, § 4). Mill's view is perhaps decisive on this point. Some, however, object to Mill's view by

saying that only variation in kind, and not variation in degree, can be a ground of generality, for the latter would lead to endless difficulties. Thus they point out that there may be various shades even in the degree of milk-whiteness, for it may be asked: 'Is buffalo's milk as white as cow's?' But to this it may be replied by saying that practical difficulty cannot modify a Logical doctrine. If milkwhiteness be of different degrees in different cases, we must consider it also as general.

A Positive term implies the presence, while a Negative term implies the absence of an attribute or a substance, a Privative term implies the present absence of an attribute in a subject capable of possessing it

§ 8. Positive, Negative and Privative Terms.

A *Positive* term implies the *presence* of an attribute or a substance; *e.g.*, 'man', 'happiness', 'kind', 'tree'. A *Negative* term implies the *absence* of an attribute or a substance; *e.g.*, 'not-man', 'not-happy', 'not-kind', 'lifeless', 'unequal'. A *Privative* term implies the *present absence* of an attribute in a subject capable of possessing it; *e.g.*, 'deaf', 'blind', 'lame'. Thus a privative term has both positive and negative elements in it; for such a term expresses that a thing has been deprived (negative) of a quality which it once possessed, or was capable of possessing, or normally does possess (positive). When we call a man 'lame', we mean the present absence in the man of the power of walking, but the man in normal condition was capable of walking, and either he had the capacity to walk, or if the defect be removed he might have. This is the reason why we do not apply a privative term to an object which has not at all the capacity for the particular attribute. Thus we never call a table blind, or a tree deaf, or a stone dead. Again, whether a particular term is negative or privative depends upon the possibility of future existence of the particular quality in question. When positive proof is not forthcoming, we are to presume such possibility. For example, the term 'barren' is to be regarded as privative, for the defect may be remedied in future. Further, while adjectives such as blind, barren and idle are to be treated as

How to determine the privative character of a term

privative, the corresponding abstract terms such as blindness, barrenness and idleness are to be treated as negative. Thus 'blind' may refer to a person's capacity for sight, but 'blindness' can never imply the presence of such a capacity.

How to determine the positive and negative character of a term.

Whether a term is positive or negative is not to be judged by reference to its form only, but by reference to its meaning also. Thus there are terms which though they seem to be negative from their form, are positive in meaning and therefore positive. For example, the terms 'unhappy', 'immoral', 'unkind', though negative in form, are to be considered as positive, as they imply positive elements of misery, vice and cruelty respectively. Again, there are terms which seem to be positive from their form, but are negative in meaning and therefore negative. For example, the terms 'idleness' and 'doubt', though positive in form, are to be considered as negative, as they respectively imply absence of work and absence of belief. Hence, for purposes of Formal Logic, it would be safe to use the prefixes 'not' and 'non' for negative forms.

Note. It may be noticed that one and the same term is sometimes regarded as positive and sometimes as negative Thus the term 'inconvenience' is placed by Mill under the head of positive terms, for it implies the presence of pain, while the same term is spoken of by Jevons as negative This may be accounted for by the fact that some terms may be regarded as positive, negative or privative according to the standpoints from which we consider them Thus 'vice' is positive if considered as implying the *presence* of a morally bad quality, it is negative if considered as implying the *absence* of virtue, and it is privative if considered as implying the *present absence* of virtue, which might have been or might be present in the same subject under different circumstances. Such terms may, therefore, be brought under different heads according to the different standpoints.

Negative terms having the force of

But undoubtedly there are terms like 'not-man', 'not-good,' 'not-black,' which cannot possibly have any positive significance, and which may, therefore, be said to have a perfectly negative character. Such terms, *i.e.*, negative

contradictories are called Infinite terms.

terms having the force of contradictories, are called by Aristotle *Infinite* terms, since their range of application is practically infinite or unlimited.

An absolute term can be understood by itself without reference to any other term, while a Relative term cannot be understood without reference to some other term.

§ 9. Absolute and Relative Terms. An *Absolute* term is one which can be understood without reference to any other term. For example, the terms 'man', 'water', 'flower' can be understood by themselves without understanding the meaning of any other term. A *Relative* term, on the other hand, is one which cannot be understood without reference to some other term. For example, the terms 'father' (implying child), 'teacher' (implying pupil), 'master' (implying servant) cannot be understood by themselves without understanding the terms they imply. In fact, the very meaning of father is that he has a child, of teacher that he has pupils, of master that he has servants. Each of the two terms constituting pairs like 'father-child', 'teacher-pupil', 'master-servant' is called a relative term, but the terms constituting each pair are known as *correlatives*. It appears from what has been said above that negative and privative terms may be regarded as relative, for they always have reference to their corresponding positive terms. 'Not-white' is not intelligible without reference to 'white'; such, however, is not always the case with positive terms such as 'man', which can be understood without reference to 'not-man'.

A Connotative term denotes a subject and implies an attribute; while a Non-connotative term signifies a subject only or an

§ 10. Connotative and Non-connotative Terms. "A *Connotative* term is one which denotes a subject, and implies an attribute." "A *Non-connotative* term is one which signifies a subject only, or an attribute only" (Mill, *Logic*, I, 2, § 5).

According to the definition given above, a *Connotative* term has two meanings—a meaning in denotation and a meaning in connotation ; it directly

signifies subjects and indirectly signifies attributes possessed in common by the subjects. In other words, a connotative term has both denotation and connotation. Thus the term 'man' is connotative, inasmuch as it denotes John, James, Peter and an indefinite number of individuals coming under the class 'man', and connotes the attributes of animality and rationality possessed in common by all men. A *Non-connotative** term, on the other hand, has one meaning only—either a meaning in denotation or a meaning in connotation. In other words, a non-connotative term has either denotation only or connotation only, but not both. Thus the term 'squareness' is non-connotative, inasmuch as it signifies merely an attribute, and no subject ; it has only connotation and no denotation. Similarly, a proper name is non-connotative, because it signifies merely a subject, and no attribute ; it has only denotation and no connotation (see below).

attribute only.

Let us note below the kinds of terms that are connotative and those that are not :

I. Connotative Terms.

Connotative Terms. (1) All general terms ;

(1) All general terms—whether concrete, abstract or collective—are connotative, for they always denote subjects and connote attributes. We have already taken an example of a concrete general term, viz.,

* Following Fowler, Adamson and Mellone, we accept this as the most logical way of expressing what Mill means by a non-connotative term, without altering the meanings which are given to "connotation" and "denotation" in connection with class names. We cannot, without inconsistency, agree with those who call a singular abstract term "denotative", for although it merely marks off a particular attribute from other attributes, yet properly speaking, here the reference to concrete cases vanishes (*vide* Fowler, *Deduction*, pp. 19-20; Mellone, *Text-Book of Logic*, pp. 34-35).

'man'. The term 'virtue' is a general abstract term, it is connotative because it denotes the several virtues of courage, wisdom, generosity, benevolence, etc., and connotes the attribute of moral goodness common to these several virtues. Similarly, general collective terms like 'army', 'library' are connotative. The term 'army', for example, implies the attributes of being composed of armed men, trained and maintained for military purposes, and further denotes each collection of men having these attributes. All adjectives are general and therefore connotative.

(2) significant proper names, designations and singular collective terms.

(2) Some singular terms—significant individual (proper) names, designations or descriptive singular terms and singular collective terms—are connotative. For example, 'the Sun', 'the Earth', 'the present Governor of Bengal', 'the author of Paradise Lost', 'the Calcutta University', 'the British Parliament' are connotative, for such terms denote particular subjects and connote attributes on account of which they are so named.

Non-connotative Terms.

II. Non-connotative Terms.

(1) Singular abstract terms;

(1) Singular abstract terms such as 'squareness', 'circularity', 'snow-whiteness', 'the colour of the orange before me' are non-connotative, for they only connote definite individual attributes and do not denote anything.

(2) proper names.

(2) Proper names such as John, James, Peter, etc., are non-connotative, for they have denotation only and no connotation.

Are proper names Connotative?

There is, however, a great controversy among Logicians with regard to the question *whether proper names are connotative or non-connotative*. Significant proper names and proper names used to designate a certain type of person (*e.g.*, a Diogenes,

a Shakespeare, a Socrates) are, indeed, connotative. This is because when they are so used they are no longer proper names, but assume the nature of general names. The question here is about insignificant proper names such as John, James, Peter, etc.

According to *Mill*, such insignificant proper names are non-connotative. "They (proper names) are not connotative ; they denote the individuals who are called by them ; but they do not indicate or imply any attribute as belonging to those individuals. When we name a child by the name Paul, or a dog by the name Cæsar, these names are simply marks used to enable those individuals to be made subjects of discourse. The only names which connote nothing are proper names ; and they have, strictly speaking, no signification. A proper name is but an unmeaning mark which we connect in our minds with the idea of the object, in order that whenever the mark meets our eyes or occurs to our thoughts, we may think of that individual object" (*Logic*, I, pp. 36-37).

According to Mill they are not.

The above view of Mill is opposed by *Jevons*, who says: "Surely no one who uses the name England, and knows what it denotes, can be ignorant of the peculiar qualities and circumstances of the country, and these form the connotation of the term" (*Elementary Lessons*, pp. 42-43). Jevons' meaning is that a proper name not only denotes the individual, but it also implies the peculiar features, form and character of the individual, which constitute the connotation of the term. Moreover, the same writer (in his *Principles of Science*, p. 72) points out that as the denotation decreases the connotation increases ; so in the case of an individual, where the denotation is the least, the connotation, instead of ceasing altogether, gets intensified to the highest limit.

According to Jevons, they are

According to Dr. Ray, proper names are at first non-connotative, but gradually become connotative as we know more and more the individuals denoted by them.

Dr. Ray attempts to reconcile these extreme views by holding that proper names are at first non-connotative, but through our increased knowledge of the individuals, they become connotative. He says: "A proper name would appear to be at first without any connotation or signification of attributes, but it seems to acquire this signification as our knowledge of the individual becomes more and more definite, as its name becomes associated in our mind with its attributes, and as the attributes become a means of distinguishing that individual from others belonging to the same class or species" (*Deductive Logic*, pp. 40-41).

Jevons' view criticized

Criticism. It must be admitted that there is some truth in Jevons' view. Proper names may be said to be connotative, if by connotation we mean the sum total of the attributes, known and unknown, of the individuals denoted by the names. Again, proper names may be said to be connotative, if by connotation of a name we mean the attributes suggested by it. But this is certainly not what we mean by connotation in Logic. We must carefully distinguish between what a proper name *suggests* and what it *implies*. Jevons seems to mistake the attribute suggested by a proper name for its connotation. A proper name, once given to a thing to mark it off from other things, becomes associated with the peculiar attributes and circumstances, and afterwards suggests those attributes. But the proper name cannot be said to connote or imply these attributes, inasmuch as the name is not given to the individual by virtue of possessing these attributes. Of course, it may be that proper names are originally given to individuals by reason of certain attributes possessed by them. Thus, 'Wednesday' originally meant the

day dedicated to the worship of Woden. The first 'John Smith' was probably so called because he took to the work of a smith. But the *history* of a name is not what we mean by its connotation. There might have been some reason for which a particular name was originally given to an individual, but the application of the name does not in any way depend on the continuance of the original meaning. As Mill says, "the name once given is independent of the reason". Hence it follows that Mill is right in holding that proper names are non-connotative.

Dr. Ray's view criticized.

Dr. Ray's view can scarcely be said to be an improvement upon Mill's, for Mill might have anticipated such an answer and pointed out its absurdities by saying that the attributes signified by proper names may vary in different minds according to the degree of acquaintance with the individuals represented by those names. To one the connotation of John might consist of cruelty, dishonesty, faithlessness, etc., but to another the connotation of the same man might consist of kindness, honesty, faithfulness, etc. Even to one and the same person the connotation might be different on different occasions. But such connotation has no place in Logic, for Logic, as we have noted before (Ch. VII, § 3, *Note*), *deals with the scientific or conventional connotation* (*i.e.*, the connotation determined by scientific research or accepted by general agreement), which is always fixed and general, and not with the variable subjective connotation. The utmost that we can concede to Dr. Ray's view is that it is psychologically correct, that it gives a correct account of how ideas develop in our mind ; but the standpoint of Logic is entirely different, it deals with ideas, not in their way of development, but as products of

Logic takes note of the scientific connotation of a term, but not of the subjective connotation.

Mill's view upheld

regularly conducted thought. Hence Mill's view as to the nature of proper names seems to be decisive.*

§ 11. Directions for working out Exercises.

How to describe the Logical characters of a term

(1) First determine whether what is given to be described is a term or not, *i.e.*, whether it is categorematic or syncategorematic. If syncategorematic like 'in spite of', 'by the by', etc., it cannot have any logical character. Do not classify terms as categorematic and syncategorematic, the distinction of categorematic and syncategorematic applies to words, and not to terms, for a categorematic term means a *term-term*—an absurd repetition.

(2) If the given expression is categorematic, *i.e.*, a term, determine whether it is univocal or equivocal; if it is equivocal, it is to be regarded as equivalent to two or more terms corresponding to its different meanings, and there will be different descriptions under different heads. When doubt arises whether a term belongs to one head or the other, the sense determining its Logical characters should be mentioned.

(3) Next, describe the Logical characters of a term by stating:

(*a*) Whether it is simple or composite;
(*b*) Whether it is singular or general;
(*c*) Whether it is collective or not;
(*d*) Whether it is concrete or abstract;
(*e*) Whether it is definite or indefinite;
(*f*) Whether it is positive, negative or privative;
(*g*) Whether it is absolute or relative;
(*h*) Whether it is connotative or non-connotative.

Examples

Son—Simple (univocal), general, non-collective, indefinite, concrete, positive, relative, connotative.

White—Simple (univocal), general, non-collective, definite, concrete, positive, absolute (relative, if it is regarded as implying black), connotative.

Blind—Simple (univocal), general, non-collective, definite, concrete, privative, relative, connotative.

The moon—Composite (univocal), singular, non-collective, definite, concrete, positive, absolute, connotative.

The present King of England—Composite (univocal), singular non-collective, definite, concrete, positive, absolute, connotative.

John—Simple (univocal), singular, non-collective, definite, concrete, positive, absolute, non-connotative—its logical

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* In this connection one may read with much advantage Minto's *Logic*, p. 57.

characters are the same as those of "the present King of England," except that it is non-connotative.

Virtue—Simple (univocal), general, non-collective, definite, abstract, positive, relative, connotative.

Justice—Simple (univocal), singular, non-collective, definite, abstract, positive, absolute, non-connotative.

The present Senate of the Calcutta University—Composite (univocal), singular, collective, definite, concrete, positive, absolute, connotative.

Light—It is equivocal—as a noun, it is opposed to 'darkness,' as an adjective, it is opposed to 'heavy'. Hence we are to describe its logical characters separately, thus:

Light (as noun)—Simple, general, non-collective, indefinite, concrete, positive, relative, connotative.

Light (as adjective)—Same as those of 'Light' as noun.

§ 12. **Exercises.**

1. What are the three parts of Logic? What do you mean by subject, predicate, copula, and proposition?

2. Distinguish Thing, Idea and Word Also distinguish Words and Terms. What reasons are there for discussing Terms in Logic?

3 Exhibit the relation between Terms and Names and sum up the characteristics of a Term.

4. What do you understand by the Denotation, and what by the Connotation, of a term, and what relation do they bear to each other? Illustrate your answer by examples.

5. Arrange the following terms in order of their Denotation, and again in order of their Connotation: quadruped, animal, substance, elephant, organic thing, Indian elephant, vertebrate, mammalian.

6. "The Denotation and the Connotation vary inversely". Explain and criticize the statement.

7. Give examples of terms whose Denotation may increase without any change in the Connotation, and also of terms whose Connotation may increase without any change in the Denotation.

8. What determines the Denotation and the Connotation of a term? Has every term a Denotation and a Connotation?

9. Describe the change in the Denotation and Connotation of the following terms as you pass from one end to the other of each series:

(1) Element, metal, gold.
(2) Book, printed book, dictionary, Latin dictionary.
(3) Solid, stone, precious stone, ruby.
(4) Honesty, virtue, moral goodness.
(5) Creature, animal, man, Englishman.

10. Define Genus, Species, Summum genus, Infima species, and Differentia. Arrange the following terms, (1) in

the order of their extension, (2) in the order of their comprehension: Rapacious animal—Lion—Vertebrate animal—Black lion—Animal—Quadruped—Sensible object—Substance—Native of Africa. What do you mean by Co-ordinate, Sub-ordinate, and Super-ordinate Terms? Has every term a Genus and a Species?

11. Illustrate the relation between Genus and Species by the Tree of Porphyry.

12. Give the Genus, Species, and Differentia of the following terms:- triangle, gold, house, virtue, flower, mind, student, book, element.

13. Give a sub-ordinate, a super-ordinate, and a co-ordinate of the following terms:-substance, animal, solid, virtue

14 What do you mean by Opposition in Terms? Explain and illustrate different forms of Opposition in Terms. Distinguish between Contrary and Contradictory terms.

15 What do you understand by Distribution of Terms? Give the meaning of *Suppositio* or Universe of Discourse

16. Give the Contradictory and a Contrary of the following terms. bad, white, cold, poor, clear, mortal, solid, mind, virtue, beautiful.

17. Explain and illustrate different kinds of Terms.

18. Explain the division of Terms into—

(a) Singular and general,
(b) Collective and distributive uses of terms;
(c) Concrete and abstract;
(d) Positive, negative and privative,
(e) Absolute and relative;
(f) Connotative and non-connotative

19. Discuss the following:-

(a) Are Proper names connotative?
(b) Are Abstract terms divisible into connotative and non-connotative? Are they singular or general?
(c) Are adjectives concrete or abstract?

20. Describe the Logical characters of the following terms:-

Lame, unholy, non-Christian, the present Governor of Bengal, James, emperor, library, the Imperial Library, book, brilliance, weight, sensation, gold, Her Majesty, the government, wealth, virtue, triangle, forest, army, the army, manhood, idleness, idle, dead, the colour of this flower, the present Emperor of China. Calcutta, Viceroy, individual, Asutosh College students, the candidates for the examination, circle, figure, honesty, human, humanity, the humanities, Cæsar, Cæsarism, the Cæsars, Napoleon, a Napoleon, conception, genus, sovereign, the many, the unthinkable, George II, a George II, the Georges, nation, a nation, national, nationality, nationalities.

CHAPTER IX

DEFINITION OF TERMS: ITS LIMITS AND FORMAL CONDITIONS

§ 1. Nature of Logical Definition. A *Logical Definition* is the explicit statement of the entire connotation of a term, *i.e.*, the essential attributes possessed in common by the things denoted by the term. Thus, we define 'man' by saying 'man is a rational animal'; here we explicitly state the full connotation of the term, *viz.*, animality and rationality, which are the essential attributes possessed in common by all men. It must not be thought, however, that in defining a term we are to state all the attributes common to the class of things denoted by it. We are to state only those fundamental and essential attributes which are the bases of many other common attributes and which serve to distinguish the particular class of things from other similar classes.

A Logical Definition is the explicit statement of the connotation of a term.

§ 2. The Method of Definition: Definition 'per genus et differentiam.' Definition, as stated above, is the statement of the essential attributes implied by a term. But it is rather difficult to enumerate all the essential attributes and to distinguish between those that are essential and those that are inessential. This difficulty has been got over by the Scholastic Method of defining *'per genus et differentiam'* *i.e.*, the method of defining by giving the *proximate genus* or the next higher class of the species denoted by the term to be defined, and the *differentia* or the attribute or attributes

Definition *'per genus et differentiam.'*

distinguishing it from the genus and other species under the same genus. Thus in the definition 'Man is a rational animal', we first state the proximate genus 'animal', and then add the differentia of man, *viz.*, 'rationality', which distinguishes man from the genus 'animal' as well as from other species—birds, reptiles, beasts, etc.,– under the genus 'animal'. It is to be remembered that in stating the genus we must select a proximate genus, otherwise our definition will omit part of the connotation of the term we are defining. Thus if we define 'man' as 'rational being', our definition will include in its scope other possible beings than man, for it omits the attribute of having a body. And it is easy to see that definition *per genus et differentiam* clearly indicates what the thing denoted by the term is like, and what it is not like. Man, for instance, is like animals in having animality, but is unlike them in having rationality.

Definition is an important preliminary requisite to correct reasoning, inasmuch as it renders our ideas clear and precise

§ 3. **Use or Function of Definition.** Definition renders our ideas clear, distinct and definite; hence it is a preliminary requisite to correct thought. To think and infer correctly, we must avoid using vague and indefinite ideas of things. We are often apt to confound one thing with other similar things; for instance, many wrongly think that bats are birds and whales are fish; definition serves to correct such wrong ideas of things by fixing the meanings of terms. Again, definition ensures correctness of inference and saves us from many verbal disputes by guarding us against ambiguous use of terms.

There can be no Logical

§ 4. **Limits of Definition.** As has been said above, a Logical Definition requires us to state the proximate genus and the differentia of the species denoted by the term to be defined. But in certain

cases, this condition cannot be fulfilled. Thus the *summum genus* or the highest class cannot be defined, for there is no higher class to which it can be referred. Again, *an individual thing* cannot be defined, for its differentia or distinctive characteristics cannot be summed up in a Logical definition, although we can describe it by pointing out a sufficient number of its attributes to distinguish it from other individuals of the same kind. Hence it has been said that 'the highest genus is above, and the individual thing below, the sphere of Logical definition'. It is also for the same reason that terms denoting objects which are *sui generis, i.e.,* of their own class, such as 'space', 'time', 'life', etc., cannot be defined, for they cannot be referred to any higher class; and names of *simple* and *elementary attributes*, such as consciousness, feeling, sweetness, pleasure, colour, smell, etc., as well as *singular abstract* terms cannot be defined, for they cannot be analysed into more elementary attributes and thus cannot be brought under simpler and more general classes.

Definition of (a) summum genus, (b) individual objects, (c) objects which form classes by themselves, (d) elementary attributes, (e) singular abstract terms, and (f) proper names

Again, as definition unfolds the connotation of a term, it follows that all connotative terms (*i.e.*, all general terms and significant individual names) are definable, while all non-connotative terms (*i.e.*, *singular abstract terms* and *proper names*) are incapable of definition.

Definition and Description.

A true Logical definition must carefully be distinguished from what has been called a mere **Description,** which is only a statement of some of the 'inseparable accidents', with or without a few of the 'properties'. Thus the propositions 'Man is a featherless biped', 'Diamond is the most precious substance in nature' are mere Descriptions, which

enable us to identify the objects denoted by the term, but do not give us any knowledge about their connotation or essential attributes. On the usefulness of Description Welton says: "Still, though in no sense definition, description is by no means useless; it serves the very useful function of enabling us to easily identify anything which bears the name. We must not, however, speak of describing a word; we *define the name*, and *describe the thing* which bears the name. The main object of the former is to make distinct our concepts of things, and so to lead to a greater clearness and definiteness of thought and language; that of the latter is to furnish a rough and ready means of making others recognize the objects of which we are speaking" (*Manual of Logic, I*, p. 122). It thus appears that *the limits of definition are not the limits of description*. Where definition is not possible, description is our only resource. A description is sometimes called a Physical Definition, 'which enumerates such parts of the objects as are *actually* separable e.g., the boiler, mechanism, etc., of a steam engine' (Emmens' *Treatise on Logic*).

Use of Description

Rules of Definition stated:

§ 5. Formal Conditions or Rules of Definition. A Logical Definition should conform to the following rules or conditions:—

A definition should be (1) adequate and complete, (2) equal in extent, and neither (3) tautologous, nor

1. *It should state exactly the connotation of the term defined, i.e., it should contain neither more nor less than the connotation of the term defined.*

2. *It should exactly coincide in extent with the denotation of the term defined, i.e., it should neither include things not denoted by the term nor exclude things denoted by the term.*

3. *It should not be tautologous*, i.e., *it should not contain the term defined nor any of its synonyms.*

4. *It should be clearer than the term defined, i.e., it should not be expressed in unfamiliar, figurative or ambiguous language.* (4) obscure, nor

5. *It should not be negative when it can be affirmative.* (5) negative

Let us now discuss these rules in detail and point out the *faults that arise from their violation.* *Fallacies arising from the violation of the rules :*

Rule I. This rule includes the Scholastic method of defining *per genus et differentiam, i.e.,* by a statement of the genus and the differentia of the term, for the genus and differentia constitute the connotation of the term. *The violation of this rule gives rise to the following faulty definitions :—* The violation of Rule I gives rise to

(a) **Accidental Definiton.** If the definition states some accidental feature not included in the connotation of the term defined, it is called an Accidental Definition, as when 'man' is defined as 'a featherless biped', or as 'a cooking animal', or when 'crow' is defined as 'a black bird'. (a) Accidental Definition,

(b) **Description.** If the definition states the peculiar features or superficial qualities of a thing without giving its connotation, or merely enumerates the several parts of things, it is called a mere Description, as when 'plant' is defined as 'an organism having a stem, roots, leaves, etc.' It thus appears that all accidental definitions are descriptions. (b) Description,

(c) **Incomplete Definition.** If the definition states only a part of the connotation of the term defined, and not the whole of it, it is called an Incomplete or Partial Definition, as when we define 'water' as 'a liquid substance', or 'triangle' as 'a plane figure', or 'man' as 'an animal'. When the connotation of a term is decreased, its denotation increases, and for this reason an Incomplete Defini- (c) Incomplete Definition,

tion is always *too wide*. An *Essential* Definition is one which states the whole, or at least a part, of the connotation of the term defined. It thus appears that an incomplete definition is an essential definition though not a true Logical definition, which must state the whole of the connotation of the term defined.

(d) Redundant Definition.

(*d*) **Redundant Definition.** If the definition states more than the connotation of the term defined, it is called a Redundant, Superfluous or Over-complete Definition, as when we define 'man' as 'a rational animal having the power of locomotion,' or 'triangle' as 'a rectilineal figure bounded by three straight lines and having three angles.'

Thus it is clear that a Logical definition should not mention either an accident or a proprium, but only the exact connotation of the term defined. (Read in this connection Welton, *Manual of Logic*, I, § 52.)

The violation of Rule 2 gives rise to

Rule 2. This rule guards us both against including in our definition things not denoted by the term, and against excluding things denoted by the term defined. *The violation of this rule gives rise to (a) too wide or (b) too narrow a definition.*

(a) Too wide Definition, and

(*a*) **A too wide Definition** includes things that are not contained in the denotation of the term defined. For example, the definition of 'metal' as 'a solid substance,' and the definition of an 'equilateral triangle' as a 'plane rectilineal figure having three sides' are too wide definitions.

(b) Too narrow Definition.

(*b*) **A too narrow Definition** excludess things that are contained in the denotation of the term defined. For example, the definition of 'man' as 'a civilized rational animal', and the definition of 'Logic' as 'the Art of Reasoning' are too narrow definitions.

Rule 3. The definition of a term by itself or by any of its synonyms is useless, for such a definition does not convey any real information about the term which we attempt to explain by the definition. *The violation of this rule gives rise to* **Circle in Definition,** which brings us round again to the very term to be defined, without, in fact, explaining it at all. Thus the definitions 'Man is a human being', 'A plant is a vegetable organism', 'Life is vitality' are examples of circle in definition.

The violation of Rule 3 gives rise to Circle in Definition.

Rule 4. If the definition of a term is expressed in obscure, figurative and ambiguous language, the end of definition, *viz.*, a clear understanding of the term defined, is evidently frustrated; such a definition will attempt to explain the unknown by the more or equally unknown. *The violation of this rule gives rise to* **Obscure and Figurative Definition,** which very often leads to much misunderstanding and confusion. Thus the definitions—'A net is a reticulated fabric decussated at regular intervals', 'Mind is a *tabula rasa'*, 'Bread is the staff of life', 'A periphrasis is a circumlocutory cycle of oratorical sonority circumscribing an atom of ideality lost in verbal profundity' —are examples of obscure and figurative definition.

The violation of Rule 4 gives rise to Obscure and Figurative Definition.

Rule 5. *The violation of this rule gives rise to* **Negative Definition.** This rule really follows from Rule 1, as the connotation of a positive term cannot be expressed by means of negations. Moreover, the possibilities of negation are infinite, hence negative definitions are indefinite and vague, and therefore, meaningless and useless. Thus the definitions—'Virtue is not vice', 'Mind is not matter', 'A man is not an ass'—are examples of negative definition. Negative definitions are, however, better and simpler

The violation of Rule 5 gives rise to Negative Definition.

than affirmative ones, when the terms defined are themselves negative or privative. Thus the following negative definitions—'An alien is one who is not a citizen of the British Empire', 'Darkness is the absence of light' are quite correct and preferable.

To sum up: A definition should be (1) adequate and complete, (2) in extent equal, and neither (3) tautologous, nor (4) obscure, nor (5) negative.

§ 6. Kinds of Definition.

A Nominal Definition explains the meaning of a name, while a Real Definition explains the nature of a thing. *Do we define name, concept, or things?*

(1) *Nominal and Real definitions.* A *Nominal* or Verbal definition merely explains the meaning of a term without any reference to the existence of the corresponding object, while a *Real* definition states the real or essential nature of some object.

In this connection we may consider a question which has been the subject of much dispute among Logicians in all ages. The question is: *What is it that we define—name, concept, or thing?* We have seen above that only universals or general terms admit of true Logical definition. We have also seen (Ch. V, § 3) that different Schools of Logic hold different views of universals. Hence their views regarding *definition* (like other logical processes—*vide* Ch. V, § 4) must also be different. Thus according to Nominalism, definition is the unfolding of the connotation of a *name*. According to Conceptualism, definition is the exposition of the meaning of a *concept* or notion. According to Realism, definition is the statement of the real nature of a *thing*. But we have already found (Ch. II, § 9) that none of the above extreme views gives the whole truth by itself. The true view must combine them all. We have also pointed out (Ch. II, § 8) the close relation between language, thought and things, so that whichever of the three (name, concept and things) is regarded as the object of our definition, the other two follow as necessary supplements. Thus, if it be said that we define a *name* only, we may at once ask the question: Do we define a *name* for its own sake merely? Certainly not. When we define a *name*, we always attempt to understand the nature of the thing to which it refers, or to render clear our *idea* or the meaning for which the name stands. The name is merely the verbal symbol, either of the concept or of the thing. But this much can be said in favour of Conceptualism (as we have already seen in Ch. II, § 9, that Logical processes are primarily concerned with concepts and secondarily with language and things) that the main aim of definition is to make our conceptions clear, for we cannot make *things* clear, for things themselves remain just the same after our

The best answer seems to be that in defining any one of these we define all the three, inasmuch as all the

definition as they were before we defined them. Thus Ueberweg says, "Every definition defines not the name, nor the thing, but the *notion* and with it the name and the thing as far as this is possible" (*Logic, Eng. Trans.*, p. 167). The tendency of the modern writers has been towards holding that no real difference exists between these extreme views with regard to definition, and consequently towards discarding the distinction between nominal and real definitions. Thus Boyce Gibson says, "All definition of meaning is at the same time verbal definition, and *vice versa*. The distinction between Nominalism and Conceptualism in definition is a distinction without difference." And Welton also remarks, "It would seem better, then, to finally discard this distinction from Logic; for it simply tends to confuse the whole object of logical definition by importing into it considerations with which the process of framing a definition is not rightly concerned."

three are inseparable.

(2) *Substantial and Genetic definitions.* Definitions have also been divided into Substantial and Genetic, according to the method employed in framing them. Thus, a definition framed by the Scholastic Method, *i.e.*, by giving Genus and Differentia, or, in other words, by stating the connotation or the essential attributes of the term defined, is called a *Substantial* or *Essential* definition. The nature of such definitions has been fully explained in § 2. And the definition framed by indicating the process by which the attributes of the term defined may be conceived as coming into existence is called a *Genetic* definition. In other words, a Genetic definition tells us how we can frame an idea of the thing corresponding to the term defined by showing the mode of its genesis or production. For example, the definition of a circle as 'a plane figure generated by revolving a straight line about one of its extremities which remains fixed' is a Genetic definition.

A Substantial Definition gives the genus and differentia, while a Genetic Definition describes how a thing comes into existence.

Some have limited the use of Genetic definition to the realm of Mathematics, but Creighton rightly points out that "it is often true, especially in the natural sciences, that a thing may be better defined by telling how it comes into being than by giving it a place in a fixed scheme of classification," which is done by giving genus and specific differentia. He adds, "Its use is frequent where we are concerned with processes and the laws of their action, and it often represents an advance in knowledge upon classificatory definition. To define 'heat,' for example, as 'a force in nature recognized in the phenomena of fusion and evaporation, etc.,' tells us less about its real nature than the statement that it is a 'form of energy possessed by bodies *derived from* an irregular motion of their molecules" (*Introductory Logic*, pp. 74-75).

(3) *Analytically-formed and Synthetically-formed definitions.* Welton thus explains the distinction between the two: "The former is the giving clearness and exactness

Analytically-formed

and Synthetically-formed Definitions. Provisional Definitions are incomplete definitions of terms, whose connotation is not completely known; while Final Definitions are complete definitions of terms, whose connotation is completely known

to the commonly received meaning of a word, which is the ordinary work of definition; the latter is the giving a new and arbitrary meaning to an old term, or the equally arbitrary fixing of the connotation of a newly invented term, to serve the purposes of some special discussion."

(4) *Provisional and Final definitions. Provisional* definitions are definitions of terms whose connotation is not rigidly fixed. Definition of a term depends upon our knowledge of the connotation of the term, and the incomplete definitions of terms, the whole connotation of which we do not know, are called provisional, *i.e.*, complete for the time being. Such terms are 'animal life', 'vegetable life', 'heat', 'electricity', 'galvanism', 'oxygen', 'hydrogen', etc. Provisional definitions are always liable to change, for with the progress of our knowledge, fresh marks or attributes of things denoted by the terms may be discovered resulting in the modification of the definitions. *Final* definitions are definitions of terms whose connotation, consisting of a limited number of attributes, is completely known and unalterably fixed for ever, and is not liable to change Such are the definitions of 'circle', 'straight line', 'square', 'school', 'dispensary', 'church', etc

§ 7. Directions for working out Exercises.

In testing any definition we have to see if it violates any of the rules of definition. Thus we have to remember the following rules :

(1) Define by giving the genus and the specific differentia.

(2) Always select a proximate genus.

(3) See that the definition is neither an Accidental definition nor a Description, nor an Incomplete definition, nor a Redundant definition.

(4) See that the definition is neither too narrow nor too wide, nor a Circle in definition.

(5) See that the definition is neither Obscure and Figurative nor a Negative one.

(6) Do not state a property (or it will be a Distinctive Explanation), or an accident (or it will be a Description).

(7) Remember the limits of definition; do not try to define what cannot be defined. For example, singular abstract terms and elementary notions can be described, but cannot be defined.

Examples :

Test the following definitions :

(1) Rice is an article which is used as food in India.

(2) An acute-angled triangle is that which has an acute angle.

(3 Man is a learned animal.

(4) A right-angled triangle is that which has a right angle and two acute angles.

(5) Oxygen is a gas.

(6) Force is that which produces motion.

(7) An equilateral triangle is a triangle with three equal angles.

(8) Pain is the absence of pleasure.

(9) Copper is an orange-coloured metal, more sonorous than any other, and the most elastic of any, except iron.

(10) Life is the sum of vital functions.

(11) Eccentricity is a peculiar idiosyncracy.

(12) A triangle is a plane figure enclosed by three equal straight lines.

All the above definitions are faulty for the reasons given below :—

(1) This is only an Accidental definition or Description, 'used as food in India' is only an accident and no connotation.

(2) This definition is too wide, as it includes right-angled and obtuse-angled triangles, which have also an acute angle.

(3) This definition is too narrow, for all men are not learned.

(4) This is a Redundant definition, for 'having two acute angles' is not an essential attribute of a right-angled triangle, inasmuch as it may be deduced from the connotation 'having a right angle'. It thus states more than the connotation.

(5) This definition is too wide, and incomplete.

(6) 'Force' being one of the most elementary notions cannot be defined. Again, 'force' does not actually produce motion in every case, thus the differentia stated in the definition is not adequate.

(7) 'Having three equal angles' follows from 'having three equal sides'; hence the definition, which gives the 'property' and no 'connotation', is only a Distinctive explanation or a Description.

(8) It is a Negative definition Moreover, negation of pleasure is no explanation of the nature of pain

(9) It is not a Logical definition, inasmuch as in the definition a genus has been combined, not with differentia, but with property and accident. It is only a Description (sufficient, however, to distinguish the term from all other terms).

(10) It is an attempt to define 'Life', which, being something unique, cannot be Logically defined. Moreover, the definition contains a synonym (*viz.*, *vital*) of the term to be defined. Hence it involves the fault of 'Circle in definition.'

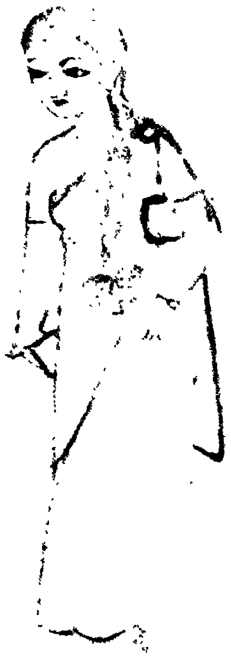
(11) It is an Obscure definition.

(12) This definition is too narrow, for it applies only to one particular species of triangles, *viz.*, equilateral triangles.

§ 8. Exercises.

1. What is Definition? Describe the nature of Logical Definition.
2. Explain the Method and Function of Logical Definition.
3. State and illustrate the Limits of Definition.
4. What are the Formal conditions or rules of Definition?
5. State and exemplify the various kinds of faulty Definitions.
6. Distinguish between :—
   (a) Definition and Description.
   (b) Provisional and Final Definitions.
   (c) Nominal and Real Definitions.
   (d) Substantial and Genetic Definitions.
   (e) Analytical and Synthetical Definitions.
7. Fully explain the bearing of Nominalism, Conceptualism, and Realism on the process of Definition.
8. Define the following terms : –
   Student, matter, mind, plant, book, soldier, library, university, virtue, work, logic, animal, triangle, house, metal, water.
9. Test the following definitions :—
   (1) Man is a featherless animal.
   (2) Man is a cooking animal.
   (3) Man is an animal that makes clothes for himself.
   (4) Man is a laughing animal.
   (5) Rice is an article which is used as food in India.
   (6) Humour is thinking in jest while feeling in earnest.
   (7) A plant is a being possessing vegetable life.
   (8) Knowledge is power.
   (9) Life is the continuous adjustment of inner to outer relations.
   (10) Logic is the science of thought.
   (11) Logic is the science of reasoning.
   (12) A triangle is a figure which is bounded by three straight lines, and which has all its angles together equal to two right angles.
   (13) Iron is the cheapest metal.
   (14) Man is a sentient being.
   (15) A metal is a solid substance.
   (16) A metal is a heavy element.
   (17) Mind is non-extended.
   (18) Man is a civilized animal.
   (19) Evil is that which is not good.
   (20) The intuitive reason is the eye of the soul.
   (21) An equilateral triangle is a plane rectilineal figure having three sides.
   (22) A violin is a musical instrument with four strings.
   (23) Fluency is the exuberance of verbosity.
   (24) Life is vitality.
   (25) Pain is the discipline of character.
   (26) Truth is veracity in speech and act.

(27) Health is the absence of sickness.
(28) Necessity is the mother of invention.
(29) Light is the opposite of darkness.
(30) A just man is one who practises justice.
(31) Porosity is the property which bodies possess of having pores.
(32) A glacier is a river of ice.
(33) Diamond is a kind of carbon.
(34) Mind is a thinking substance.
(35) Oxygen is a supporter of combustion.
(36) The lion is the king of beasts.
(37) A judge is one who exercises judicial powers.
(38) A point is that which has no parts and which has no magnitude.
(39) A camel is the ship of the desert.
(40) The sun is a star that shines by day.
(41) Words are the signs of thought.



CHAPTER X

Logical Division and its Conditions

Logical Division is the breaking up of a genus into its constituent species

§ 1. **Nature of Logical Division.** The term *Division*, as technically used in Logic, may be defined as the analysis of the denotation of a term. Logical Division of a term, however, is to be understood not as an *enumeration* of the individuals making up the class denoted by the term, but as the breaking up of the class (genus) into its constituent sub-classes (species) according to the presence or absence, or varying degree of an attribute. In short, Division is the Logical process of analysing a genus into its constituent species. Thus we divide the genus 'animal' into the species 'man' and 'brute', according to the presence or absence of *rationality;* or, we divide the genus 'book' into the species 'folio', 'quarto', 'octavo', etc., according to the variation in *size*. The genus or the large class (animal) which is divided is called the *totum divisum* (divided whole), the species or the smaller classes ('man' and 'brute') into which it is divided are called the *membra dividentia* (dividing members). The attribute (rationality) upon whose presence or absence, or modification the division is based is called *fundamentum divisionis* (the principle or ground of division). The same genus can be divided into various sub-classes according to different principles of division. Thus we may take 'equality of length of sides' as *fundamentum divisionis*, and divide triangles into three sub-classes —equilateral, isosceles and scalene; or, we may take the 'size of the largest angle' as *fundamentum divisionis*, and divide triangles into three other sub-classes —obtuse-angled, right-angled and acute-angled.

Totum Divisum.

Membra Dividentia.

Fundamentum Divisionis.

This process of dividing the same genus in different ways according to different grounds of division is called *Co-division*. The sub-classes resulting from an act of division may again be logically divided, according to new principles of division, into other narrower sub-classes, and so on, until we reach *infimæ species* or lowest sub-classes which do not admit of further Logical Division, but only of distribution into individuals. This continued application of the process of Division is called *Sub-division*. Thus, having divided 'animals' into 'rational' and 'irrational' on the basis of 'rationality', we may take a new *fundamentum* and divide 'rational animal' or 'man' into 'white' and 'not-white' on the basis of 'complexion'. Next, we may choose another *fundamentum* and divide 'white man' into 'learned' and 'not-learned' on the principle of 'learning', and so on.

Logical Division is to be carefully distinguished from Enumeration,

§ **2. Logical Division distinguished from Some Other Kindred Forms.** From what has been said above it is clear that the process of Logical Division cannot be carried further than *infimæ species*, for we cannot subdivide the lowest sub-class into narrower classes, but can only enumerate individuals under it. Hence Logical Division is to be distinguished at the outset from **Enumeration.** Logical Division is further to be distinguished from three other processes, *viz.*:—

Physical Partition,

(1) **Physical Division** (or **Partition**). This is the breaking up of an individual into its component parts, *e.g.*, the division of a tree into its root, trunk, branches, leaves, etc., or of a man into head, hands, legs, feet, eyes, etc.

Metaphysical Analysis, and

(2) **Metaphysical Division** (or **Analysis**). This is the enumeration (or mental analysis) of the attributes of a thing, e.g., the division or analysis of

orange into its different attributes of colour, taste, smell, shape, etc., of man into his attributes of animality and rationality, or of mind into its different functions of thinking, feeling and willing. A Physical Division is *actually*, while a Metaphysical Division is only *mentally*, possible.

Verbal Division.

(c) **Verbal Division** (or **Distinction**). This consists in distinguishing between the various meanings of an ambiguous or equivocal term, as, for instance, we have to distinguish between a watch in the sense of a guard and a watch in the sense of a time-piece.

Definition and Division imply and help each other.

§ 3. **Mutual Relation of Definition and Division.** Definition unfolds the connotation of a term, while Division analyses its denotation. As we have already seen that the connotation and the denotation of a term are closely related, it follows that definition and division must also be closely related. In fact, definition and division are interdependent processes, each implying the other. Logical division involves a reference to the genus and the differentia of the dividing members, and consequently implies their definition. Thus a division of triangles, on the principle of the relation of sides, into equilateral, isosceles and scalene, implies our knowledge of the differentia or the distinguishing attributes, *i.e.*, the definitions, of the dividing members—equilateral, isosceles and scalene. Definition, similarly, implies a division of the genus which forms a part of its predicate. Thus the definition 'Man is a rational animal' implies a division of animals into 'rational' and 'irrational.'

Division helps us in framing definition and in knowt

§ 4. **Uses and Limits of Division.** Logical Division, as indicated in the previous section, helps definition. It also helps us to understand the full significance of a general term by analysing its contents.

As definition makes our ideas distinct by unfolding their connotation, so division makes our ideas clear by marking the boundaries of their denotation. By marking off the sub-classes from one another, division helps our memory, renders our knowledge of the sub-classes clear and distinct, enables us to avoid confusing one sub-class with another, and is thus a great help to clear and consistent reasoning. Moreover, it is always necessary for a scientific and methodical treatment of a subject.

the precise application of a term by marking the boundaries of its denotation.

Logical division, however, has its limits. We have already seen that the lowest sub-classes as well as individuals cannot be logically divided. A simple, unique and unanalysable phenomenon (as, for example, the sensation of milkwhiteness), on the one hand, and a composite thing (as, for example, a library), on the other,—both of which are treated like individuals—do not admit of Logical division.

Infimæ species, unanalysable phenomena, and composite things do not admit of Logical Division.

§ 6. Conditions or Rules of Logical Division.

Rules of Logical Division stated:

A Logical Division should conform to the following rules or conditions:—

1. *The term to be divided must be general.*

(1) The term to be divided must be *general*.

2. *There should be a single principle of division, i.e., each act of division must be founded on one basis, called the* fundamentum divisionis.

(2) Each act of division must have only one *basis*.

3. *The division must be exhaustive,* i.e., *the sub-classes, when taken together, must be equal in extent to the class or genus divided.*

(3) The division must be *exhaustive*.

4. *The term divided must be applicable to each of the sub-classes into which the whole is divided.*

(4) The whole divided must be *predicable* of each of the sub-classes.

(5) The sub-classes must be mutually exclusive.

5. *The sub-classes must be mutually exclusive, i.e., they must not overlap, but must completely exclude each other.*

(6) The Division must be step by step.

6. *In a continued division, each step must divide a class or a sub-class into its proximate or nearest sub-classes.*

Let us now discuss the rules in detail and point out the *faults that arise from their violation.*

Fallacies arising from the violation of the rules:

The violation of Rule 1 gives rise to Physical Partition or Metaphysical Analysis

Rule 1. This rule follows from the very definition of Logical division; hence it has often been regarded as superfluous. According to this rule, no individual can be logically divided. A singular or collective term is, therefore, incapable of Logical division. *The violation of this rule gives rise to the fallacy of* **Physical Partition** or **Metaphysical Analysis.** Thus the division of a man into hands, feet, etc., and the division of a regiment into soldiers are only examples of Physical Partition. Similarly, if we enumerate the constituent attributes of an individual object, this will be only a Metaphysical Analysis.

Rule 2. This rule is not to be understood as condemning the processes of *sub-division* and *co-division.* The first act of division may be based on one principle, and the next sub-division may be based on another principle, and so on indefinitely. Again, the same class may be divided according to different principles for different purposes. What this rule enjoins is that one and the same act of division shall be based on one principle only. *The violation of this rule gives rise to the fallacy of* **Cross Division.** If, for instance, we divide men into 'Hindu', 'Christian' and 'illiterate', we commit the fallacy of Cross Division, for some Hindus and some Christians are illiterate, so that the sub-classes

The violation of Rule 2 gives rise to Cross Division.

'Hindu' and 'Christian' cross the sub-class 'illiterate'. Again, a man may be both a Hindu and also illiterate, so that we cannot place him under any of these sub-classes, nor can we place him under both. The fallacy here is due to our taking two principles in one and the same act of division, *viz.*, religion and education. Similarly, the division of books into 'folio', 'quarto', 'scientific', and 'Sanskrit' illustrates the fallacy, for here three principles of division, *viz.*, size, subject and language, have been adopted.

Rule 3. Division is the splitting up of a genus into its constituent species. Hence it is obvious that the sum of the denotations of the species must be equal to the denotation of the genus, that all the species included in the genus must be mentioned and none of the species should be omitted. *The violation of this rule gives rise to the fallacy of* **Incomplete Division,** *or the fallacy of* **too narrow or too wide Division.** If we omit some of the species or sub-classes of the genus, the division is too narrow. For instance, the division of triangles into equilateral and isosceles is a too narrow division, for here the sub-class 'scalene' has been omitted. If we include some that are not species of the genus, the division is too wide. For instance, the division of coins into gold, silver, copper, bronze, and banknotes is a too wide division, for the sub-class 'banknotes' is not included in the genus 'coin'.

The violataion of Rule 3 gives rise to a too wide or a too narrow Division.

Rule 4. If, for example, we divide triangles into equilateral, isosceles and scalene, the term 'triangle' can be predicated of each of the sub-classes; thus equilateral triangles, isosceles triangles and scalene triangles are triangles. *The violation of this rule gives rise to the fallacy of* **Physical** or **Metaphysical Division.** When, for example, we divide

The violation of Rule 4 gives rise to Physical Partition

or Metaphysical Analysis.

'man' into hands, feet, head, liver, etc. (physical partition), or into animality and rationality (metaphysical analysis), the term 'man' cannot be predicated of any of the dividing members, viz., hands, feet, head, liver, animality or rationality.

The violation of Rule 5 gives rise to Overlapping Division

Rule 5. This rule implies that when a genus is divided into species, no individual must have a place under more than one of the species. *The violation of this rule gives rise to the fallacy of* **Overlapping Division.** Thus, we commit this fallacy if we divide books into quarto, folio and scientific, inasmuch as any particular book may, at the same time, come under two of the sub-classes - quarto and scientific, or folio and scientific. Here the sub-classes are not mutually exclusive, but they overlap each other. Welton has rightly pointed out that this rule follows from Rule 2, which is the principal rule of division, "for if there is only one basis of division it is impossible for any individual to fall into more than one sub-class." Cross division, arising from the violation of Rule 2, is the same thing as Overlapping division.

The violation of Rule 6 gives rise to Division by a leap.

Rule 6. A continued division, *i.e.*, a division involving more than one step, must be progressive, *i.e.*, must proceed one step at a time. In other words, a genus should be divided into its proximate or next species. Hence the old Logical rule—*Divisio non faciat saltum, i.e.*, division must not make a leap. *The violation of this rule gives rise to the fallacy of* **Division by a leap,** and usually makes the division *too narrow* by leaving out some intermediate species. The division of animals into invertebrates, Englishmen, fish, crow, horse, and dog is an example of the violation of this rule. Such divisions are confusing and worthless, and serve no useful

purpose. Again, an immediate division of animals into 'honest' and 'not-honest' makes a leap by leaving out the intermediate species 'man', and is thus meaningless and useless.

Logical Division is never purely formal, but always partly material—hence the difficulty of a purely formal division.

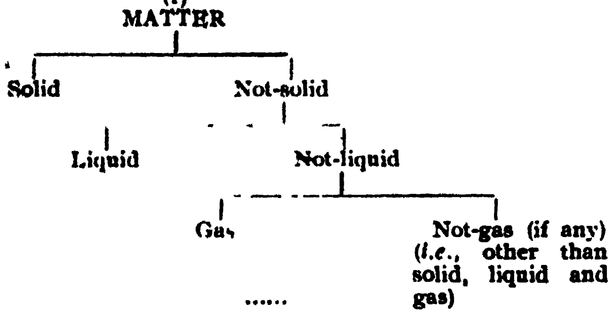
§ 6. **Difficulties of Logical Divisions: Division by Dichotomy.** From a consideration of the rules of Logical or Formal Division given in the last section, it is evident that we cannot be sure whether an act of division has been carried on in conformity with the given rules, unless we have material knowledge of the things falling under the class divided. For this reason it has been held that the *process of Logical Division is never purely formal, but always partly material.* Logicians have, however, invented a kind of Logical Division, called **Division by Dichotomy** or simply **Dichotomy,** which, it has been said, conforms to all the rules of Logical Division, and may, at the same time, be carried on in a purely formal way, without any appeal to matters of fact. Dichotomy (from Gr. *dicha,* 'in two', and *temno,* 'to cut') literally means cutting or dividing in two, and division by dichotomy consists in the simple process of dividing a class into those members which have, and those which have not, a given attribute. And this is evidently done by means of a pair of contradictory terms, as when we divide 'animals' into 'man' and 'not-man', or 'colour' into 'red' and 'not-red'. And, as already explained, two contradictory terms, like 'man' and 'not-man', are mutually exclusive (according to the Principle of Contradiction), and taken together, they exhaust the whole universe of discourse (according to the Principle of Excluded Middle). It thus follows that *dichotomy* is based on the Principles of Contradiction and Excluded

Division by Dichotomy, however, may be carried on in a purely formal way. It is the division by means of a pair of contradictory terms.

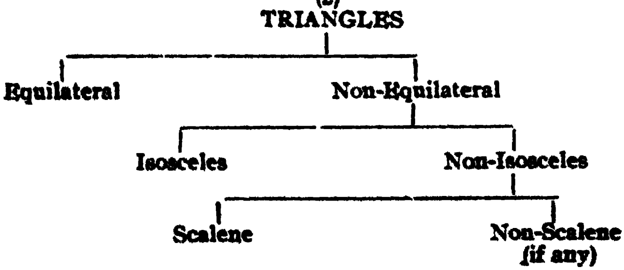
Middle, and it fulfils the two most important conditions of Logical division, *viz.*, exclusiveness of the sub-classes (Rule 5) and exhaustiveness of the division (Rule 3), and thus enables us to avoid the errors of cross division or overlapping division and incomplete division. Hence it has been held that division by dichotomy is the only form of Logical division that can have a place in Formal or Deductive Logic. Dichotomy has also been called Bifid (from Lat. *bi*, 'twice', and *findere*, 'to cleave') or Exhaustive Division. A good example of division by dichotomy is furnished by the Tree of Porphyry (*vide* Ch. VII, § 4). We give below three more illustrations :—

Illustrations of Dichotomy.

(1)



(2)



(3)

ALL THINGS

- Phenomenal
  - Extended
    - Resistant
      - Gravitating
        - Simple
        - Not-simple (compound)
      - Non-gravitating
    - Non-resistant (space)
  - Unextended (*e.g.*, pleasure and pain)
- Non-Phenomenal

§ 7. Directions for working out Exercises.

A. In testing any division we have to remember the following points :—

(1) It must not be an enumeration, or a physical partition, or a metaphysical analysis, or a verbal distinction.

(2) The term divided must be a general term, and not a singular term—whether concrete, abstract or collective.

(3) Each act of division should be based on one *fundamentum divisionis.*

(4) The sub-classes must be mutually exclusive and, taken together, must exhaust the whole divided.

(5) There shall be no leap from a high genus to a low species.

B. In dividing a term we may proceed by dichotomy, if we have no material knowledge of the term to be divided.

Examples :

Q. 1. Test the following divisions :—

(*a*) A Horse into head, neck, body, tail, etc.

(*b*) Water into liquidity, colourlessness, power of quenching thirst, etc.

(*c*) Man into John, James, Socrates, etc.

(*d*) Charm into 'sweetness of manner' and incantation.

(*e*) Indian into Hindu, Mahomedan, Bengalee and educated.

(*f*) Government into Anarchy, Monarchy and Republic.

(g) Sciences into physical, moral and medical
(h) Geometrical figures into Equilateral triangles and Not-equilateral triangles

Ans

(a) It is a Physical partition violates Rule 1
(b) It is a Metaphysical analysis—violates Rule 4
(c) It is an Enumeration and no *Logical* division
(d) It is a Verbal distinction
(e) It is a Cross division—violates Rule 2
(f) It is a too wide division violates Rule 3
(g) It is a too narrow division violates Rule 3
(h) It is a too narrow division or a Division by a leap violates Rule 6

Q 2 Divide 'Men,' 'Things,' 'Quadrilateral' and 'Colour'

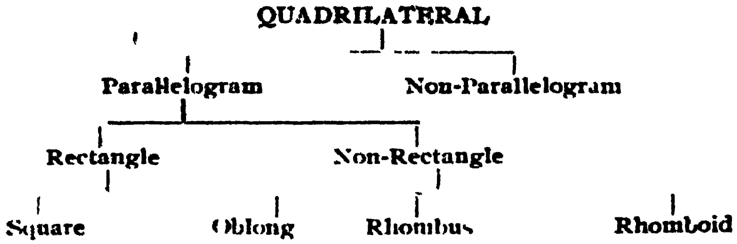
Ans

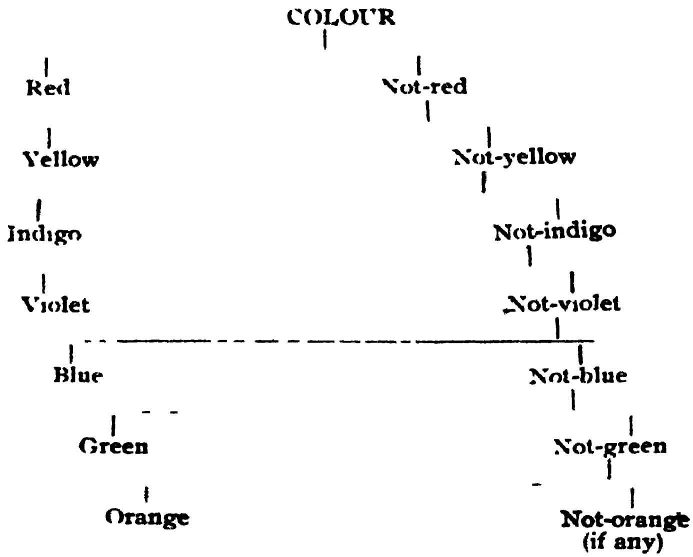
MEN

- Asiatics
- Non-Asiatics
  - Europeans
  - Non-Europeans
    - Africans
    - Non-Africans
      - Americans
      - Non-Americans
        - Australians
        - Non-Australians (and so on)

THINGS

- Material
  - *Organized* (Plants and animals)
    - Sentient (Animals)
      - Rational (Man)
      - Irrational (Lower animals)
    - Insentient (Plants)
      - Flowering
      - Flowerless
  - Not-organized (Inorganic things)
- Immaterial





§ 8. Exercises.

1. Explain the nature of Logical Division. Illustrate your meaning by an example.

2. Distinguish Logical Division, Physical Partition, Metaphysical Analysis, Enumeration and Verbal Distinction, giving examples.

3. Explain the relation between Definition and Division, and point out the uses and limits of the latter.

4. State and explain the rules of Logical Division, and point out the faults that arise from their violation.

5. Explain and exemplify:—Enumeration, Cross division, too wide and too narrow Divisions, Division by a leap, Overlapping division.

6. In writing an essay you apply both Definition and Division: Explain for what purposes you apply them.

7. Is Logical Division possible in the true sense of the term, if so, in what form and under what conditions?

8. Explain and exemplify Division by Dichotomy.

9. State the advantages and disadvantages of Dichotomy.

10. Test the following divisions:—

(1) The inhabitants of Calcutta into men, women, educated, Bengalees, and members of a university.

(2) Men into those that lend and those that borrow.

(3) Nations into Progressive and Indo-Aryan.

(4) World into Europe, Asia, Africa, America and Australia.

(5) Metals into white, heavy, and precious.

(6) Books into cheap, historical, poetical, and French.

(7) Man into civilized, uncivilized, clergyman, and layman.

(8) Quadrilateral figures into square, rectangle, parallelogram, and rhombus.

(9) Governments into monarchies, tyrannies, oligarchies, and democracies.

(10) Great Britain into England, Scotland, and Wales.

(11) Men into white, black, educated, intelligent, and dull.

(12) English poetry into lyrical, descriptive, didactic, and the writings of the Lake School.

(13) Man into mind and body.

(14) Nation into progressive and stationary.

(15) Living beings into mortal and immortal.

(16) Man into civilized and black.

(17) Books into Poetry, Geography, History, Philosophy and Drama.

(18) Students into idle, athletic and diligent.

(19) Animals into rational, biped, quadruped, and flying.

(20) A book into pages, letters, and covers.

(21) Material bodies into solid, liquid, heavy and green.

(22) Logic into terms, propositions and arguments.

(23) Grammar into Orthography, Etymology, Syntax and Prosody.

(24) Gold into yellow colour, a certain specific gravity, a certain form, size, solidity, etc.

(25) Religion into Christian, Mahomedan, Hindu, and Parsi.

(26) Chalk into whiteness, extension, solidity and weight.
(27) A person into bones, flesh, stomach and head.
(28) Quinine into bitterness, whiteness and fineness.

11. Divide the following terms :—
(1) World, (2) man, (3) animal, (4) science, (5) virtue, (6) government, (7) logic, (8) book, (9) student, (10) paper, (11) phenomenon, (12) rectilineal figure, (13) virtue, (14) chair, (15) substance.

Part II

PROPOSITIONS

CHAPTER XI

JUDGMENT AND PROPOSITION

Judgment is the most elementary act of thought

§ 1. **Nature of Judgment.** We have already noticed the importance of Judgment and its bearing on the problem of Logic (Ch. VII, § 1). It has also been observed (Ch. II, § 1) that although from the Logical point of view concepts precede judgments, for a judgment is the combination of two concepts, yet from the psychological point of view judgment is the simplest process, for even a concept is arrived at by many acts of judgment. It thus appears that *the unit* (or the most elementary act) *of thought is the judgment*, and not the concept; for the judgment, by itself, conveys a clear and complete sense, but the concept, outside the judgment, is only an abstraction. This is the reason why some of the modern writers* have maintained that Logic should begin with a discussion of judgment. Thus it has been said that Logic is only a *doctrine of judgment*, for concepts are arrived at by way of abstraction from judgments and supply us with materials for other judgments, and inference is the process of arriving at new judgments by means of given judgments.

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* Mill and Carveth Read define Logic as the Science of the Proof of Propositions. Jones defines Logic as the *Science of the Import and Relations of Propositions.*

But all scientific study has recourse to abstraction, and thereby separates in thought what cannot actually be separated. Hence, although concepts have no meaning outside a judgment, yet we have arranged for a separate treatment of the three parts of Logic, *viz.*, Concepts (Term), Judgment (Proposition) and Reasoning (Argument), in order to attend to them conveniently and to understand their nature clearly. And, as we have begun with the study of concepts (terms) as constituent elements of judgments, it will not be improper now to define a judgment, by reference to concepts, as *the mental operation of recognizing a relation between two concepts*. In other words, it is an act of thought in which we compare two concepts with each other for the purpose of finding a relation of agreement or disagreement between them. The product of this mental act is also called a judgment, just as the product of conception is called a concept. By a judgment, therefore, we affirm or deny one concept of another. For instance, we have in our minds the concepts of 'man', 'mortality' and 'perfection'; we may establish a judgment of agreement or affirmation between 'man' and 'mortality', and may also establish a judgment of disagreement or denial between 'man' and 'perfection', and may thus arrive at the results—'Man is mortal' and 'Man is not perfect'.

For Logical purpose, however, we may consider concepts as preceding judgments, and may accordingly define a judgment as the mental operation of recognizing a relation between two concepts.

§ **2. Nature of Proposition.** A Proposition is an expression of a Judgment in language. In other words, it is the affirmation or denial (by means of language) of a certain relation (of agreement or disagreement) between two terms (which are verbal expressions of concepts). A judgment being a purely mental act admits of Logical treatment only

A Proposition is a judgment expressed in language.

when it is expressed as a proposition. This is the reason why in Logic we often meet with indiscriminate use of the two expressions—judgment and proposition, for in treating of the one we necessarily treat of the other. From the above definition of a proposition it follows that a Logical proposition consists of three parts, *viz.*, (1) the subject, (2) the predicate, and (3) the copula or the sign of a relation of agreement or disagreement between the subject and the predicate. The term of which something is predicated, *i.e.*, affirmed or denied, is called the **subject.** The term which is predicated, *i.e.*, affirmed or denied, of the subject is called the **predicate.** The sign which expresses the relation of affirmation or denial is called the **copula.** Thus in the proposition 'Men are mortal', 'men' is the subject, 'mortal' is the predicate, for here 'mortality' is predicated, *i.e.*, affirmed, of 'men', and 'are' is the copula, for it expresses the relation of agreement between 'men' and 'mortal'. Similarly, in the proposition 'John is not good', 'John' is the subject, 'good' is the predicate, and 'is not' is the copula.

Analysis of a Proposition. Three parts of a proposition—the subject, the predicate, and the copula.

Distinction between a Logical proposition and a Grammatical sentence

A Logical proposition must, at the outset, be distinguished from a Grammatical sentence. Every logical proposition is a grammatical sentence, for 'what the logician calls a proposition, the grammarian calls a sentence' (Jevons), but every sentence is not a proposition. A sentence in the indicative mood is the typical form of a logical proposition. Sentences which express a wish or a command or an interrogation or an exclamation cannot be regarded as logical propositions, unless and until they are reduced to the form of a sentence in the indicative mood. "Again, even indicative sentences frequently require to be rewritten in order

to reduce them to the form of a logical proposition, which demands two terms and a copula. The sentence, 'the sun shines', must, therefore, for purposes of logical treatment, be reduced to, 'the sun is a body which shines' " (Creighton). Another point of distinction consists in this that, unlike a logical proposition which has three parts, *viz.*, the subject, the predicate and the copula, a grammatical sentence has only two parts, *viz.*, the subject and the predicate. Thus in a grammatical sentence the copula forms part of the predicate. We shall see later on (Ch. XIII, § 2) that a grammatical sentence must be reduced to the proper form of a logical proposition before it can enter into any logical argument at all.*

The Logical copula always consists of some form of the verb 'to be' with or without the negative particle 'not,' according as the predicate denies or affirms something of the subject.

§ 3. Nature of Logical Copula. We have already seen that the copula is the sign of affirmation or denial of a relation between the subject and the predicate. The Logical copula consists of some form of the verb 'to be' with or without the negative particle 'not', according as the predicate denies or affirms something of the subject. In other words, the forms 'is' or 'is not', 'are' or 'are not' are usually employed as the logical copula. And it has already been pointed out that a sentence or a proposition in any other form must first be reduced to its proper logical form by means of the logical copula, before we can attempt to treat it logically. Thus the logical forms of the propositions 'John deserves praise', 'Misers envy others' wealth', 'The sun shines' are respectively 'John is a person deserving praise', 'Misers are persons envious of

* For directions for discriminating between the subject and the predicate of a proposition, students may read Coffey, *Science of Logic*, I, pp. 155-7.

A proposition in any other form must first be reduced to its proper Logical form before it can be treated logically

others' wealth', 'The sun is a body that shines'. The use of the verb 'to be' as indicating the existence or non-existence of the subject or the predicate should be distinguished from the use of the same verb as the logical copula, which only serves to express a relation of agreement or disagreement. Thus in the propositions 'God is' and 'Mermaid is not', 'is' is not the copula, but the copula and the predicate combined, for it means 'exists', and so the logical forms of the propositions are 'God is existent' and 'Mermaid is not existent.'

There has been a good deal of discussion among Logicians on the following questions regarding the nature of the copula.* Thus it has been asked: (1) Is the copula always affirmative? (2) Of what tense should the copula be? (3) Does modality belong to the copula or to the predicate? Let us discuss these questions one by one.

The Logical copula may be either affirmative or negative

(1) According to some Logicians (*e.g.*, Hobbes), the Logical copula should always be affirmative. In other words, the logical copula should always consist of the verb 'to be' without the negative particle 'not'. In case of negation or denial of a relation, the negative particle 'not' should, according to them, be attached to the predicate and not to the copula. They hold, for example, that the proper logical form of 'Some men are not good' is 'Some men are not-good', so that 'are' is the copula and 'not-good' the predicate. This view makes all propositions affirmative, for the copula always affirms and never denies. This view, however, cannot be

* In connection with the nature of the copula, students may consult Mill, *System of Logic*, I, Ch. IV, § 1; Mansel, *Prolegomena Logica*; Hamilton, *Discussions*; Fowler, *Deductive Logic*; Joseph, *Introduction to Logic*.

accepted, because when we affirm a negative term, we really affirm the absence, not the presence, of something, not that something *is*, but that it *is not*. The negative force being still there, it is clear that we think by negation as well as by affirmation. Hence, a negation or denial will be more naturally and directly expressed by including the negative sign in the copula, which is the sign of a relation between the subject and the predicate. Again, negative terms being most indefinite in meaning and extent, the use of such terms in propositions will unnecessarily make their meaning vague and indefinite. Hence we conclude that *the copula may be either affirmative or negative.*

The Logical copula should always be in the present tense.

(2) According to Hamilton, Fowler and others, the logical copula should always be in the present tense of the verb 'to be' with or without the negative particle 'not', as the case may be. They hold that the copula is merely the sign of relation, and so it must be free from all reference to time (except the time of one's judgment)—present, past or future. But Mill holds that the copula is the sign of predication, and should, therefore, include in it whatever affects the applicability of the predicate to the subject; and so, according to Mill, the copula may be of any tense of the verb 'to be'. In order to decide between these two opposite views, we have to recall what has already been said in connection with the Principle of Identity (*vide* Ch. VI, § 2), *viz.*, that the element of time or change has no place in Deductive Logic. In the external world things are constantly changing, but Logic takes no notice of such changes. Strictly speaking, then, the copula has no time at all, but expresses a timeless relation. The time belongs properly to the predicate, and

hence *the copula should always be in the present tense*. Thus, the proper logical form of 'James was a king' is 'James is a person who was a king'. (Read Welton, *Manual of Logic*, I, p. 158.)

The words expressing modality belong to the copula rather than to the predicate.

(3) Modality consists in the degree of certainty or probability with which the predicate is affirmed or denied of the subject. Thus in the propositions (*a*) 'Three and three *must be* six', (*b*) 'All men *are* mortal', and (*c*) 'John *may be* good', the modality is expressed by (*a*) 'must be' (implying that the proposition is certain under all circumstances), (*b*) 'are' (implying that the proposition is certain as far as experience goes), and (*c*) 'may be' (implying that the proposition is only probable). Now, the question is whether modality belongs to the copula or to the predicate. Here also there is difference of opinion among Logicians. Thus, some Logicians (*e.g.*, Fowler) maintain that the words expressive of the modality of a proposition (*e.g.*, must, certainly, may, possibly, probably, etc.) should be taken along with the predicate. According to this view, the logical forms of 'Three and three must be six', John may be good' are 'Three and three are numbers which must be six', 'That John is good is probable.' Others, on the contrary, maintain that the words expressing modality should be attached to the copula. According to this view, there are as many different kinds of copula as there are different degrees of certainty with which a statement may be made. The latter view seems to be more reasonable, for modality consists in the degree of assurance with which the predication is made, and the copula, being the mark of predication, should include in itself the signs of modality. Hence we conclude

that *the words expressing modality belong to the copula rather than to the predicate.*

Summary.

To sum up, the Logical copula always consists of some form of the verb 'to be', is either affirmative or negative, is always in the present tense and includes in it the words expressing modality.

CHAPTER XII

KINDS OF PROPOSITIONS

Table of Classification of propositions.

§ 1. **Classification or Division of Propositions.** Propositions may be divided into various classes acording to different principles, as shown in the following table:—

Propositions.

1. According to Composition	Simple, *e.g.*, S is P; S is not P. Compound, *e.g.*, S as well as Q is P; Neither S nor Q is P.	
2. According to Relation	Categorical, *e.g.*, S is P; S is not P.	
	Conditional	Hypothetical, *e.g.*, If S is P, Q is R. Disjunctive, *e.g.*, S is either P or Q.
3. According to Quality	Affirmative, *e.g.*, S is P. Negative, *e.g.*, S is not P.	
4. According to Quantity	Universal, *e.g.*, All S is P; No S is P. Particular, *e.g.*, Some S is P; Some S is not P.	
5. According to Modality	Necessary, *e.g.*, S must be P. Assertory, *e.g.*, S is P. Problematic, *e.g.*, S may be P.	
6. According to Significance	Verbal, *e.g.*, All men are rational. Real, *e.g.*, All men are mortal.	

Let us now proceed to explain these various classes or divisions in order.

A Simple proposition expresses a single judgment, while a Compound proposition consists of two or more inde-

§ 2. **Division according to Composition.** According to the principle of *Composition*, propositions may be divided into Simple and Compound. A *Simple* proposition is one which expresses a single judgment only; in other words, a simple proposition contains one subject and one predicate, *e.g.*, 'All men are mortal', 'Some men are not wise'. A *compound* proposition, on the other hand, is one

which consists of two or more independent propositions combined together, *e.g.*, 'John as well as James is good,' which is a combination of two propositions, *viz.*, 'John is good' and 'James is good'. Compound propositions, again, are of two kinds, *viz.*, (*a*) Copulative and (*b*) Remotive. A *Copulative* proposition is one which consists of two or more affirmative propositions, *e.g.*, 'Gold and silver are precious metals', which is equivalent to two affirmative propositions, *viz.*, 'Gold is a precious metal' and 'Silver is a precious metal'. A *Remotive* proposition is one which consists of two or more negative propositions, *e.g.*, 'Neither John nor Peter is good', which is equivalent to two negative propositions, *viz.*, 'John is not good' and 'Peter is not good'.

pendent propositions combined together.

A Compound proposition has different forms *viz.*, (*a*) Copulative, (*b*) Remotive and (*c*) Exponible.

Besides copulative and remotive, sometimes a third kind, *viz.*, Exponible, is included in compound propositions. *Exponible* propositions are those 'whose composition is not obvious from their form, and which, therefore, require explanation to show what this hidden composition really is'. Such propositions are, therefore, compound in meaning, and not in form, for an adequate explanation of its meaning requires us to resolve it into two or more propositions. Exponible propositions are usually sub-divided into (*a*) Exclusive and (*b*) Exceptive. An *Exclusive** proposition is one which contains such words as *only*, *alone*, *none but*, etc., and in which the predicate is limited to the subject to the exclusion of others, *e.g.*, 'The virtuous alone are happy', which is equivalent to two propositions, *viz.*,

Exponible propositions, again, fall under two heads, *viz.*, (*a*) Exclusive and (*b*) Exceptive.

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* Logicians differ as to whether such propositions are compound or simple. *Vide* Welton, *Logic*, I, p. 179; Keynes, *Formal Logic*, pp. 104, 205; Mellone, *Text-Book of Logic*, p. 61.

'Some virtuous are happy' and 'No non-virtuous are happy'. An *Exceptive* proposition is one in which something is predicated of the whole of the subject with some exception, e.g., 'All metals except mercury are solid', which is equivalent to two propositions, *viz.*, 'All metals are solid' and 'Mercury is not solid'. We shall consider later on how to reduce exponible propositions into logical forms. (*Vide* Ch. XIII, § 2.)

A Categorical proposition expresses a relation between the subject and the predicate absolutely, while a Conditional proposition makes a statement subject to a condition.

§ [illegible]. **Division according to Relation.** According to the principle of *Relation*, propositions may be divided into (1) Categorical and (2) Conditional. (1) A *Categorical* proposition is one in which the predicate is affirmed or denied of the subject absolutely, *i.e.*, *without any condition or restriction*. For example, the proposition 'Man is mortal' is a categorical proposition, for the predicate 'mortality' is affirmed of the subject 'man' unconditionally. (2) A *Conditional* proposition, on the other hand, is one in which the predicate is affirmed or denied of the subject *under a certain condition*. For example, the proposition 'If God exists, the virtuous must be rewarded' is a conditional proposition, for the predicate 'rewarded' is affirmed of the subject 'the virtuous' not absolutely, but under the condition that God exists. Thus a conditional proposition expresses a relation between the subject and the predicate, dependent on some condition. Conditional propositions are, again, divided into (*a*) Hypothetical and (*b*) Disjunctive.

Conditional propositions are of two kinds, *viz.*, (*a*) Hypothetical and

(*a*) A *Hypothetical* proposition is one in which two (or more) predications are so conjoined together by some such words as *if*, *when*, etc., that the truth of the one depends upon the truth of the other. In short, a hypothetical proposition is of the form 'If

......then'. It may also be expressed by such words as *provided*, *supposing*, *granted*, *where*, *that*, *unless*, *until*, *till*, *on condition*, *although*, and the like. The following are examples of some of its various forms: If A is then B is; If A is B, it is C; If A is, B is not; If A is not B, it is not C; If A is B then C is D; If A is not B, C is not D; If A is B and C is D, E is not F; If A is B, either C is D, or E is F, or G is H. To take concrete examples: If the sun rises, there is light; When a lighted match-stick is applied to gunpowder, there will be an explosion; The people are happy provided the government is good; Where there is a will, there is a way; If metal is heated, it is softened; We shall have no rain until (or unless) the barometer falls. In such propositions the part which contains the condition (on which a relation between the subject and the predicate depends) is called the *antecedent*, and the part which contains the result (the relation itself) which so depends, is called the *consequent*. Thus in the above illustrations—If A is B then C is D; if the sun rises, there is light; If metal is heated, it is softened;—'if A is B', 'if the sun rises' and 'if metal is heated' are antecedents, while 'C is D,' 'there is light' and 'it is softened' are consequents. The logical form of a hypothetical proposition requires us to put the antecedent first and the consequent next, but in ordinary language this order is often reversed. Thus, in the above illustrations—The people are happy provided the Government is good; We shall have no rain until the barometer falls;—the consequents are put first and the antecedents last; and the logical forms of these propositions are 'If the government is good, the people are happy' and 'If the barometer does not fall, we shall have no

rain.' The antecedent and the consequent of a hypothetical proposition correspond respectively to the subject and the predicate of a categorical proposition. A hypothetical proposition is thus reducible to the categorical form and *vice versa* (*vide* Ch. XVI, § 7). Thus, for example, the propositions 'If A is B, C is D' and 'If metal is heated, it is softened' may be reduced to categorical forms 'The case of A being B is a case of C being D' and 'The case of metal's being heated is that of its being softened.'

(c) Disjunctive.

(c) A *Disjunctive* proposition is one in which two (or more) predications are disjoined by 'either... or' in such a way that the falsity of one alternative implies the truth of the other. In short, a disjunctive proposition is of the form 'either......or,' and makes an alternative predication. The following are examples of some of its various forms: A is either B or C; Either A or B is C; Either A is B, or C is D; Either A is B, or C is not D; Either A is B, or C is D, or E is F; Either A is not B, or C is not D. To take concrete examples: He is either truthful or a liar; He is either a knave or a fool; Either John or James will be present; Either the government will be considerate, or the people will revolt; He is either at home or abroad; Every swan is either black or white. It is thus clear that in a disjunctive proposition, at least two alternatives are mentioned, of which one must be accepted as true. It must be remembered in this connection that a proposition of the form 'A is neither B nor C' is not a disjunctive proposition (for there is no offer of choice between alternatives), but only a compound categorical (*i.e.*, remotive) proposition, equivalent to two negative propositions, *viz.*, 'No A is B' and 'No A is C.'

Import of Disjunctive Propositions. There has been a difference of opinion among Logicians with regard to the interpretation of a disjunctive proposition. Thus, according to Ueberweg (as well as Hamilton, Bain, Fowler and others), the alternatives of a disjuncttive proposition are *mutually exclusive* (like contradictory terms), so that the falsity of one alternative implies the truth of the other, and the truth of one alternative implies the falsity of the other. According to this view, therefore, from the disjunctive proposition 'John is either truthful or a liar,' we may get *four* hypothetical propositions, *viz.*,—

Logicians differ as to the interpretation of a disjunctive proposition. ***Is a disjunctive proposition resolvable into two or four hypothetical propositions?***

(1) If John is not truthful, he is a liar ;
(2) If John is not a liar, he is truthful ;
(3) If John is truthful, he is not a liar ;
(4) If John is a liar, he is not truthful.

But according to Mill (as well as Whately, Jevons and others), the alternatives of a disjunctive proposition are *not necessarily exclusive*, so that the falsity of the one implies the truth of the other, but not *vice versa*. According to this view, therefore, we may get *only two* hypothetical propositions from a disjunctive proposition. Thus the disjunctive proposition 'He is either a fool or a knave' is reducible to two hypothetical propositions, *viz.*,—

(1) If he is not a fool, he is a knave ;
(2) If he is not a knave, he is a fool.

As the two alternatives 'fool' and 'knave' are not mutually exclusive, the truth of the one does not imply the falsity of the other ; we cannot thus say 'If he is a fool, he is not a knave' or 'If he is a knave, he is not a fool', for it is possible for a man to be both a fool and a knave.

If the alternatives mentioned in a disjunctive are mutually exclusive, it is resolvable into four hypotheticals; if not, it is resolvable into two only.

To settle the dispute, we are to consider whether the alternatives of a disjunctive proposition are mutually exclusive or not. Now, it is clear that the alternatives are mutually exclusive, if they contain terms that are mutually incompatible (like contradictory or contrary terms), as in the illustrations—'Every swan is either black or white,' 'He is either at home or abroad,' 'He is either honest or dishonest,' 'Man is either mortal or immortal.' In such cases Ueberweg's view is right. But Mill's view is right, when the alternatives do not contain incompatible terms, and are not, therefore, mutually exclusive, as in the illustrations—'He is either a fool or a knave,' 'He is either intelligent or dishonest,' 'He is either fond or foolish.' In Formal Logic, however, it is safer to adopt Mill's view, for, without material knowledge of the alternatives, we can never be sure whether they are mutually exclusive or not.

Hypothetical and disjunctive propositions are really simple, though apparently consisting of several propositions.

Note I. "Disjunctive and hypothetical propositions* have been also called *Complex* and even *Compound*, because they apparently consist of more than one proposition. In reality, however, they are as simple as categorical propositions, and express each but one affirmation or denial—the affirmation or the denial of the dependence of one assertion upon another, or, more properly, of one many-worded term upon another. The two clauses of a hypothetical proposition are really equivalent to two many-worded terms, and not to two categorical propositions as in the case of a compound proposition. In the proposition 'If A is, B is,' the antecedent 'A is,' and the consequent 'B is' are not two independent assertions in which the existence of A and that of B are, respectively, affirmed, but parts of a conditional affirmation, the truth of the one part depending upon that of the other......The disjunctive proposition may likewise be shown to be really simple, though apparently consisting of several propositions"—Dr. Ray, *Deductive Logic,* pp. 68-9.

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* For a full discussion of the nature of Hypothetical and Disjunctive Propositions, see Keynes, *Formal Logic;* Coffey, *Science of Logic;* Venn, *Empirical Logic;* Welton, *Manual of Logic;* Bradley, *Principles of Logic;* and Joseph, *Introduction to Logic.*

Note 2. A hypothetical proposition may be said to involve a categorical assertion, for it implies that a certain consequent follows a certain antecedent. Again, a categorical may be said to be really a hypothetical in disguise, for there is often an element of uncertainty as to the conditions on which the truth of a categorical proposition depends. Thus the categorical proposition 'All men are mortal' will be more accurately stated in the hypothetical form 'If a being is a man, then it is mortal.' Further, as to the relation between the disjunctive and the categorical, it has been said that the categorical proposition 'All S is P' may be more definitely expressed by means of the disjunctive form 'All S is either *a*, or *b* or *c*', if we know that the department P includes *a*, *b* and *c*. Regarding the relation between the hypothetical and the disjunctive, it has just been shown that the disjunctive proposition is really a combination of two (or four) hypotheticals.

Categorical hypothetical and disjunctive propositions in their relation to each other.

It must, however, be observed that although a categorical and a hypothetical proposition may involve each other, yet the two forms cannot be regarded as interchangeable, for "the element of supposal which is prominent in the hypothetical disappears if the judgment is written in the categorical form; and on the other hand it is introduced *ab extra* (from without) when a categorical proposition is translated into the hypothetical form. Though the one essential nature of judgment pervades both, yet each emphasises just that aspect which is only implicit—and often but vaguely so—in the other" (Welton, *Manual of Logic*, *I*, p. 185). The same may also be said of the relation between the disjunctive and the categorical as well as that between the hypothetical and the disjunctive. Though one may involve the other, yet the peculiar nature of each cannot be got rid of. "Thus disjunctive judgment at once includes and goes beyond hypothetical, in the same sort of way as hypothetical includes and goes beyond categorical. An hypothetical judgment makes an assertion, like a categorical; but what it asserts is a relation of a consequent to a condition. A disjunctive judgment involves hypotheticals; but it presents them as alternatives and asserts the truth of one or other of them" (Coffey, *Science of Logic*, *I*, p. 287).

§ **4. Division according to Quality.** According to the principle of *Quality*, propositions may be divided into (1) Affirmative and (2) Negative. (1) An *Affirmative* proposition is one in which the predicate is affirmed of the subject. For example, the propositions 'Men are mortal,' 'Some men are wise' are affirmative propositions. Such propositions express

An Affirmative proposition expresses a relation of agreement, while a Negative proposition expresses

a relation of disagreement, between the subject and the perdicate.

a relation of agreement between the subject and the predicate. (2) A *Negative* proposition, on the other hand, is one in which the predicate is denied of the subject. For example, the propositions 'No men are perfect,' 'Some men are not wise' are negative propositions. Such propositions express a relation of disagreement between the subject and the predicate. The quality of a proposition is to be determined by its copula. If the copula contains the negative particle 'not,' then the proposition is negative; if the copula does not contain the negative particle, the proposition is affirmative. If the negative particle is attached either to the subject or to the predicate and not to the copula, the proposition is no other than affirmative. Thus the propositions 'Men are not-perfect' and 'Not-man is irrational among animals' are affirmative, for the copula does not contain the sign of negation.

Hypothetical propositions admit of distinctions of quality, and the quality of a hypothetical proposition is determined by the quality of its consequent.

Quality of Hypothetical Propositions. Logicians differ in their opinions with regard to the quality of hypothetical propositions. Some Logicians (*e.g.*, Dr. Ray, Welton and others) are of opinion that hypothetical propositions admit of distinctions of quality, and that "the quality of an hypothetical proposition is determined by the quality of its consequent and is not affected in any way by the quality of its antecedent. Thus if the consequent clause of an hypothetical proposition is affirmative, the proposition is affirmative; and if it is negative, the proposition is negative" (Dr. Ray, *Deductive Logic*, p. 71). And these Logicians further explain the nature of a negative hypothetical by saying that when the connection between the antecedent and the consequent is denied, or, in other words, when the consequent does not depend on, or is independent

of, the antecedent, the proposition is negative. Thus Welton regards the hypothetical proposition 'If a man is honest, he will not deceive his fellows' as negative ; for, according to him, here a connection between the antecedent and the consequent is denied, or, in other words, the consequent 'deceiving his fellows' is not dependent on the antecedent 'a man's being honest.' But according to other Logicians, this can never be the true meaning of a hypothetical proposition, for, they hold that there is always a relation of dependence between the antecedent and the consequent. Even when the consequent of a hypothetical proposition is negative, its true meaning is that the consequent is dependent on the antecedent. To make the consequent independent of the antecedent is to deny any relation between the antecedent and the consequent, and this is equivalent to making a hypothetical proposition almost meaningless. Thus in the above illustration, the proposition really means that the consequent 'Not deceiving his fellows' *is* dependent on the antecedent 'a man's being honest.' In other words, the form 'If A is B, then C is not D' is really affirmative, for its force is to *affirm* that 'C's not being D' *is* a necessary consequence of 'A's being B.'

In criticism we may say that the former view (that of Dr. Ray and Welton) is right in so far as it holds that hypothetical propositions admit of distinctions of quality. But, as the opposite view rightly points out, it seems to be incorrect as regards its interpretation of the nature of a negative hypothetical. In this respect the latter view is, indeed, an improvement upon the former. But this view, again, seems to be incorrect in holding that hypothetical propositions are always affirmative, as it

fails to take into consideration one important factor in its interpretation of a hypothetical. The difference between a categorical and a hypothetical lies only in this that in the former the predicate is affirmed or denied of the subject *unconditionally*, while in the latter, *conditionally*. Hence, *affirmation* as well as *denial* being the common factor present in both, if categoricals admit of distinctions of quality, then hypotheticals also must admit of distinctions of quality. And, as the consequent of a hypothetical contains the assertion, its quality must be determined by the quality of the consequent. Thus we see that a *hypothetical proposition may be either affirmative or negative* as much as a categorical one.

Disjunctive propositions are always affirmative.

Quality of Disjunctive Propositions. Unlike hypotheticals, disjunctive propositions are *always affirmative*. Thus Welton says, "It follows from the very nature of disjunctive propositions that they can only be affirmative; for they must give a choice of predicates, one or other of which must be affirmed of the subject."

A Universal proposition has for its subject a term *distributed*, *i.e.*, taken in its entire extent; while a particular proposition has for its subject a term *undistributed*, *i.e.*, taken in its partial extent.

§ 5. **Division according to Quantity.** According to the principle of *Quantity*, propositions have been divided into (1) Universal and (2) Particular. (1) A *Universal* proposition is one in which the predicate is affirmed or denied of the entire denotation of the subject. For example, the propositions 'All men are mortal' and 'No man is perfect' are universal, for, in the first instance, the predicate 'mortal' is affirmed, and in the second instance, the predicate 'perfect' is denied, of the whole class of human beings, *i.e.*, of every individual denoted by the subject 'man'. (2) A *Particular* proposition is one in which the predicate is affirmed or denied of an indefinite part of the denotation of the subject. For

example, the propositions 'Some men are wise' and 'Some men are not honest' are particular, for, in the first instance, the predicate 'wise' is affirmed, and in the second instance, the predicate 'honest' is denied, of only a part of human beings, *i.e.*, of some of the individuals denoted by the subject 'man.' It must be carefully noted that in ordinary language the word 'some' means 'some but not all,' but in Logic it means 'at least one—it may be more, it may be all.' Thus, the logical meaning of 'Some men are wise' or 'Some men are mortal' is that at least one man is wise or at least one man is mortal, it may be that all men are wise or all men are mortal. Hence, in Logic, the word 'some' does not necessarily exclude 'all.'

The Logical meaning of 'some' is 'at least one—it may be all'.

Quantity of Singular Propositions. A *Singular proposition* is one of which the subject is a singular term. A singular proposition is regarded in Logic as *universal*, because the subject definitely denotes one specified individual or group of individuals, and the predicate is, in such a case, affirmed or denied of the whole of the subject: *e.g.*, 'Shakespeare is a poet,' 'The man who followed you is present here,' 'The present Viceroy of India is a great statesman,' 'The Imperial Library of Calcutta is open to the pubiic,' 'That man is a villain,' 'The Romans conquered Gaul,' 'The following twenty students have passed the examination,' 'All metals except mercury are solid substances,' 'These few men were present in the meeting.' It must, however, be noted that when the subject of a proposition refers to an indefinite or unspecified individual or group of individuals, *i.e.*, when the subject is a general term limited in application by such qualifying words as *'a'* (meaning 'some *one*'), *'a certain'*, *'one'*, etc., the

A Singuliar proposition has a singular term for its subject, and is to be regarded as universal.

When the subject of a proposition refers to an undedermined individual, the

proposition is to be treated as particular.

proposition is to be treated as *particular.* In other words, an undetermined subject, even though singular in reference, will make the proposition particular ; *e.g.*, 'An Englishman wrote this book,' 'A boy came to me yesterday,' 'A Viceroy of India was a great statesman,' 'One army conquered Gaul,' 'One library was destroyed by fire.' Thus it is evident that 'This boy has passed the examination' is a singular, and therefore universal, proposition, while 'A boy has passed the examination' is a particular proposition ; 'one metal is liquid' is a particular proposition, while 'Mercury is a liquid metal' is a universal proposition. A singular proposition is regarded as universal, because the *whole* extent of the subject is referred to, for it is absolutely indifferent whether the subject be great or small or even an individual, so long as the predication is *both definite and applies to the whole denotation of the subject.*

An Indesignate proposition contains no sign to indicate its quantity. Its quantity is to be determined by examining its meaning with reference to the context.

An *Indesignate* proposition* is one which contains no sign to indicate its quantity, or in which the quantity of the subject is not explicitly stated. For example, 'Material bodies have weight,' 'Man is mortal,' 'Metals are useful,' 'Men are selfish' are indesignate propositions. An indesignate proposition has also been called *Indefinite,* because its quantity is left undefined. But the word 'indefinite' is ambiguous, as it has sometimes been used to mean particular propositions as well. The quantity of such a proposition cannot be determined without examining its meaning in the context in which we find it. Thus, in the above illustrations, the pro-

* For directions to determine the quantity of Indesignate propositions, *vide* Welton, I, p. 170, and Coffey, I, pp. 201-2.

positions 'Material bodies have weight' and 'Man is mortal' are universal, while the propositions 'Metals are useful' and 'Men are selfish' are particular. It may, however, be laid down as a general rule that in Logic such a proposition is to be regarded as particular, unless it is known to be universal.

Hypothetical propositions admit of distinctions of quantity. The signs of quantity may be attached either to the antecedent or to the consequent.

Quantity of Conditional Propositions. Hypothetical propositions admit of distinctions of quantity. As Dr. Ray says, "The hypothetical proposition is universal, when in every case, the antecedent is followed by the consequent; and it is particular, when the consequent follows the antecedent in some cases, or in at least one case." For example, the propositions 'In all cases, if A is B, C is D' and 'In all cases, if a body is heated, it expands' are universal; while the propositions 'In some cases, if A is B, C is D' and 'In some cases, if a man is punished, he is corrected' are particular. Thus we see that the quantity of a hypothetical proposition depends on the nature of the relation between the antecedent and the consequent. Hence it is not quite correct to say that the quantity of a hypothetical proposition is determined by the quantity of its antecedent, inasmuch as the signs of quantity (*e.g.*, 'in all cases,' 'sometimes,' 'always,' etc.) may be attached either to the antecedent or to the consequent. Thus, in the propositions 'If the sun shines, there is always heat and light' and 'If a man falls from a height, he sometimes breaks his limbs,' the signs of quantity are attached to the consequent.

Disjunctive propositions admit of distinctions

Disjunctive propositions of the form 'Either A is B or C is D' are to be treated as universal. They, however, admit of distinctions of quantity, if the

of quantity, if the several alternative predications have the same term as their subject.

several alternative predications have the same term as their subject. Thus propositions of the form 'A is either B or C' may be either universal or particular. For example, the proposition 'All men are either honest or dishonest' is universal, while the proposition 'Some men are either wicked or foolish' is particular.

Modality indicates the degree of certainty or probability with which the predicate is affirmed or denied of the subject.

§ 6. Division according to Modality. According to the principle of *Modality*, propositions may be divided into (1) Necessary, (2) Assertory, and (3) Problematic. As already indicated (Ch. XI, § 3), modality consists in the degree of certainty or probability with which the predicate is affirmed or denied of the subject. Thus, while the quality of a proposition simply states a relation of agreement or disagreement between the subject and the predicate, the modality of a proposition goes further and expresses the mode or manner of relation. The modality of a proposition is expressed by such words as 'must be,' 'necessarily,' 'certainly,' 'most probably,' 'may be,' 'perhaps,' etc.

A Necessary proposition expresses a relation which is universally and necessarily true.

(1) A *Necessary* proposition is one in which the relation between the subject and the predicate is founded on their very nature and constitution, and is consequently stated with the greatest possible degree of certainty. Such a relation must always be true and it cannot possibly be otherwise ; in other words, it is ***universally and necessarily true***. For example, 'Two and two must be four,' 'Two straight lines cannot enclose a space,' 'Things which are equal to the same thing are equal to one another' are necessary propositions, inasmuch as the relations stated herein are unalterably fixed and no one can even conceive their opposites. These propositions are most certain.

(2) An *Assertory* proposition is one in which the relation between the subject and the predicate is founded on our experience, and is consequently stated to be *true so far as our experience goes*. For example, 'All men are mortal,' 'The sun will rise to-morrow,' 'Some men are wise', are assertory propositions, inasmuch as the relations stated herein are certain in point of fact, but which might have been otherwise. These propositions are less certain than necessary ones.

An Assertory proposition expresses a relation which is true so far as our experience goes.

(3) A *Problematic* or contingent proposition is one in which the relation between the subject and the predicate is founded on our inadequate knowledge of the real state of things, and is consequently stated to be merely *probable* or *uncertain*. For example, 'He may not be a good man,' 'John is perhaps honest,' 'A falling barometer probably indicates a coming storm' are problematic propositions, inasmuch as the relations stated herein, though not inconsistent with the actual state of things, are not positively known to be true, and so they may, or may not be true. Such propositions have the lowest degree of assurance.

A Problematic proposition expresses a relation which may or may not be true.

§ 7. **Division according to Significance.** Accodring to the principle of *Significance* or import or meaning, propositions may be divided into (1) Verbal and (2) Real. . The significance or meaning of a proposition is determined by the relation of the connotation of the predicate to that of the subject.

In a Verbal proposition the predicate merely states the whole or part of the connotation of the subject.

(1) A *Verbal* proposition is one in which the predicate merely states the whole or part of the connotation of the subject. For example, the propositions 'All men are rational animals,' 'All men are animals,' 'All men are rational,' 'Matter is extended,'

'A triangle is a three-sided figure' are verbal, because they give no new knowledge or information about the subject. Such propositions have also been called *Analytical*, inasmuch as they are the results of the analysis of the connotation of the subject. They have also been called *Explicative*, because they merely explain the meaning of the subject. They have further been called *Essential*, for their formation requires a knowledge of the essence or the connotation of the subject.

In a Real proposition the predicate states about the subject some fact not contained in its connotation.

(2) A *Real* proposition is one in which the predicate states about the subject some fact not contained in its connotation. For example, the propositions 'All men are mortal,' 'Some men are learned,' 'All ruminants are herbivorous' are real, because they give some new knowledge or information about the subject, and are not merely statements of the whole or part of the connotation of the subject. Such propositions have also been called *Synthetical*, inasmuch as they are the results of the synthesis or combination of two distinct ideas represented by the subject and the predicate. They have also been called *Ampliative*, because they amplify or add to our knowledge of the subject, *i.e.*, they tell us something more about the subject than what is implied in its connotation. They have further been called *Accidental*, for their formation requires a knowledge of such relations between the subject and predicate as can be established only by accident or experience of fact.

An Identical proposition has the same term as subject and predicate.

In this connection we may also note that an *Identical* (or *Tautologous* or *Truistic*) proposition means a proposition that has the same term as subject and predicate; as, 'Whatever is, is,' 'Man is man.' Sometimes, however, a proposition of this form has more meaning than what appears at first

sight ; *e.g.*, the proposition 'What is done, is done' implies that what is done will not be altered. An identical proposition is distinguishable from a verbal proposition, inasmuch as the latter unfolds the meaning of the subject, while the former only repeats the subject, giving the same matter in the same form.

CHAPTER XIII

Four-fold Scheme of Propositions

According to the combined principle of quality and quantity, propositions have been divided into four forms, *viz.*, A (Universal affirmative), I (Particular affirmative), E (Universal negative) and O (Particular negative).

§ 1. Division according to the Combined Principle of Quality and Quantity. According to the principle of quality, propositions are divided into affirmative and negative; each of these two may, again, be divided into universal and particular according to the principle of quantity. Thus, according to the combined principle of quality and quantity we may divide propositions into the following four forms:—

1. **Universal affirmative**—*e.g.*, All S is P, All men are mortal.
2. **Particular affirmative**—*e.g.*, Some S is P, Some men are wise.
3. **Universal negative**—*e.g.*, No S is P, No men are perfect.
4. **Particular negative**—*e.g.*, Some S is not P, Some men are not learned.

It is customary to represent these four forms of propositions by the symbols **A, I, E,** and **O,** respectively. **A** and **I** stand for affirmative propositions, **A** for the universal and **I** for the particular form. **E** and **O** stand for negative propositions, **E** for the universal and **O** for the particular form. The first two vowels in the Latin word *affirmo* (meaning 'I affirm') are said to have suggested the symbols **A** and **I,** while the two vowels in the Latin word *nego* (meaning 'I deny') are said to have

suggested the symbols **E** *and* **O.** This division according to the combined principle of quality and quantity has been called the *four-fold scheme of propositions*.

Simplification of propositions means reduction of ordinary propositions into their strict Logical forms.

§ 3. **Simplification of Propositions.** We have already indicated (Ch. XI, § 2) that a logical proposition must consist of three parts, *viz.*, the subject, the predicate and the copula ; that the subject should be stated first, next the copula, and last of all the predicate ; and the logical propositions are different from grammatical sentences and ordinary statements, which may be expressed in any order and mode. It has also been pointed out (Ch. XI, § 3) that the logical copula must always consist of some form of the verb 'to be'. Hence, for the purpose of Logical treatment, we are required to change or reduce ordinary sentences and statements to their Logical forms. In reducing a proposition to the Logical form, the sense of the original proposition should not be altered and the quality and quantity of the proposition should be clearly indicated. This reduction of ordinary propositions into their strict logical forms is known as *Simplification of Propositions*, which is of utmost importance in Logic, for without it we can neither describe the logical characters of propositions, nor draw any inference from them and test any argument of which they form parts.

The Logical form of a proposition is to be determined by its meaning.

Let us now consider *how to determine the quality and quantity of a proposition ; i.e.*, how to determine whether a proposition is **A, I, E** or **O.** As it is not always easy to say whether a proposition is **A, I, E** or **O** from its form, the first thing will be to clearly understand the meaning of the proposition, for both *form* and *meaning* are to be taken into consideration

CHAPTER XIII

FOUR-FOLD SCHEME OF PROPOSITIONS

According to the combined principle of quality and quantity, propositions have been divided into four forms, *viz.*, A (Universal affirmative), I (Particular affirmative), E (Universal negative) and O (Particular negative).

§ 1. **Division according to the Combined Principle of Quality and Quantity.** According to the principle of quality, propositions are divided into affirmative and negative ; each of these two may, again, be divided into universal and particular according to the principle of quantity. Thus, according to the combined principle of quality and quantity we may divide propositions into the following four forms :—

1. **Universal affirmative**—*e.g.*, All S is P, All men are mortal.
2. **Particular affirmative**—*e.g.*, Some S is P, Some men are wise.
3. **Universal negative**—*e.g.*, No S is P, No men are perfect.
4. **Particular negative**—*e.g.*, Some S is not P, Some men are not learned.

It is customary to represent these four forms of propositions by the symbols **A, I, E,** and **O,** respectively. **A** and **I** stand for affirmative propositions, **A** for the universal and **I** for the particular form. **E** and **O** stand for negative propositions, **E** for the universal and **O** for the particular form. The first two vowels in the Latin word *affirmo* (meaning 'I affirm') are said to have suggested the symbols **A** and **I,** while the two vowels in the Latin word *nego* (meaning 'I deny') are said to have

suggested the symbols **E** *and* **O.** This division according to the combined principle of quality and quantity has been called the *four-fold scheme of propositions*.

§ 2. **Simplification of Propositions.** We have already indicated (Ch. XI, § 2) that a logical proposition must consist of three parts, *viz.*, the subject, the predicate and the copula ; that the subject should be stated first, next the copula, and last of all the predicate ; and the logical propositions are different from grammatical sentences and ordinary statements, which may be expressed in any order and mode. It has also been pointed out (Ch. XI, § 3) that the logical copula must always consist of some form of the verb 'to be'. Hence, for the purpose of Logical treatment, we are required to change or reduce ordinary sentences and statements to their Logical forms. In reducing a proposition to the Logical form, the sense of the original proposition should not be altered and the quality and quantity of the proposition should be clearly indicated. This reduction of ordinary propositions into their strict logical forms is known as *Simplification of Propositions*, which is of utmost importance in Logic, for without it we can neither describe the logical characters of propositions, nor draw any inference from them and test any argument of which they form parts.

Simplification of propositions means reduction of ordinary propositions into their strict Logical forms.

Let us now consider *how to determine the quality and quantity of a proposition ; i.e.*, how to determine whether a proposition is **A, I, E** or **O.** As it is not always easy to say whether a proposition is **A, I, E** or **O** from its form, the first thing will be to clearly understand the meaning of the proposition, for both *form* and *meaning* are to be taken into consideration

The Logical form of a proposition is to be determined by its meaning.

in order to determine whether a particular proposition is **A, I, E** or **O.** Thus the proposition 'All men are not learned' seems to be **E** from its *form*, but from its *meaning* we know that it is really **O**, for it means that some men are not learned. Hence it may be laid down as a general rule that *the Logical form of a proposition is to be determined by its meaning.* The following directions will be of much help in determining the quality and quantity of a proposition:

Directions to determine the quality and quantity of propositions.

(1) **Propositions with 'Every,' 'All,' 'Any,' 'Anyone,' 'Each,' 'The whole.'** *Affirmative* propositions beginning with any one of these words are to be regarded as universal, while *negative* propositions beginning with any one of these words are to be regarded as particular. For example, the propositions 'All men are mortal,' 'Every miser envies others' wealth,' 'Anyone of these books will do,' 'Each member of the House will speak' are universal, while the propositions 'All that glitters is not gold' (= some glittering things are not gold), 'Every man is not rich' (= some men are not rich), 'Every one of your friends will not help you' (= some of your friends are not persons who will help you) are particular. Similarly, affirmative propositions with their predicates qualified by 'always,' 'absolutely,' 'necessarily,' or their equivalents are to be treated as universal, while negative propositions with similar qualifications are to be treated as particular. For example, 'Sages are always respected' (= all sages are respected) is universal, while 'The virtuous are not always rewarded in this life' (= some virtuous men are not rewarded in this life) is particular. 'Cleanliness is absolutely necessary for health' is universal, while 'Women are not necessarily inferior to men' (= some women are not inferior to men) is particular.

(2) **Propositions with 'No,' 'None,' 'Nothing,' 'No one,' 'Nobody.'** Propositions of which the subjects contain any of these words, or the grammatical predicates contain such words as 'No,' 'Not,' 'Never,' etc., clearly making the copula negative, are to be regarded as universal negative. For example, the propositions 'None need apply who has not passed the examination,' 'Nobody could say where he was gone,' 'Men are never perfect' are universal negative.

(3) **Propositions with 'Some,' 'A few,' 'Certain,' 'Several,' 'Many,' 'Most,' 'Almost all.'** Such propositions are to be treated as particular. For example, the propositions 'Some men are wise,' 'A few men are learned' (= some men are learned), 'Most boys are well-behaved' (= some

boys are well-behaved) are particular. Propositions of which the quantity is expressed by 'most' are sometimes called *plurative*.

(4) **Propositions whose predicates are qualified by 'often,' 'mostly,' 'sometimes,' 'generally,' 'hardly,' 'scarcely,' 'rarely,' etc.** Such propositions are to be treated as particular. The words 'seldom,' 'hardly,' 'scarcely,' 'rarely,' etc., have a negative force, so when they occur in affirmative propositions, they make the propositions negative ; and when they occur in negative propositions, they make the propositions affirmative. For example, 'Students are mostly well-behaved' (=some students are well-behaved), 'Books' are generally readable' (=some books are readable), 'The vicious are not seldom punished' (=most vicious persons are punished=some vicious persons are punished), 'Summer days are generally (or often) hot' (=some summer days are hot), 'Earnest students seldom waste their time' (=most, *i.e.*, some, earnest students are not those who waste their time), 'Men are hardly (or rarely or scarcely or seldom) wise' (=most men are not wise= some men are not wise) are particular propositions.

(5) **Propositions with 'Few,' 'Hardly any,' 'Scarcely any.'** Affirmative statements with these words have a negative force and must be reduced to negative propositions, while negative statements with these words have an affirmative force and must be reduced to affirmative propositions. Thus 'Few men are really good,' 'Scarcely any students could answer 'correctly' are **O** propositions, for they mean 'Most men are not really good' (- some men are not really good), 'Most students could not answer correctly' (=some students are not those who could answer correctly). Similarly, 'Few graduates are not eligible,' 'Hardly any Hindu will not worship his God' are **I** propositions, for they mean 'Most graduates are eligible' (=some graduates are eligible), 'Most Hindus will worship their God' (=some Hindus are those who will worship their God).

(6) **Exclusive Propositions.** Exclusive propositions or propositions with 'None but,' 'Only,' 'Alone' are universal, and are to be treated as affirmative or negative according to the form to whom we may reduce them. Thus the propositions 'None but the brave deserve the fair,' 'Only the brave deserve the fair,' 'The brave alone deserve the fair' may all be reduced either to 'No non-brave are persons who deserve the fair,' (an **E** proposition—the primary meaning), or to 'All who deserve the fair are brave' (an **A** proposition—the secondary implication). It should be remembered that in reducing an exclusive proposition to its logical form we must take its meaning into consideration. It thus appears that an exclusive proposition may be reduced to an **A** proposition with its terms transposed, *i.e.*, the original predicate should be made the subject, and

the original subject, the predicate. It may also be noted here that, when propositions with 'only,' 'alone,' etc., are negative, they are to be treated as exceptive and not as exclusive propositions. Thus the proposition 'Only fools do not believe in God' is equivalent to 'All believe in God except fools.' (See below.)

(7) **Exceptive Propositions.** General statements with indefinite exceptions are to be regarded as particular. For example, 'All metals except one are solid' is logically equivalent to 'Some metals are solid' and consequently it is an **I** proposition. But general statements with definite exceptions are to be regarded as universal and resolved into two propositions. Thus the proposition 'All metals except mercury are solid' is equivalent to two propositions, viz., 'All metals are solid' **(A)** and 'Mercury is not solid' **(E)**.

(8) **Infinite Propositions.** An Infinite proposition is one of which the predicate is an infinite term. An Infinite proposition is to be treated as affirmative, if there is no sign of negation in the copula. Thus the proposition 'All men are not-perfect' is affirmative, while the proposition 'Some animals are not non-carnivorous' is negative.

(9) **Impersonal Propositions.** In an Impersonal proposition like 'It rains,' 'It is day,' 'It is hot,' the real subject is not made definite at all. Such a proposition may, however, be reduced to the logical form by supplying the real subject and thereby making the indefinite 'it' definite. Thus 'It rains,' 'It is day,' 'It is hot' may be reduced to 'The weather is rainy,' 'The time is day,' 'The weather is hot.'

(10) For Simplification of Indesignate and Singular propositions, *vide* Ch. XII, § 5.

Logical Opposition is the relation between any two propositions which have the same subject and predicate, but which differ in quality, or quantity, or both.
Kinds of Opposition?

§ 3. **Mutual Relation or Opposition of Propositions.** Two propositions having the same subject and predicate, but differing in quality or in quantity or in both, are said to be opposed to each other, and their mutual relation is called the **Logical Opposition** of propositions. In other words, the Logical Opposition is the relation between two propositions which are identical in matter but different in form. Thus, of the four forms of propositions, *viz.*, **A, I, E** and **O**, any two are said to be opposed to each other, provided their subject and predicate are the same. There are four kinds of Opposition, *viz.*, Contrary, Sub-contrary, Contradictory and Subaltern.

(1) **Contrary Opposition** *is the relation between two universal propositions that have the same subject and predicate that differ in quality.* Thus the relation between **A** (*e.g.*, 'All men are mortal') and **E** (*e.g.*, 'No men are mortal') is contrary opposition. (1) Contrary.

(2) **Sub-contrary Opposition** *is the relation between two particular propositions that have the same subject and predicate but differ in quality.* Thus the relation between **I** (*e.g.*, 'Some men are honest') and **O** (*e.g.*, 'Some men are not honest') is sub-contrary. (2) Sub-contrary.

(3) **Contradictory Opposition** *is the relation between two propositions that have the same subject and predicate but differ both in quality and in quantity.* Thus the relation between **A** (*e.g.*, 'All men are mortal') and **O** (*e.g.*, 'Some men are not mortal') as well as the relation between **E** (*e.g.*, 'No men are perfect') and **I** (*e.g.*, 'Some men are perfect') is contradictory opposition. (3) Contradictory.

(4) **Subaltern Opposition** *is the relation between two propositions that have the same quality and the same subject and predicate but differ in quantity.* Thus the relation between **A** (*e.g.*, 'All men are mortal') and **I** (*e.g.*, 'Some men are mortal') as well as the relation between **E** (*e.g.*, 'No men are perfect') and **O** (*e.g.*, 'Some men are not perfect') is subaltern opposition. Here the universal proposition is called *subalternant* and the particular one is called *subalternate*; while both in relation to each other are called *subalterns*. (4) Subaltern.

According to some Logicians, Subaltern Opposition cannot be regarded as opposition at all, for propositions not differing in quality, such as **A** and **I**, or **E** and **O**, cannot be said to be opposed to, or in- ***Is Subalternation a kind of Opposition?***

compatible with, each other, inasmuch as two subalterns may both be true or false. Hence they limit the term 'opposition' to inconsistent or incompatible relations, and treat subaltern opposition as a kind of compatible relation and call it **Subalternation.** Other writers hold that the difference in quantity is surely a difference, though, of course, it is not so thorough-going as the one in quality. Hence they maintain, on the contrary, that in subaltern opposition, in which the propositions differ in quantity, there is some amount of opposition at any rate. And this view is further supported by the facts that the universal may be false without the particular being false, and that the particular may be true without the universal being true (*vide* Ch. XVI, § 6).

It is true, indeed, that according to the ordinary sense of the word 'opposition', it seems to be a violence to thought and language to speak of **A** and **I**, or **E** and **O** propositions as being opposed to each other, for no real opposition can ever be said to exist between two propositions, both of which can be true together. Hence, to avoid confusion, we have preferred the expression *Logical Opposition,* and have used it in a technical sense to include all possible relations of propositions, whether of compatibility or of incompatibility.

The Common Square and Aristotle's Square of Opposition.

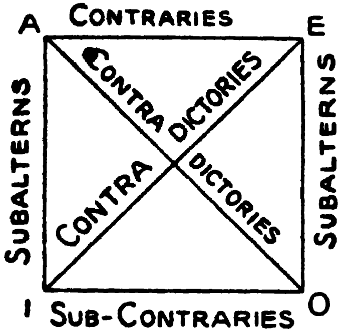
The opposition of propositions is usually indicated by the diagram, called the **Common Square of Opposition,** which, again, is different from **Aristotle's Square of Opposition.**

Aristotle's Square has the advantage of representing

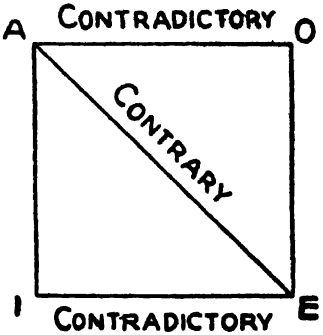
From a comparison of the Common Square with that of Aristotle, it is obvious that Aristotle's Square possesses the diagrammatic advantage of representing the strongest or the most complete opposition, *viz.,*

the contrary opposition between the universals **A** and **E**, by the longest line, the diagonal of the square. Partial opposition, *viz.*, the contradictory opposition between **A** and **O** and between **I** and **E**, is repre-

Common Square of Opposition



Aristotle's Square of Opposition



sented by a shorter side, *viz.*, a side of the Square. The expression 'diametrically opposite' owes its origin to this arrangement of exhibiting the fullest contrast by the longest line. Further, in Aristotle's Square there is no mention of subaltern opposition, and the second diagonal connecting **I** and **O** is not drawn at all. Hence it follows that Aristotle did not recognize sub-contrary opposition and subaltern opposition as forms of opposition, for **I** and **O** may both be true together, as well as **A** and **I**, and **E** and **O** (*vide* Ch. XVI, § 6).

the complete opposition by the longest line. It also indicates that there is no opposition between subalterns and sub-contraries.

Note. **Opposition in Singular Propositions.** As the subject of a singular proposition cannot have particular quantity, there can be no sub-contrary or subaltern opposition in the case of the singular proposition; but only contrary opposition, *i.e.*, opposition between **A** (*e.g.*, 'Socrates is wise') and **E** (*e.g.*, 'Socrates is not wise'). But although only contraries in form, they have the force of contradictories for here not only can we proceed from the truth of the one to the falsity of the other but also conversely, *i.e.*, from falsity to truth. Hence it has been said that *in the case of the singular proposition, the con-*

tradictory and the contrary coincide. The opposition of singulars is called *secondary opposition.*

Two rules regarding the distribution of terms.

§ 4. **Distribution of Terms in Propositions.** We have already explained the meaning of Distribution of Terms (Ch. VII, § 5). Here we are to consider the distribution of each of the terms (subject and predicate) in each of the four forms of propositions, **A, I, E** and **O.** There are two general rules regarding the distribution of terms in a proposition:

(1.) *Only universal propositions distribute their subjects.* **A** and **E** are universal propositions; the signs of an **A** proposition, *viz., 'All', 'Every'*, etc., and the signs of an **E** proposition, *viz., 'No', 'None'*, etc., indicate that the subjects are taken in their entire extent. In a universal proposition, something is predicated of an *entire class denoted by the subject.* Thus **A** and **E** propositions distribute their subjects. **I** and **O** are particular propositions; the signs of a particular proposition, *viz.,* 'Some', etc., indicate that the subject is taken only in its partial extent. In a particular proposition, something is predicated of only a *part of the class denoted by the subject.* Thus **I** and **O** propositions do not distribute their subjects.

(2.) *Only negative propositions distribute their predicates.* **E** and **O** are negative propositions. In 'No man is perfect' and 'Some men are not wise', we necessarily exclude everything denoted by the predicates 'perfect' and 'wise' from the subjects 'all men' and 'some men'. The predicates in these cases are *wholly* denied of the subjects. Thus **E** and **O** propositions distribute their predicates. **A** and **I** are affirmative propositions; in 'All men are mortal' (**A**) and 'Some men are wise' (**I**) we do not necessarily include in the class of human beings all mortal beings or all wise beings. In these cases

only a part of the class denoted by the predicate is coincident with the class denoted by the subject. Thus **A** and **I** propositions do not distribute their predicates.

By applying the above two rules in the case of each of the four forms of propositions, we can easily determine which terms are distributed and which not. Thus **A**, being a universal proposition, distributes its subject (Rule 1) ; and being an affirmative proposition, it does not ditsribute its predicate (Rule 2). **E**, being a universal proposition, distributes its subject (Rule 1) ; and being a negative proposition, it distributes its predicate (Rule 2). **I**, being a particular proposition, does not distribute its subject (Rule 1). ; and being an affirmative proposition, it does not distribute its predicate (Rule 2). **O**, being a particular proposition, does not distribute its subject (Rule 1). ; and being a negative proposition, it distributes its predicate (Rule 2).

A distributes its subject only, **E** distributes both its subject and predicate, **I** distributes no term and **O** distributes its predicate only.

To sum up, **A** distributes *subject only*, **E** distributes *both subject and predicate*, **I** distributes *neither subject nor predicate*, and **O** distributes *predicate only*. The Symbolic word A*s*E*b*I*n*O*p* may help the students to keep the above results in memory. These results may further be verified by means of diagrams, which we shall consider in the last section (§ 6).

Affirmative propositions distribute their predicates, when the subject and the predicate are co-extensive and identical in meaning.

Note. It follows from Rule 2 that affirmative propositions do not distribute their predicates. But there are certain exceptions to this rule. (1) If the predicate of an affirmative proposition is a singular term, either a proper name or a designation, it is distributed; *e.g.*, 'Lord Canning is the first Viceroy of India.' (2) If the whole of the predicate agrees with the subject as in the case of definitions; *e.g.*, 'All men are rational animals.' In such cases the term standing for subject and predicate are co-extensive and therefore convertible. Hence we can express the above examples thus—"The first Viceroy of India is Lord Canning'; "All rational animals are men.'

Hamilton's eight-fold scheme of propositions obtained by quantifying the predicate, *i.e.*, by stating explicitly whether the whole or the part of the predicate agrees with, or differs from, the subject

§ 5. Quantification of the Predicate.* The doctrine of the Quantification of the Predicate is connected with the name of Hamilton, according to whom, in every judgment we not only think of the quantity of the subject, but implicity think of the quantity of the predicate as well. And if we state explicitly in language what is implicitly contained in thought, we get *eight* forms of propositions instead of the four ordinary forms, *viz.*, **A, I, E & O.** Thus, to quantify the predicate is simply to state explicitly the quantity of the predicate in propositions of different qualities. Hence Hamilton divided propositions into eight forms by dividing each of **A, I, E** and **O** into two forms,

A { *afa*—Toto-total affirmative, *e.g.*, All S is *all* P.......... **U**
afi Toto-partial affirmative, *e.g.*, All S is *some* P......**A**

I { *ifa* -Parti-total affirmative, *e.g.*, Some S is *all* P.........**Y**
ifi -Parti-partial affirmative, *e.g.*, Some S is *some* P...**I**

E { *ana*—Toto-total negative, *e.g.*, No S is *any* P...........**E**
ani—Toto-partial negative, *e.g.*, No S is *some* P.........*η*

O { *ina* -Parti-total negative, *e.g.*, Some S is not *any* P...**O**
ini Parti-partial negative, *e.g.*, Some S is not *some* P..*ω*

To understand Hamilton's symbols *afa*, *afi*, etc., we have to remember that *f* stands for the affirmative copula, *n* for the negative coupla, *a* for a distributed term and *i* for an undistributed term.

Hamilton's scheme, however, has been rejected by the majority of Logicians, inasmuch as it rests on a false psychological basis, is cumbrous and inconsistent with the general rules of distribution of terms.

According to Hamilton, this eight-fold scheme of propositions has the merit of removing all difficulties with regard to distribution of terms, and consequently of reducing all Conversion to simple Conversion (*vide* Ch. XVI, § 1), and of considerably simplifying the different forms of Syllogistic inference (*vide* Welton, I, p. 201). But this doctrine is now generally discarded by Logicians for the following reasons: (1) This doctrine, it has been said, rests on a basis which is psychologically false. The predicate of a proposition is not always thought of in its denotation (as this doctrine upholds), but is often thought of in its connotation. Thus in the proposition 'All men are mortal,' we never think of the extension of mortal beings and never interpret the proposition by thinking that all men are some mortals. We rather think of the attribute of mortality as belonging to the class of human beings. (2) Some of the above propositions are really complex, as, for example, the proposition 'All S is all P' is equivalent to two ordinary propositions—'All S is P' and 'All P is S'; the proposition 'Some S is all P' is equivalent to two ordinary propositions 'Some S is P' and 'All P is S'. These forms are, therefore, cumbrous and are not of common usage. Some of the above forms, again,

* For a detailed account, *vide* Welton, *Manual of Logic*, *I*, p. 200.

are meaningless and hence useless, as, *e.g.*, 'No S is some P' and 'Some S is not some P'. Moreover, the Toto-partial and Parti-partial negative forms are inconsistent with the general rules of distribution of terms, for we know that all negative propositions distribute their predicates. Hence the majority of Logicians have rejected Hamilton's eight-fold scheme in favour of the traditional four-fold scheme of propositions.

Various attempts have been made to represent propositions by means of diagrams.

5. Diagrammatic Representation of Propositions. Different attempts have been made by different Logicians such as Euler, Jevons, Ueberweg, Lambert, Venn and Welton to represent propositions by diagrams. Propositions may be represented by means of diagrams only when both the subject and the predicate are understood in denotation or extension (*vide* Ch. XIV, § 2). As we shall see later on, representation of propositions by diagrams is very useful to beginners for understanding the relations between the terms of a proposition, and consequently for testing arguments. An account of the various schemes of diagrams will be found in Welton's *Manual of Logic*, Vol. I, p. 216. We give below the most commonly employed scheme of diagrams, known as Euler's Circles:—

The most commonly accepted scheme is that of Euler.

Euler's Circles.

Fig. I

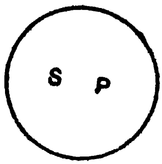


Fig. II

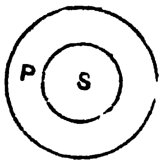
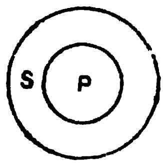


Fig. III



Euler's Circles.

Fig. IV

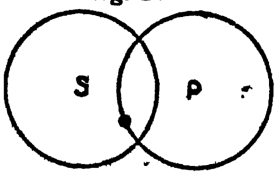
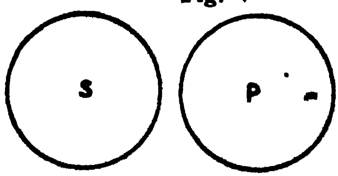


Fig. V



In the above figures, a circle represents things denoted by a term ; 'S' stands for the subject and 'P' for the predicate ; all possible relations between S and P in the recognized four forms of propositions have been represented by five possible combinations of two circles. Thus :—

A proposition is represented by Figs. I and II.

A—An **A** proposition establishes a relation of co-extension or inclusion. Thus "All triangles are three-sided figures' may be represented by Fig. I, for the subject and predicates in this case are co-extensive with each other. An **A** proposition may also be represented by Fig. II, as in the example 'All men are mortal', where the subject, a smaller class, is included in the predicate, a larger class, or, in other words, the whole of the subject coincides with a part of the predicate.

I proposition is represented by Figs. I, II, III and IV

I—An **I** proposition may be represented by the diagrams representing **A**, for 'some', in Logic, means 'at least one, it may be all'. Thus if 'some' means 'all', the propositions 'Some men are rational animals' and 'Some men are animals' may respectively be represented by Figs. I and II. Again, if 'some' means 'a part' only, then an **I** proposition may also be represented by Figs. III and IV. If a part of S coincides with the whole of P, it is represented by Fig. III, as in the example 'Some men are philosophers'. If a part of S coincides with a part of P, it is represented by Fig. IV, as in the example 'Some philosophers are mathematicians.'

E proposition is represented by Fig. V.

E—An **E** proposition (*e.g.*, 'No S is P') is represented by Fig. V, where two circles, standing for S and P, wholly exclude each other.

O proposition is represented

O—An **O** proposition may be represented by the diagram representing **E**, *i.e.*, by Fig. V, for 'some',

occurring in an **O** proposition, may not exclude 'all', as in the example 'Some men are not brutes'. Again, if 'some' means 'a part' only, then an **O** proposition may also be represented by Figs. III and IV, as in both these Figs., a part of S is excluded from the whole of P. Thus the proposition 'Some philosophers are not mathematicians' may be represented either by Fig. III, or by Fig. IV.

by Figs. III, IV and V.

To sum up, **A** is represented by Figs. I and II ; **I** is represented by Figs. I, II, III and IV ; **E** is represented by Fig. V ; **O** is represented by Figs. III, IV and V.

CHAPTER XIV

Import of Propositions

The Predicables are so many classes of the different relations of the predicate to the subject.

§ 1. **The Predicables.** We know that a predicate is that which is said, *i.e.*, affirmed or denied, of a subject. All terms that can thus be employed as predicates have been grouped into several classes, according to the different relations they bear to the subjects of which they happen to be predicated. These various classes of relations which a predicate may bear to a subject are known as the *Predicables.* Thus while a predicate is a term which is *actually* affirmed or denied of a subject, a Predicable is a class of terms that *may be* affirmed or denied of a subject. The commonly accepted scheme of Predicables is that of Porphyry, who classified the Predicables under five heads, *viz.*, **Genus, Species, Differentia, Proprium** and **Accidens.** The nature of these five Predicables has already been explained in Ch. VII, § 4. The predicate of a proposition in relation to its subject must belong to one of these five classes. For exmaple, in the proposition 'Man is an animal', the predicate 'animal' is a genus in relation to the subject 'man'; in the proposition 'Animals having rationality are men', the predicate 'man' is a species in relation to the subject 'animal'; in the proposition 'Man is rational', the predicate 'rational' is a differentia in relation to the subject 'man'; in the proposition 'Man is susceptible to pleasure and plain', the predicate 'susceptible' to pleasure and pain' is a

They are Genus, Species, Differentia, Proprium and Accidens.

Illustrations.

proprium in relation to the subject 'man'; in the proposition 'Man is social', the predicate 'social' is an accidens in relation to the subject 'man'.

We may note in this connection that in the case of an **A** proposition, (1) if the predicate be a common term, the subject may be either (*a*) a common term, or (*b*) a singular or collective term; (2) if the predicate be a singular or collective term, the subject must be the same; (3) if the predicate be an abstract term, the subject must be the same; and (4) if the predicate be an attributive, the subject may be either (*a*) a common term, or (*b*) a singular or collective term, or (*c*) an abstract term (Fowler).

'Essence' of a thing is that attribute or combination of attributes by virtue of which that thing and all other things of the same kind are what they are.

Essence. Some Logicians mention 'Essence' as one of the predicables. 'Essence', like concept, has been interpreted from different points of view, and the controversy as to the meaning and nature of 'essence' turns up to be the same as that among Realists, Conceptualists and Nominalists as to the nature of Concept. We have already noted the identity of 'universal' (concept) and 'essence', and have given an account of the controversy in Ch. V, §§ 3 and 4. Avoiding all metaphysical discussions here, we may note the meaning in which the word 'essence' has been used in Logic. By 'essence' we mean 'the attributes which we would assign to explain the thing if asked *what the thing is.*' In other words, the 'essence' of a thing is that attribute or combination of attributes by virtue of which that thing and all the things of the same kind are what they are. But we know that the fundamental and important attributes which determine the nature of a thing constitute its connotation. Hence in Logic, the 'essence' of a term means its connotation. The 'essence' of man, for example, is the connotation of the term 'man', *viz.*, the attributes of animality and

rationality combined. If a being possesses only one of these two attributes, he will not be a man. It is the combination of the two, it is the entire connotation, that constitutes the essence of a man or makes him to be what he is (*i.e.*, a 'man') as distinguished from all other beings. As connotation consists of the generic and the specific attributes, it follows that if we combine genus and differentia, we get the essence. This is the reason why Porphyry omitted essence from his list of predicables.

Distinction between Essence, on the one hand, and Proprium and Accidens, on the other.

It is in no way difficult now to distinguish between essence, on the one hand, and proprium and accidens, on the other. 'Essence' stands for the entire connotation of a term, 'proprium' for the attributes necessarily connected with its connotation, and 'accidens' for attributes other than those contained in, or necessarily connected with, its connotation. And it may easily be understood that the predicables determine the distinction between a verbal and a real proposition (Ch. XII, § 7). Any predication of a genus, species, differentia, or essence is a verbal proposition, and any predication of a proprium or accidens is a real proposition. In a verbal proposition, the connotation of the predicate is a part or the whole of the connotation of the subject; and the subject being a species, the predicate, if genus or differentia, expresses a part, and if essence, expresses the whole, of the connotation of the subject. Thus all definitions are verbal or essential propositions. In a real proposition, the predicate states an attribute not contained in the connotation of the subject; and the subject being a species, the predicate in relation to the subject is either a proprium or an accidens.

The way in which the different heads of predicables are obtained may be exhibited as follows:—

The Subject being a Species
Every Predicate expresses either

- The whole Connotation of the Subject—**Essence**
- Or not the whole
  - A Part
    - **Genus**
    - **Differentia**
  - Or not a Part
    - Connected with Connotation—**Proprium**
      - Connected with genus (*Generic Property*)
      - Connected with species (*Specific Property*)
    - Or not—**Accidens**
      - If true at all times (*Inseparable Accident*)
      - If not (*Separable Accident*)

Note. **Aristotle's Scheme of Predicables.**

Aristotle mentions four predicables, *viz.*, Definition, Proprium, Genus and Accidens. Aristotle's scheme is based on the Laws of Contradiction and Excluded Middle, and is therefore necessarily exhaustive of all common terms used as predicates. The scheme may be shown in a tabular form as follows:— **Aristotle's four-fold scheme of Predicables.**

In every Proposition
The predicate either

- agrees with the subject in denotation
  - as well as in connotation and is thus—a *Definition* in relation to the subject (*e.g.*, Man is a rational animal)
  - but disagrees with it in connotation, and is a *Proprium* in relation to the subject (*e.g.*, Man is an animal that cooks his food)
- or disagrees with the subject in denotation
  - but partially agrees with it in connotation, and is a *Genus* in relation to the subject (*e.g.*, Man is an animal)
  - as well as in Connotation, and is an *Accidens* in relation to the subject (*e.g.*, Some men are fair-complexioned)

Aristotle's four-fold scheme has, however, been practically superseded by Porphyry's five-fold scheme. Some modern Logicians have made further addition. Fowler, for example, adds 'synonym' and 'designation'. Thus, in the proposition 'Tully is Cicero,' the predicate is a synonym, and in the proposition 'Lord Canning was the first Viceroy of India,' the predicate is a designation.

Theories of Predication are the different views as to the meaning of the subject and the predicate of a proposition and of the relation between them.

§ 2. Theories of Predication.* By **'Import of Propositions'** we understand the meaning or significance of propositions. And as a proposition is nothing but a predication which establishes a relation between the subject and the predicate, the import or the meaning of a proposition will require an understanding of the exact nature of the predication or the manner of the relation between the subject and the predicate. Different views have been held by different Logicians as to the interpretation of the import or meaning of a proposition or predication, giving rise to different *Theories of Predication*. The questions to be discussed, therefore, in this connection are:—

Two questions to be discussed in this connection.

(I) What do the terms of a proposition, the subject and the predicate, stand for? This is the question of the Theory of Predication from the **Psychological standpoint.**

(II) What sort of relation is asserted between these terms? This is the question of the Theory of Predication from the **Logical standpoint.** We shall treat of these two questions separately, briefly noting the leading theories in this connection.

From the Psychological standpoint, the question is—Do the

I. The question here is: *Do the terms of a proposition stand for names, or concepts, or things?* This question has already been dealt with, and we have seen (Ch. II, § 9) that there are three theories

* For a fuller and masterly account, one may read Dr. Ray, *Deductive Logic*, p. 93.

in this connection, *viz.*, (1) Nominalism, (2) Conceptualism, and (3) Realism. It has also been pointed out (Ch. II, § 9) that none of these different views, taken separately, gives the whole truth. In a proposition, we usually think of three sorts of relations corresponding to one another, *viz.*, a relation between ideas corresponding to a relation between actually existing things as well as a relation between words. In some cases, however, a relation between ideas may not correspond to a relation between things, but there must always be a double relation corresponding to each other, *viz.*, a relation between ideas corresponding to a relation between words. Hence we see that a proposition, besides expressing a relation between terms or words, also expresses a relation between ideas, and, sometimes, a relation between real things.

terms of a proposition stand for names or concepts or things? There are three theories in this connection, *viz.*, Nominalism, Conceptualism, and Realism. The three views, taken together, express the whole truth.

II. The other question, *viz.*, the question of the Theory of Predication from the purely Logical standpoint, is of greater importance in this connection. Here the question is *whether the subject and the predicate of a proposition are to be understood in denotation, or in connotation, or in both, or one in denotation and the other in connotation.* Let us consider here the four possible theories one by one.

From the Logical standpoint, the question is—What sort of relation is asserted between the terms of a proposition? Four possible theories have been advanced.

1. The Predicative or Common or Ordinary View (held by Martineau, Venn, and others). According to this view, the subject is taken in denotation and the predicate in connotation, and the relation between them is one of *substance* and *attribute*. In other words, according to this view, the subject stands for things and the predicate for attributes possessed by the things, and a proposition is regarded as affirming or denying an attribute or attributes of a subject. Thus the proposition 'All

1. The Predicative view, according to which, the subject is taken in denotation and the predicate in connotation, and the relation between

them is one of substance and attribute.

men are mortal' means that the attribute of mortality is affirmed of all human beings. Similarly, the proposition 'No man is perfect' means that the attribute of perfection is denied of all human beings. This is the common and natural way of interpreting propositions, for, generally speaking, the subject of a proposition is a substantive, while the predicate is an adjective. The four-fold scheme of propositions is based on this view.

2. The Denotative view, according to which, both the subject and the predicate are taken in denotation, and the relation between them is one of inclusion or exclusion or co-extension.

2. The Denotative or Class View. According to this view, both the subject and the predicate are taken in denotation, and the relation between them is one of *inclusion* or *exclusion* or *co-extension*. Thus the proposition 'All men are mortal' means that the class of men is included in the class of mortal beings. Similarly, the proposition 'No men are perfect' means that the class of men is excluded from the class of perfect beings. Again, the proposition 'All men are rational' means that the class of men is co-extensive with the class of rational animals. Diagrammatic representation of propositions, and, as we shall see later on, Conversion and Syllogistic inference are all based on this denotative interpretation of propositions.

Hobbes' view and the Equational view are forms of the Denotative view.

Note. The Denotative view includes (*a*) *Hobbes' view*, according to which, the predicate is said to be, or not to be, the name of that of which the subject is a name, *e.g.*, the proposition 'All men are mortal' means that the name 'mortal' is applicable to all those beings that are called 'man'; (*b*) the *Equational view*, according to which, the things denoted by the subject are held to be, or not to be, the same as those denoted by the predicate. This doctrine is the outcome of Hamilton's Doctrine of the Quantification of the Predicate (Ch. XII, § 5). Thus the relation between the subject and the predicate turns out to be one of *equality* or *inequality*. For example, the proposition 'All S is some P' means 'All S = Some P,' the proposition 'No S is some P' means 'No S = Some P.'

3. The Connotative or Attributive View (held by Mill and others). According to this view, both the subject and the predicate of a proposition are to be taken in connotation, and the relation between them is one of *concomitance* or *non-concomitance*. Thus the proposition 'All men are mortal' means that the attributes connoted by 'man' are accompanied by the attributes connoted by 'mortal'. Similarly, the proposition 'No men are perfect' means that the attribute of perfection does not accompany the group of attributes known as humanity. According to Mill, every real proposition expresses one or other of the five relations, *viz.*, existence, co-existence, succession, causation and resemblance. Bain adopts this view, but reduces the five relations to three, *viz.*, co-existence, succession, and equality or inequality. The quantitative propositions dealing with relations of equality or inequality belong to Mathematics, while Logic is chiefly concerned with qualitative propositions dealing with relations of co-existence (*e.g.*, All men are mortal) and succession (*e.g.*, Water quenches thirst).

3. The Connotative View, according to which, both the subject and the predicate are taken in connotation, and the relation between them is one of concomitance or non-concomitance.

4. The Denotative-Connotative or Comprehensive View (held by Hamilton and others). According to this view, both the subject and the predicate may be taken either in denotation or in connotation, and the relation between them is a twofold one. Thus, when both the terms are taken in denotation, as in the Denotative View, the subject is either included in, or excluded from, or co-extensive with, the predicate. And, when both the terms are taken in connotation, as in the Connotative View, the predicate is included in, or excluded from, the subject. Here the relation is one of concomitance or non-concomitance, as in the Conno-

4. The Comprehensive view, according to which, both the subject and the predicate may be taken either in denotation or in connotation, and the relation between them is a twofold one.

tative View. This view is, therefore, a combination of the Denotative and Connotative Views. Thus the proposition 'All men are mortal', when understood in denotation, means that the class of men is included in the class of mortal beings; and when understood in connotation, it means that the attribute of mortality is included in the group of attributes constituting humanity.

As different thought-relations are expressed in different kinds of propositions, different theories are to be adopted in interpreting different classes of propositions.

Note. **Interpretation of Propositions.** A question may now be raised: Of the four theories considered above, which one represents the correct view of interpreting propositions? To this we may answer by saying that no one of the above views, taken by itself, gives us satisfactory explanations of the different kinds of propositions, and that different theories hold good in interpreting different thought-relations expressed in different kinds of propositions. To attempt to explain all kinds of propositions by means of any particular theory would amount to taking up an extreme position, and this will scarcely succeed in rendering clear and explicit the diverse relations implied in different classes of propositions. The right view seems to be that in interpreting different kinds of propositions, different theories are to be adopted. Thus the Denotative Theory may be conveniently adopted when the subject and the predicate are respectively the species and the genus; for example, in the proposition 'Man is an animal,' we naturally think of the classes 'man' and 'animal'. Universal propositions arrived at by means of complete examination of all the individuals of a class (*e.g.*, 'All the boys of the first-year class are well-behaved'), singular and particular propositions with concrete terms as subjects (*e.g.*, 'Socrates is a philosopher,' 'Some boys are well-behaved') may be conveniently interpreted either according to the Predicative Theory (if the predicate is an attributive), or according to the Denotative Theory (if the predicate is a class-name). Again, universal propositions arrived at by an examination of some individuals of a class on the basis of a necessary relation between the subject and the predicate (*e.g.*, 'All men are mortal'), as well as propositions with abstract terms as subjects (*e.g.*, 'Virtue is divine') may be better interpreted by the Connotative Theory. Conditional propositions, as already discussed, clearly express a relation of dependence of one quality on another, and so they are most appropriately explained by the Connotative Theory. But we have already seen that it is the Predicative Theory which represents the most common and natural way of interpreting propositions, and, as Coffey points out, this view alone "seems appropriate for judgments that are particular, collectively or concretely universal, based on observation and experience,

synthetic, contingent—as opposed to strictly universal, necessary, abstract or generic judgments." Broadly speaking, however, the Denotative interpretation is adopted in Deductive Logic, while the Connotative interpretation is adopted in Inductive Logic.

§ 3. Directions for working out Exercises.

In describing the Logical characters of propositions, we have first to simplify them (Ch. XIII, § 2), *i.e.*, we have to reduce them to their strict Logical forms by

(*a*) Considering what the propositions mean :
(*b*) Resolving compound propositions into their constituent simpler propositions :
(*c*) Carefully distinguishing between the subjects and predicates :
(*d*) Making the copula consist of some form of the verb 'to be.'

After having reduced them to Logical forms, we have next to state whether they are (*a*) simple or compound; (*b*) categorical, hypothetical or disjunctive; (*c*) universal or particular; (*d*) necessary, assertory or problematic; and (*e*) verbal or real.

The following illustrations will be instructive :

(1) 'God is'—Its logical form is 'God is existent'—It is simple, categorical, universal, affirmative (A), assertory, and real.

(2) 'All mangoes are not sweet'—Its logical form is 'Some mangoes are not sweet'—It is simple, categorical, particular, negative (O), assertory, and real.

(3) 'Milton is the only poet, except Dante, who has written religious poetry of the highest order'—It is a compound proposition made up of :—

(*a*) 'Milton is a writer of religious poetry of the highest order'—It is simple, categorical, universal, affirmative (A), assertory, and real.
(*b*) 'Dante is a writer of religious poetry of the highest order'—It is simple, categorical, universal, affirmative (A), assertory, and real.
(*c*) 'No other poet is a writer of religious poetry of the highest order'—It is simple, categorical, universal, negative (E), assertory, and real.

(4) 'Only the Hindus worship Shiva'—Its logical form is either (*a*) 'All who worship Shiva are Hindus'—It is simple, categorical, universal, affirmative (A), assertory, and real; or (*b*) 'No non-Hindus are worshippers of Shiva'—it is simple, categorical, universal, negative (E), assertory, and real.

(5) 'Not being rich is not always an evil'—Its logical form is 'Some cases of not being rich are not an evil'—It is simple, categorical, particular, negative (O), assertory, and real.

(6) 'All men die'—Its logical form is 'All men are necessarily mortal'—It is simple, categorical, universal, affirmative (A), necessary, and real.

(7) 'Few men are wise'—Its logical form is 'Some men are not wise'—It is simple, categorical, particular, negative (O), assertory, and real.

(8) If a man falls down, he does not always break his limbs'—Its logical form is 'In some cases, if a man falls down, he does not break his limbs'—It is simple, hypothetical, particular, negative (O), assertory, and real.

(9) 'Three and four are seven'—Its logical form is 'The sum of the numbers three and four must be seven'—or 'All additions of three and four must be equal to seven'—It is simple, categorical, universal, affirmative (A), necessary, and verbal.

§ 4. Exercises.

1. Define a Proposition and indicate its relation to a Judgment.
2. Clearly explain the characteristics of a Logical Proposition and indicate the mental process of which a Proposition is the product.
3. What is the nature of the Logical Copula? To what part of the proposition do Modality, Tense and the Negative sign belong?—Discuss.
4. What are the various ways of classifying Propositions? Explain and exemplify each class.
5. Classify Propositions according to Composition. Explain and illustrate the different kinds of Compound Propositions.
6. Classify Propositions according to Quality. Explain the nature of Hypothetical and Disjunctive Propositions.
7. What do you consider to be the precise significance of a Disjunctive Proposition, and why?
8. Indicate the relation between the Hypothetical and the Categorical Propositions. Discuss whether Categorical and Hypothetical forms can be reduced to each other without change of meaning.
9. Do Hypothetical and Disjunctive Propositions admit of the distinctions of Quality? Discuss.
10. Classify Propositions according to Quantity. How would you determine the Quantity of (*a*) Singular Propositions, (*b*) Indesignate Propositions and (*c*) Conditional Propositions?
11. Classify Propositions according to Modality.
12. Explain and exemplify Verbal and Real Propositions.
13. Indicate the four-fold Scheme of Propositions. How would you represent A, E, I and O by means of diagrams?
14. What terms are distributed in the propositions represented by the symbols A, E, I and O respectively? State the rules of the Distribution of terms in Propositions. Does an A proposition ever distribute the predicate?

15. How many propositional forms are ordinarily recognized by Logic? What addition to the ordinary scheme was proposed by Hamilton, and on what grounds? Justify or controvert the Hamiltonian Scheme.

16. What do you mean by the Simplification of Propositions? What is its use?

17. Explain what is meant by the Opposition of Propositions? State and illustrate the different forms of Opposition.

18. Draw the Common Square of Opposition and explain it. How does it differ from Aristotle's Square of Opposition? What is the advantage of the latter over the former?

19. Wherein is Sub-contrary Opposition defective? Are I and O propositions opposed to each other? Is Subalternation a kind of Opposition? Discuss.

20. Compare Contrary Opposition and Contradictory Opposition. Which is the most perfect form of Opposition, and why? (*Vide* Ch. XVI, § 6, *Note*.)

21. What is meant by Predicable? Distinguish between a Predicate and Predicable.

22. Explain and illustrate the different relations in which the predicate can stand to the subject.

23. Distinguish between Essence, Property, and Accidens in the Logical senses of these words, giving examples of each, and apply this distinction to explain the difference between Analytical and Synthetical Propositions.

24. Compare Aristotle's and Porphyry's classifications of Predicables, explaining the different points of view from which they arrived at their classifications. Which classification do you consider superior, and on what grounds?

25. In what propositions may the predicate be (*a*) a species, (*b*) a property, (*c*) an inseparable accidens, and (*d*) a separable accidens? In each of these instances state what kinds of terms may appear as subject and predicate respectively.

26. Refer the predicates of the following propositions to their heads:—

(1) Gold is a metal.
(2) Gold is heavier than iron.
(3) Gold is found in Australia.
(4) Gold is yellow.
(5) Gold is, in Latin, Aurum.
(6) Man is capable of progress.
(7) Oxygen is an elementary gas.
(8) Crows are animals.
(9) Crows are black.
(10) Squares are rectangles having their adjacent sides equal.
(11) Lightning precedes thunder.
(12) Chloride of sodium is common salt.
(13) Philosophy is the pursuit of reasoned truth.
(14) All men are able to rectify their errors.
(15) Some historians are philosophers.

(16) An isosceles triangle is a triangle having two sides equal.
(17) Men are mortal.
(18) A triangle has the interior angles equal to two right angles.
(19) Shakespeare was a native of Stratford-on-Avon.
(20) He has got his clothes on.
(21) Courage is a moral quality.

27. Explain the question regarding the Import of Propositions or Theory of Predication. State and explain the different views taken by Logicians on this subject. Which do you consider to be the correct view of interpreting propositions, and why? Which do you believe to represent the common and natural way of understanding Propositions, and why?

28. Give the Genus or Species, the Differentia, a Property, and an Accidens of the following. Also frame analytical and synthetical judgments with these as subjects :—

Triangle, Man, Plant, Animal, Registrar of the University of Calcutta, Teacher, Virtue, Book, Star, Beast of burden, Material body, Knowledge.

29. Reduce the following propositions to their Logical forms and describe their Logical characters :—

(1) Matter cannot change its own state of motion or rest.
(2) Familiarity breeds contempt.
(3) Every man is not learned. (6
(4) One page was overlooked.
(5) All except John tried.
(6) Few fled.
(7) No one is free who doth not command himself.
(8) All are not happy that seem so.
(9) Natives alone can stand the climate of Africa.
(10) Every mistake is not culpable. (C
(11) He jests at scars who never felt a wound.
(12) Nothing is beautiful except truth.
(13) The hand of the diligent maketh rich.
(14) Not to go on is to go back.
(15) All swans are not white. (O
(16) It is mostly the boastful who fail.
(17) Uneasy lies the head that wears a crown.
(18) Where there's a will there's a way.
(19) Whosoever is delighted in solitude is either a wild beast or a god.
(20) Fixed stars are self-luminous.
(21) Few of the passengers escaped.
(22) None but Englishmen fought in that battle.
(23) Not a few women have been great writers.
(24) All is not lost.
(25) No one is always happy.
(26) To be or not to be, that is the question.
(27) The virtuous alone are truly happy.
(28) Only law can give us liberty.

(29) Nothing is annihilated.
(30) Waste not, want not.
(31) Rxercise aids health.
(32) Slow rises worth by poverty depressed.
(33) Very few of these elements occur in Nature in free state.
(34) None think the fools great but the fools themselves.
(35) Anybody can do that.
(36) One kind of metal is at least liquid.
(37) Sixty *per cent.* of the candidates have passed.
(38) There is no man that is not naturally good.
(39) All but a few were taken prisoners.
(40) Many are the deserving who are unfortunate.
(41) Only ignorant persons hold such opinions.
(42) Every term has not connotation.
(43) It never rains but it pours.
(44) In man there is nothing great but mind.
(45) One may be happy without being rich.
(46) Where there is no law, there is no transgression.
(47) They must have come by this time.
(48) I hope to succeed.
(49) Hardly any situation in life is quite free from temptation.
(50) If mercury is heated, it rises in temperature.
(51) Some men are either philosophers or poets.
(52) The soul is either mortal or immortal.
(53) If the wind blows from the north, it will be hot.
(54) The earth moves round the sun.
(55) The trade of a country does not always suffer, if its exports are hampered by foreign duties.
(56) If a man is honest, he will not deceive.
(57) Though a country be well-governed, nevertheless its people need not be happy.
(58) Two srtaight lines cannot enclose a space.
(59) If the barometer falls, we shall have rain.
(60) He is neither wicked nor foolish.

30. Give the contradictory, the contrary or sub-contrary, the subalternant or subalternate of the above propositions.

Part III

INFERENCES

CHAPTER XV

INFERENCE IN GENERAL

Inference is the process of passing from given to new truths.

§ 1. **Nature and Kinds of Inference.** We have already attempted to understand the nature of Inference in Ch. II, §§ 1, 2, 3 and 4. Thus we have seen that Inference is the process by which we pass from given and known truths to some other truth not already known. The new truth arrived at by the process is called the *conclusion*, and the known truths with which the process starts are called the *data* or *premises*.

Different senses in which the term 'Inference' has been used.

The term **'Inference'** has been used at least in *three different senses, viz.*, (1) the mental process by which the conclusion is drawn from the premises; (2) the mental product consisting of the premises and the conclusion; and (3) the conclusion only. But as Logic is not concerned with the mental process, the term is very rarely used in Logic in the first sense. It is more properly used in Logic in the second sense. When we use the term in the second sense, we generally speak of 'an inference' or 'inferences'. An Inference expressed in language is called an Argument. But the terms 'Inference' and 'Argument' have their special uses also. Thus the term 'Argument' is sometimes used in the sense of 'reasons advanced', but in this sense the term 'Inference' is never used. Again, the word 'Inference' sometimes signifies the conclusion only, but the word 'Argument' never does so. Further,

Inference and Argument.

the word 'Argument' is also used at times for a *series of inferences* advanced to prove or disprove a certain position.

Importance of Inference.

The importance of Inference is, indeed, very great. Knowledge, we have already seen (Ch. II, § 6), is either immediate or mediate. By immediate knowledge we come to know what is particular and what is present before the senses, and by mediate or inferential knowledge we come to know what is general and what is past, distant and future. Thus, if we are to acquire general or scientific knowledge, and if we are to know what is distant from us either in space or in time, we must take the help of Inference. It is through Inference, therefore, that by far the greater and more important part of human knowledge is obtained, and that systematization and rationalization of knowledge become at all possible. Had we been confined to immediate knowledge only, had there been no extension of knowledge beyond the immediate present, our knowledge would certainly have remained detached and unconnected, and we could scarcely have progressed far above the level of lower animals.

***Similarity is the ground of Reasoning,* which may be either from Particulars to Particulars (Analogy),**

Bain distinguishes three kinds of Reasoning or Inference, and maintains that "Reasoning, in every form, supposes the operation of Similarity—the assimilating of one thing to some other thing."

"The most general type of Reasoning is to infer from one particular fact to another particular fact of the same kind; the likeness being both the means of suggestion, and the justification of the transfer of properties. We throw a stone into a pool; it makes a splashing noise, sinks to the bottom, and diffuses a series of waves from the point where it fell. We infer or reason, or presume, that another stone thrown into the same pool will be followed by the same series of effects; and we may extend the inference to another pool, or to any mass of liquid. This is to infer ....to make an affirmation respecting the unknown. This mode of Reasoning is in constant use, and extends to

the animal intelligence. An animal accustomed to find a shelter under a bush, reasons from one bush to another bush, being moved solely by the resemblance of the second to the first.

or from Particulars to Generals (Induction),

A second mode of Reasoning is when by the help of general language, we infer from one or a few cases, to all cases of the kind; as when we conclude, after a certain number of trials, that all stones sink in water........... This is *Induction*........... The mental process is still Similarity, or the process whereby one thing suggests other resembling things. It is by similarity that we assemble in the mind all kindred facts that have ever come under our knowledge; we then are able to compare the points of agreement, with a view to an accurate general statement, in other words, an Inductive proposition.

or from Generals to Particulars (Deduction).

The third kind of Reasoning, called *Deductive*, is also based on the tracing of resemblance. When we infer that, because all stones sink in water, a certain body will sink (which is Deduction), it is because that body resembles the rest, or has the points of community indicated by the general word 'stone'. When we have mastered a general principle, it is by similarity that we discover cases to apply it to, and so extend our knowledge deductively." (*Deduction*, Intro., § 14.)

Classification of Inferences.

We have also indicated before (Ch. II, § 2) that Inference may be either immediate or mediate according as the conclusion is drawn from a single premise or from more. Inference has also been shown (Ch. II, §§ 2 and 4) to fall under two great divisions, *viz.*, Deductive and Inductive. Hence Inference may be divided in two ways thus:—

I

- Inference
  - Immediate
  - Mediate
    - Deductive
    - Inductive.

II

- Inference
  - Deductive
    - Immediate
      - From a Term
      - From a Proposition
    - Mediate
      - Syllogistic
        - Pure
        - Mixed.
      - Non-Syllogistic
  - Inductive

The second of these two divisions seems to be preferable, for the contrast between Deductive and Inductive Inferences is stronger and more prominent than that between Immediate and Mediate Inferences. Hence it is more appropriate to express the contrast by representing Deduction and Induction as two main members, rather than as subordinate members, of the division. Again, we know that not only Mediate Inference is Deductive, but Immediate Inference may also be regarded as Deductive. The distinction between Deductive and Inductive Inferences has already been indicated in the Introduction. Inductive Inference is usually Mediate. If, however, a general conclusion could be drawn from a single instance, *i.e.*, from a *single premise*, it would certainly be an Immediate Inductive Inference. We shall treat of Induction fully in Book III. Book II deals with Deduction. The nature and forms of Immediate Inference have been considered in this Chapter and in the next. The rest of Book II will be devoted to the exposition of the nature of Syllogism. Some forms of Non-Syllogistic arguments will also be mentioned towards the end of Book II (*vide* Ch. XVII, § 20).

Immediate Inference defined. *Two kinds*:

§ 2. **Character and Kinds of Immediate Inference.** An *Immediate Inference* is an inference in which a conclusion is directly drawn either from a single term or from a single proposition, without the help of any other term or proposition. Thus it is evident that immediate inference is of *two kinds*, *viz.*, (1) immediate inference from a single term, and (2) immediate inference from a single proposition.

(1) Inference from a term. (2) Inference from a proposition.

The first kind consists in analysing the connotation of a given term in verbal propositions with the term as the subject and the connotation as the predicate.

Thus, given the term 'Man' we know that its connotation consists of the two attributes of animality and rationality, and hence we may at once infer the following two verbal propositions: (*a*) 'Man is animal', (*b*) 'Man is rational'. The second kind consists in directly deducing from a single proposition given as true, other propositions necessarily following from it. This is illustrated when from the given proposition 'All men are mortal' we infer such propositions as 'Some mortal beings are men', 'No men are immortal', 'No immortal beings are men', 'Some not-men are immortal', and so on. There are nine different forms of immediate inference from a given proposition, *viz.*, (1) Conversion, (2) Obversion, (3) Contraposition, (4) Inversion, (5) Opposition, (6) Change of Relation, (7) Modal Consequence, (8) Inference by Complex Conception, and (9) Inference by Added Determinants. The name **Eduction** has been given to the first four forms, because they *draw out* or unfold the various implications of a given proposition. We shall consider the nature of these different forms in the next Chapter.

Different forms of the latter.

Character of Immediate Inference. **Mill and Bain contend that Immediate Inference is no inference at all.**

A question has often been asked: Is immediate inference a form of inference at all, or is it merely a case of verbal transformation? Mill and Bain contend that immediate inference is no inference at all, for "there is in the conclusion no new truth, nothing but what was already asserted in the premises, and obvious to whoever apprehends them" (Mill, *System of Logic*, II, i, § 2). And, as all inference consists in passing from the known to the unknown—from a given fact to a new fact, immediate inference cannot be regarded as a true form of inference, for, here "there is merely the transition from one wording to another wording of the same fact" (Bain, *Deduc-*

tion, p. 108). This view has, however, been rejected by others, who maintain that the conclusion of an immediate inference is implicitly contained in the premise and is thus *not known* before it is made explicit and *known* by the process of immediate inference. Hence, immediate inference enables us to pass from a given fact to a *new* fact, and thus extends our knowledge, and is, therefore, a true form of inference. And Welton has truly remarked that Mill's objection "would be fatal to all inference ; for in every valid inference the conclusion must be a necessary consequence of the premises, and, therefore, potentially known as soon as these are fully apprehended. The step from premise to conclusion in an Immediate Inference is small; but this does not prove that it is no step at all, or that it is unnecessary to take it" (Welton, *Manual of Logic*, I, pp. 226-7).

But in Immediate Inference also, there is a passage from the known to the unknown

Hence it is a true form of inference.

CHAPTER XVI

IMMEDIATE INFERENCES

Conversion is the admissible transposition of the subject and the predicate of a proposition.

§ 1. **Conversion.** 'Conversion is the admissible transposition of the subject and the predicate of a proposition' (Ray). It is a form of immediate inference in which from a given proposition we infer another having the same terms but with their order reversed, *i.e.*, the subject of the original proposition becoming the predicate of the conclusion and the predicate of the original proposition becoming the subject of the conclusion. The given premise is called the *Convertend*, and the conclusion is called the *Converse*.

Rules of Conversion.

There are three rules of Conversion: (1) The subject and the predicate of the convertend must respectively be the predicate and the subject of the converse.

(2) The quality of the converse must be the same as that of the convertend.

(3) No term should be distributed in the converse which is not distributed in the convertend.

The first rule follows from the very definition of Conversion. The second rule follows from the very meaning of affirmative and negative propositions, inasmuch as from an affirmative proposition, such as 'S is P' (which means that at least one S is included

in P), we cannot infer a negative proposition, such as 'P is not S' (which means that at least one P is excluded from S), for P and S may coincide. Similarly, it may be shown that we cannot infer an affirmative, such as 'P is S', from a negative, such as 'S is not P'. The third rule follows from the very nature of Deductive inference, in which the conclusion can never be more general than the premise or premises, and consequently no term which is undistributed in the premise can be distributed in the conclusion. We violate the rule, if we take more in the conclusion than what is given in the premise and thus make the conclusion more general than the premise. We may, indeed, take less, but never more, in the conclusion than what is given in the premise.

Application of the Rules.

By applying the above rules to the four forms of propositions, *viz.*, **A, E, I** and **O,** we may get the following results by Conversion.

The converse of **A** is **I.**

(1) The converse of **A** is **I.** The converse of **A** must be affirmative (Rule 2), so it cannot be **E** or **O,** it must be either **A** or **I**; but it cannot be **A** (Rule 3), for **A,** being universal affirmative, does not distribute its predicate; therefore, when the predicate becomes the subject in the converse (Rule 1), it must remain undistributed. In other words, the converse must be particular, for if it be **A,** Rule 3 will be violated. Hence we see that the converse of **A** cannot be **A,** it must be **I.** Thus the converse of 'All S is P' is 'Some P is S,' the converse of 'All men are mortal' is 'Some mortal beings are men.'

The converse of **E** is **E.**

(2) The converse of **E** is **E.** The convertend being negative, the converse must also be negative. **E,** being universal negative, distributes both subject and predicate. Therefore, when the subject and predicate of the convertend become respectively the predicate and subject in the converse, they may be distributed. Hence we see that the converse of **E** is **E.** Thus, the converse of 'No S is P' is 'No P is S,' and the converse of 'No men are perfect' is 'No perfect beings are men.'

The converse of **I** is **I.**

(3) The converse of **I** is **I.** The convertend being affirmative, the converse must also be affirmative; **I,** being particular affirmative, distributes no term, so no term can

be distributed in the converse. Hence we see that the converse of **I** must be **I**. Thus the converse of 'Some S is P' is 'Some P is S,' the converse of 'Some men are wise' is 'Some wise beings are men.'

O cannot be converted.

(4) **O** cannot be converted. **O** being negative, its converse must also be negative (Rule 2). Again, **O**, being particular negative, distributes only the predicate, and not the subject. Therefore, if the undistributed subject of the convertend become the predicate (Rule 1) in a negative converse, a term is distributed in the converse which is not distributed in the convertend. But this is against Rule 3. Thus we see that **O** propositions cannot be converted at all.

Conversion by Negation.

Some Logicians, however, attempt to convert an **O** proposition by a process known as **'Conversion by Negation.'** Thus they first reduce the given **O** proposition to the form of an **I** proposition by connecting the negative particle 'not' with the predicate, and then convert it as indicated above. Thus the proposition 'Some S is not P' is first reduced to the form 'Some S is not-P'. This proposition, being an **I** proposition, is next converted into 'Some not-P is S'. But Conversion by Negation cannot be said to be a true form of Conversion, inasmuch as in Conversion the quality shall remain unchanged, whereas in Conversion by Negation the quality is changed, since the convertend is an **O** proposition and the converse is an **I** proposition. Again, in Conversion the subject of the converse should be the predicate of the convertend, whereas in Conversion by Negation the subject of the converse is the contradictory of the predicate of the convertend. Thus we see that an **O** proposition cannot really be converted.

It is not a true form of Conversion.

We may now sum up the results. The converse of **A** is **I**, of **E** is **E**, and of **I** is **I**, while **O** cannot be converted.

Diagrammatic Proof of the above results.

The above results may also be proved by means of diagrams.

Fig. I

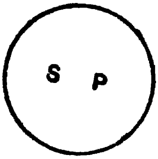


Fig. II

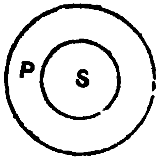


Fig. III

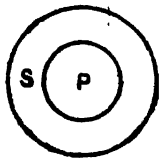


Fig. IV

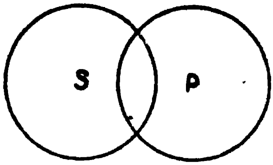
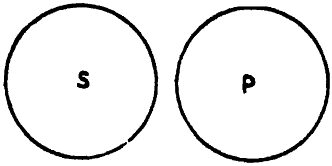


Fig. V



(1) We know (Ch. XIII, § 6) that an **A** proposition 'All S is P' is represented by diagrams I and II. From diagram I we may infer 'All P is S' and 'Some P is S.' But from diagram II we infer only 'Some P is S.' Hence the common conclusion that we can draw from both these diagrams is 'Some P is S.' (2) An **E** proposition 'No S is P' is represented by diagram V. From the diagram we can at once draw the conclusion 'No P is S.' (3) An **I** proposition 'Some S is P' is represented by diagrams I, II, III and IV. From diagrams II and IV we may infer 'Some P is S' and 'Some P is not S;' but from diagrams I and III we cannot infer 'Some P is not S,' though we can infer 'Some P is S.' Hence the common conclusion that we can draw from these four diagrams is 'Some P is S.' (4) An **O** proposition 'Some S is not P' is represented by diagram III, IV and V. From diagram V we may infer 'Some P is not S' and 'No P is S'; but none of these two conclusions can be drawn from diagram III. Again, from diagram III we may infer 'All P is S' and 'Some P is S;' but none of these conclusions can be inferred from diagram V. Hence we see that no common conclusion can be drawn from these three figures. Thus it is proved that an **O** proposition cannot be converted.

Kinds of Conversion–Simple Conversion and Conversion *per accidens*

Kinds of Conversion. There are two kinds of Conversion, *viz.*, Simple Conversion and Conversion *per accidens* (or by limitation) or Partitive Conversion. In **Simple Conversion,** the quantity of

or by Limitation.

the converse is the same as that of the convertend. In other words, a proposition is said to be converted *simply,* when its quantity remains unchanged in the converse. In **Conversion per accidens,** the quantity of the converse is different from that of the convertend. In other words, a proposition is said to be converted *per accidens* or *by limitation,* when its quantity is limited in the converse. In this kind of Conversion, the convertend is universal and its converse is particular. Thus it is obvious that **E** and **I** propositions are converted *simply,* while **A** propositions are converted by *limitation.*

*Can an **A** proposition be converted simply?*

Note 1. It has been laid down as a general rule that an **A** proposition cannot be converted *simply,* because it does not ordinarily distribute the predicate. Some cases of **A** proposition, however, may be converted *simply, viz.,* those in which the predicate and the subject are co-extensive with, or equivalent in extent to, each other. Thus we may convert *simply* (1) all logical definitions (*e.g.,* 'All men are rational animals'); (2) all those forms of **A** proposition in which the predicate and the subject are both singular terms, collective terms or abstract terms used as such (*e.g.,* 'Lord Reading is the present Viceroy of India,' 'The second legion is the only legion quartered in Britain,' 'Virtue is the condition of happiness'); (3) all propositions in which the predicate is a synonym of the subject (*e.g.,* 'The elder Pitt is Chatham'). It must also be noted that a singular proposition having for its subject a singular term and for its predicate a general term is to be converted *simply,* if the proposition is negative; and is to be converted *per accidens,* if the proposition is affirmative. Thus 'Socrates is no mathematician' is converted into 'No mathematician is Socrates,' while 'Socrates is a philosopher' is converted into 'Some philosopher is Socrates'.

Conversion of singular propositions.

Inference by Converse Relation consists in transposing the terms of a proposition and in substituting the correlative of the word which

Note 2 **Inference by Converse Relations.** Dr. Keynes describes a form of Conversion, which he calls *Inference by Converse Relation.* It consists in transposing the terms of a proposition and in replacing the word, which indicates the relation between the subject and predicate, by its correlative. It is obvious that such inference is possible when the subject and predicate of a proposition are relative terms. Thus from the propositions 'Alexander was the son of Philip,' 'Henry is the husband of Catherine,' 'A is greater than B' we may respectively infer 'Philip was the father of Alexander,' 'Catherine is the wife of Henry,' 'B is less than A.' Some Logicians, however, rightly maintain that

this form of inference falls outside the province of Formal Logic, as it requires a knowledge of the meaning of the subject and predicate (§ 10).

indicates the relation between the terms.

Ground and Importance of Conversion. Conversion, as Mill says, is based on the Principle of Identity, which he states thus: "Whatever is true in one form of words, is true in every other form of words which conveys the same meaning." Though conversion seems to be a very simple process, yet we very often find ourselves in error in connection with its application. Thus it has been pointed out that "the conversion of an **A** proposition without limitation or depression of the quantity, is one of the commonest of all fallacies; and it is particularly liable to be incurred in dealing with indesignate propositions. Thus, it is wrongly inferred that because clever people have large brains all who have large brains are clever; that because idlers are commonly out of work all unemployed people are idlers; that because pious people go to church all church-going people are pious; that because all beautiful things are agreeable all agreeable things are beautiful" (Coffey, *Science of Logic*, I, p. 237).

Conversion is based on the Principle of Identity.

Conversion of an **A** proposition without limitation is one of the commonest of errors.

Conversion of Hypothetical Propositions. We have seen that the antecedent and the consequent of a hypothetical proposition correspond respectively to the subject and the predicate of a categorical proposition. Hence a hypothetical proposition can be easily converted by transposing its antecedent and consequent, and in the case of an **A** proposition by changing the quantity as well. As the quality of a hypothetical proposition is determined by the quality of the consequent, and as in conversion there is no change of quality, the quality of the consequent of the converse must be the same as the quality of the consequent of the convertend.

Conversion of Hypothetical propositions.

Examples

Convertend		*Converse*
A. If A is B, C is D; or, In all cases if A is B, C is D.	**I.**	In some cases if C is D, A is B.

Illustrations.

Convertend	*Converse*
I. If A is B, C is sometimes D; or, In some cases if A is B, C is D.	**I.** If C is D, A is sometimes B.
E. If A is B, C is not D; or, In no cases if A is B, C is D.	**E.** If C is D, A is not B.
E. If A is not B, C is not D; or, In no cases if A is not B, C is D.	**E.** If C is D, A is not not-B.

O. If A is B, C is sometimes not D.—Cannot be converted.
A. If A is not B, C is D.—**I.** If C is D, A is sometimes not-B.

A. If any man is honest, he is trusted (*Convertend*);
I. ∴ If a man is trusted, he is sometimes honest (*Converse*).
E. If any man is happy, he is not vicious (*Convertend*);
E. ∴ If any man is vicious, he is not happy (*Converse*).
I. If a story is believed, it is sometimes true (*Convertend*);
I. ∴ If a story is true, it is sometimes believed (*Converse*).

Obversion is the process of substituting for the predicate its contradictory term.

§ 12. **Obversion.** Obversion is the legitimate substitution of the contradictory of the predicate of a given proposition for the predicate itself. This process is always legitimate in the case of a proposition of any form, if at the same time we change the quality of the proposition. Hence we may define obversion more fully thus: It is a form of immediate inference 'which consists in taking the contradictory of the predicate of the given proposition, and then changing the quality of the proposition' (Ray). Thus Obversion, being a process of double negation, involves no change of meaning, it merely expresses an affirmative proposition in a negative form, or a negative proposition in an affirmative form. It is evident that Obversion is ultimately based on the Principles of Contradiction and Excluded Middle. The Obversion of affirmative pro-

Ground of Obversion.

positions is based on the Principle of Contradiction which may be stated thus: 'Affirmation of an assertion and the denial of its contradictory are logical equivalents,' *i.e.*, if any term be affirmed of a subject, the contradictory term may be denied of the subject without any change of meaning. The Obversion of negative propositions is based on the Principle of Excluded Middle, which may be stated thus; "Denial of either of the two contradictories and the assertion of the other are logical equivalents," *i.e.*, if any term be denied of a subject, its contradictory may be affirmed of the subject without any change of meaning. The given premise is called the *Obvertend* and the conclusion is called the *Obverse*. The process of Obversion has many other names: Permutation (Fowler and Ray), Æquipollence (Ueberweg and Ray), Infinitation (Bowen), Immediate Inference by Privative Conception (Jevons), Contravension (De Morgan).

Different names.

Rule of Obversion.

It follows from what has been said above that (1) the subject of the obverse shall be the same as the subject of the obvertend; (2) the quantity of the obvertend and the obverse shall be the same; (3) the predicate of the obverse shall be the contradictory of the predicate of the obvertend; (4) the quality of the obverse shall be different from the quality of the obvertend. All these can be summed up in one simple rule, *viz.*, *negative the predicate and change the quality of the given proposition, but leave its quantity unaltered*.

Application of the rule.

By applying the above rule to the four forms of propositions we get the following results by Obversion:—

The obverse of A is E.

(1) The obverse of A is E. Take an A proposition 'All S is P.' The predicate of the obverse shall be not-P, which is the contradictory of the original predicate P. The

quality of the obverse shall be different from the quality of the obvertend, *i.e.*, the obverse shall be negative, as the obvertend is affirmative. The subject of the obverse shall remain the same, *viz.*, S, and the quantity of the obverse shall remain the same, *viz.*, universal. Hence by obverting 'All S is P' we get 'No S is not-P.' Similarly, the obverse of 'All men are mortal' is 'No men are not-mortal.'

The obverse of **E** is **A**.

(2) The obverse of **E** is **A**. The obverse of 'No S is P' is 'All S is not-P.' The obverse of 'No men are perfect' is 'All men are not-perfect'. Here the rule of Obversion has been fully observed. We have taken the contradictory of the original predicate for the predicate (the subject remaining the same), we have changed the quality from negative to affirmative and the quantity has been left unaltered.

The obverse of **I** is **O**.

(3) The obverse of **I** is **O**. The obverse of 'Some S is P' is 'Some S is not not-P.' The obverse of 'Some men are wise' is 'Some men are not not-wise.' It is easy to see that all the conditions of Obversion have been fulfilled.

The obverse of **O** is **I**.

(4) The obverse of **O** is **I**. The obverse of 'Some S is not P' is 'Some S is not-P'. The obverse of 'Some men are not wise' is 'Some men are not-wise.' Here also it is easy to see that the rule of Obversion has been fully observed.

Proof by Diagrams.

The above results may also be proved by means of diagrams, already mentioned and drawn to prove the results of Conversion (§ 1). We have only to add here that 'Not-P' and 'Not-S' mean respectively the whole space outside P (*i.e.*, everything else but P) and the whole space outside S (*i.e.*, everything else but S). (1) We have already seen (Ch. XIII, § 6) that an **A** proposition ('All S is P') is represented by Figs. I and II. In both these diagrams the whole of S is outside the whole of Not-P. Hence our conclusion is 'No S is not-P.' (2) An **E** proposition ('No S is P') is represented by Fig. V. In the diagram the whole of S is included in Not-P. Hence our conclusion is 'All S is not-P.' (3) An **I** proposition ('Some S is P') is represented by Figs. I, II, III and IV. From diagrams I and II we may infer 'No S is not-P', from which we may further infer 'Some S is not not-P.' Again, from diagrams III and IV we may infer 'Some S is not-P' as well as 'Some S is not not-P.' But the affirmative form 'Some S is not-P' cannot be inferred from diagrams I and II. Hence the common conclusion that we may infer from all these diagrams is 'Some S is not not-P.' (4) Lastly, an **O** proposition ('Some S is not P') is represented by Figs. III, IV and V. From diagram V we may infer 'All S is not-P,' but this conclusion cannot be inferred from diagrams III and IV, where only a part of S is included in Not-P. Hence the common conclusion that we may infer from all these diagrams is 'Some S is not-P.'

We may now sum up the results: The obverse of **A** is **E**, of **E** is **A**, of **I** is **O**, and of **O** is **I**.

Material Obversion.

Bain describes a process of immediate inference, which he calls Material Obversion. By Material Obversion is meant the process of drawing 'obverse inferences which are justified only on an examination of the matter of the proposition.' For examples, 'Warmth is agreeable; therefore, cold is disagreeable. War is productive of evil; therefore, peace is productive of good. Knowledge is good; therefore, ignorance is bad'.* The process thus consists in inferring from a given proposition another having for its subject the contrary of the original subject and for its predicate the contrary or contradictory of the original predicate, leaving both quality and quantity unaltered. But it is incorrect to regard the process as a form of immediate inference, for the conclusion does not in any way follow from the original proposition; it may be suggested by the original proposition, but it requires quite independent examination of the facts to establish itself. Again, it is a mistake to call this process obversion, for the conclusion neither has the same subject as the original proposition nor is its quality different. Moreover, as two opposite subjects may yet have the same predicate, this form of inference is often erroneous, *e.g.*, from 'Light is beneficial' it does not follow that 'Darkness is harmful', from 'Work is pleasurable' it does not follow that 'Rest is painful.' Lastly, as Material Obversion is based on actual experience, that is, on an examination of the matter

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* *Vide* Keynes, *Studies and Exercises in Formal Logic*, Part II, Ch. IV, § 74.

of the proposition, it cannot have a place in Formal Logic.

Obversion of Hypothetical propositions.

Obversion of Hypothetical Propositions. A hypothetical proposition may be obverted by taking the contradictory of its consequent as the consequent of the obverse, and then changing the quality of the given proposition by changing the quality of the consequent.

Examples

Illustrations

Obvertend	*Obverse*
A. If A is B, C is D . .	**E.** If A is B, C is not not-D. or, In no cases if A is B, C is not-D.
E. If A is B, C is not D . .	**A.** If A is B, C is not-D.
I. In some cases If A is B, C is D	**O.** In some cases if A is B, C is not not-D.
O. If A is B, C is sometimes not D	**I.** If A is B, C is sometimes not-D.

A. If any man is honest, he is trusted (*obvertend*);
E. ∴ If any man honest, he is never distrusted (*obverse*).
E. If any man is happy, he is not vicious (*obvertend*);
A. ∴ If any man is happy, he is not-vicious (*obverse*).
I. If a story is believed, it is sometimes true (*obvertend*);
O. ∴ If a story is believed, it is not sometimes untrue (*obverse*).
O. Sometimes if a man is intelligent, he is not wise (*obvertend*);
I. ∴ Sometimes if a man is intelligent, he is unwise (*obverse*).

Note. In some cases of Obversion the corresponding Privative term may be substituted for the contradictory term, *e.g.*, 'imperfect' for 'not-perfect,' 'immortal' for 'not-mortal.' But unless the privative term is exactly equivalent in meaning to the formal negative, we cannot substitute the one for the other, as, for example, we cannot substitute 'poor' for 'not-rich', 'unhappy' for 'not-happy'. This fact that sometimes a privative term may be used for the contradictory of the predicate has led Jevons and others to give Obversion the name of *Immediate Inference by Privative Conception*. But we should always bear in mind what Welton remarks in this connection: "Obversion is, in short, a formal process; and, therefore, if we do not use a formal negative term for our new predicate, we must make sure that the term we do use is the exact equivalent of that formal negative (*Manual of Logic*, I, p. 253).

§ 3. **Contraposition.** Contraposition is a form of immediate inference in which from a given proposition we infer another having for its subject the contradictory of the original predicate. The conclusion is called the *Contrapositive* We know that the contradictory of the original predicate appears as the predicate in the obverse of the given proposition, and in the process of first obverting the given proposition the quality is changed. Next, the contradictory of the predicate may be made the subject in the contrapositive, if we convert the obverse of the original proposition. And we know that in the case of converting an **A** proposition the quantity is changed. Thus Contraposition has been more fully described as a process of immediate inference which 'consists in taking the contradictory of the predicate of the given proposition as the subject of the inference, and the subject as the predicate, and then changing the quality or both the quality and the quantity of the proposition, if required' (Ray).

Contraposition consists in inferring from a given proposition another proposition having for its subject the contradictory of the original predicate. It is, in short, the conversion of the obversion of a proposition.

From what has been said above it is quite clear that the contrapositive of a given proposition is its *converted obverse ;* and the simple rule for contraposition is: *First obvert, then convert.* And, as Contraposition is a compound form of immediate inference, involving both obversion and conversion, it is justified by the Laws of Thought, being based on the Law of Identity (the ground of Conversion) and the Laws of Contradiction and Excluded Middle (the grounds of Obversion).

Practical Rule of Contraposition and its Ground.

In fact, Contraposition may be regarded as a simple process of immediate inference quite as much as Conversion and Obversion. Some Logicians, however, are not justified in excluding Contraposition from immedate inference on the ground that it is

Contraposition may be regarded as a simple process of Inference.

not a separate process of inference, but only a compound form of obversion and conversion. Thus we can contrapose propositions independently and directly (1) by taking the contradictory of the original predicate as the subject of the conclusion and the original subject as the predicate of the conclusion, then changing the quality and observing the rule that no term is distributed in the conclusion which is not distributed in the premise; and also (2) by diagrams.

Rule of Contraposition.

Let us now see what conclusions follow from the Contraposition of the four forms of propositions.

Application of the rule.

The contrapositive of **A** is **E**.

(1) The contrapositive of **A** is **E**. The contrapositive of 'All S is P' is 'No not-P is S'. The contrapositive of 'All men are mortal' is 'No not-mortals are men.' Here the contradictory of the original predicate has been taken as the subject of the conclusion and the original subject as the predicate; the quality of the proposition has been changed from affirmative to negative; and no term has been distributed in the contrapositive which was not distributed in the given premise. The conclusion may also be established by the help of obversion and conversion. Thus the obverse of 'All S is P' is 'No S is not-P,' and the converse of the observe is 'No not-P is S,' which is the contrapositive. This conclusion may further be proved by means of diagrams, already mentioned and drawn in § 1. An **A** proposition is represented by diagrams I and II. In both these diagrams the whole of S is excluded from the whole of Not-P. Hence our conclusion is 'No not-P is S.'

The contrapositive of **E** is **I**.

(2) The contrapositive of **E** is **I**. The contrapositive of 'No S is P' is 'Some not-P is S.' The contrapositive of 'No men are perfect' is 'Some not-perfect beings are men.' Here all the conditions of contraposition mentioned above have been fulfilled and the conclusion may also be obtained by obversion and conversion. Thus the obverse of 'No S is P' is 'All S is not-P,' and the converse of this obverse is 'Some not-P is S,' which is the contrapositive. The same conclusion also follows from diagram V, which is the only diagram representing an **E** proposition.

I cannot be contraposed.

(3) **I** cannot be contraposed. This follows from the conditions mentioned above. The contrapositive of **I** must be negative, but a negative proposition distributes its predicate. Thus the original subject which is undistributed, becomes distributed in the contrapositive, and this is absurd. Hence an **I** proposition cannot be contraposed. This can also be established by the double processes of obversion

and conversion. Thus the obverse of 'Some S is P' is 'Some S is not not-P,' and this being an **O** proposition, cannot be converted. Hence **I** cannot be contraposed. The same conclusion may further be proved by means of diagrams. An **I** proposition is represented by Figs., I, II, III and IV. From diagrams III and IV we may infer 'Some not-P is S,' but this cannot be inferred from diagrams I and II. Again, from diagrams I, II and III we may infer 'Some not-P is not S,' but this cannot be inferred from diagram IV. Thus no common conclusion can be inferred from all these diagrams. Hence **I** cannot be contraposed (*vide Note* at the end of this Section).

The contrapositive of O is I.

(4) The contrapositive of **O** is **I.** The contrapositive of 'Some S is not P' is 'Some not-P is S.' The contrapositive of 'Some men are not wise' is 'Some not-wise beings are men.' Here all the conditions of contraposition are satisfied, and the conclusion may as well be established by the double method. Thus the obverse of 'Some S is not P' is 'Some S is not-P,' and the converse of this obverse is 'Some not-P is S,' which is the contrapositive. This may further be proved by means of diagrams. Thus the conclusion 'Some not-P is S' may be inferred from each of the diagrams III, IV and V, which represent an O proposition.

. We may now sum up the results: The contrapositive of **A** is **E,** of **E** is **I,** and of **O** is **I,** while **I** cannot be contraposed. It should be noted that in every case the contrapositive differs in quality from the original proposition; and that the quantity remains unaltered, except in the case of **E.**

Contraposition of Hypothetical propositions.

Contraposition of Hypothetical Propositions. A hypothetical proposition can be contraposed by taking the contradictory of the original consequent as the antecedent of the contrapositive, and the original antecedent as the consequent of the contrapositive, and then changing the quality of the proposition in all cases, and the quantity also in the case of **E.**

Examples

Illustrations.

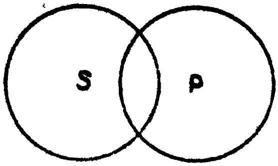
Original Proposition			*Contrapositive*
A.	If A is B, C is D ...	**E.**	If C is not-D, A is not B.
E.	If A is B, C is not D ...	**I.**	In some cases if C is not-D, A is B.
O.	In some cases if A is B, C is not D ...	**I.**	In some cases if C is not-D, A is B.

A. If any man is honest, he is trusted (*original prop.*);
E. ∴ If any man is not trusted, he is not honest (*contrapositive*).
E. If any man is happy, he is not vicious (*original prop.*);
I. ∴ If a man is not vicious, he is sometimes happy (*contrapositive*).
O. If a man is intelligent, sometimes he is not wise (*original prop.*);
I. ∴ If a man is not wise, sometimes he is intelligent (*contrapositive*).

Do the Circles of Euler pass the contrapositive of I as valid?

Note. It has sometimes been held that the invalidity of the contrapositive of I cannot be proved by means of Euler's Circles. For, the conclusion Some not-P is not S' can be inferred from all the diagrams representing an I proposition, and thus Euler's Circles which pass the contrapositive of I as valid, have been regarded as useless for testing the validity of inference. But against this view we have held that although the conclusion 'some not-P is not S' may follow from diagrams I, II and III, it does not follow from diagram IV. Let us show here at length that the conclusion 'some not-P is not S' does not follow from diagram IV. The interpretation of Euler's Circles is usually based on the assumption that every term, whether subject or predicate of a proposition, has a contradictory, *i.e.*, it does not exhaust the whole universe of thought and existence, but there also exist not-S and not-P in the universe of discourse; so that S, not-S, P, not-P all represent existing classes. This assumption leads to the result that in diagrams, I, II, III and IV, there should be some part of the universe—not-S and not-P—lying outside both the circles; so that there must be something which is both not-S and not-P, and consequently some not-P must be excluded from the whole of S. But in diagram IV, another possible alternative may be this that S, not-S, P, not-P are all of them represented within the circles, and in this case, therefore, the pair of circles may between them exhaust the whole universe, so that there may not be something which is both not-S and not-P. According to this latter alternative, a part of P coincides with a part of S; and some not-P, which lies outside P and consequently outside the coinciding part of P, lies outside the coinciding part of S and not outside the whole of S. Hence, from the two possible interpretations of diagram IV, all that is known for certain is that some not-P is excluded from a part, and not from the whole, of S; or, in other words, the proposition 'Some not-P is not S' is not true, for in an O proposition the predicate is distributed or taken in its entire extent. (*Vide* Keynes, *Formal Logic*, 4th Ed., § 130.)

Fig. IV



§ 4. **Inversion.** Inversion is a form of immediate inference in which from a given proposition we infer another having for its subject the contradictory of the original subject. The original proposition is called the *Invertend*, and the conclusion is called the *Inverse*. There are two forms of *Inversion*, *viz.*, (1) **Partial Inversion,** in which the predicate of the inverse is the same as that of the invertend, and (2) **Complete Inversion,** in which the predicate of the inverse is the contradictory of the original predicate.

Inversion consists in inferring from a given proposition another having for its subject the contradictory of the original subject. There are two forms of Inversion: (a) Partial and (b) Complete.

Inversion involves a repeated and combined application of conversion and obversion ; and hence, like contraposition, it is a compound mode of immediate inference and is ultimately based on the three Laws of Thought. In order to get the contradictory of the original subject for the subject of the conclusion, the original subject is first to be made the predicate of a proposition by conversion and then that proposition is to be obverted, and finally the contradictory of the original subject thus obtained in the position of predicate is again to be restored to the position of subject by another act of conversion. Thus the following rule of Inversion has been framed: *Convert either the Obverted Converse or the Obverted Contrapositive.* We may further simplify this rule thus: Alternately convert and obvert, or obvert and convert, the given proposition till the inverse is obtained. And we shall presently see that we are to begin with conversion in the case of **E,** and with obversion in the case of **A;** that **I** and **O** cannot be inverted ; and that the quantity in every case is changed from universal to particular. Let us illustrate the rule by applying it in the case of the four forms of propositions.

Inversion is ultimately based on the three Laws of Thought, 'as it is nothing but a compounding of conversion with obversion.'

Rule of Inversion.

Application of the rule.

(1) **A** proposition.

The Complete inverse of A is I, and the Partial inverse of A is O.

Let us begin with conversion :

A. All S is P........................*Invertend*; ... ... (*a*)
I. ∴Some P is S...............[by converting (*a*)]; ... (*b*)
O. ∴Some P is not not-S......[by obverting (*b*)]. ... (*c*)

But an **O** proposition cannot be converted. Hence we reach no inverse by beginning with conversion.

Let us begin with obversion :

A. All S is P.. ....*Invertend*; ... ... (*a*)
E. ∴No S is not-P...............[by obverting (*a*)]; ... (*b*)
E. ∴No not-P is S...............[by contraposing (*a*)]; ... (*c*)
A. ∴All not-P is not-S.........[by obverting (*c*)]; ... (*d*)
I. ∴Some not-S is not-P......[by converting (*d*)] ... (*c*)
(*Complete Inverse*);
O. ∴Some not-S is not P......[by obverting (*c*)]
(*Partial Inverse*).

Or, take a concrete example :

A. All men are mortal.........*Invertend*; ... ... (*a*)
E. ∴No men are immortal...[by obverting (*a*)]; ... (*b*)
E. ∴No immortal beings are men...[by converting (*b*)]; ... (*c*)
A. ∴All immortal beings are not-men.. [by obverting (*c*)]; ... (*d*)
I. ∴Some not-men are immortal. .[by converting (*d*)] ... (*c*)
(*Complete Inverse*);
O. ∴Some not-men are not mortal...[by obverting (*c*)]
(*Partial Inverse*).

Thus we see that in the case of the inversion of **A**, we are to begin with obversion, and the quantity is changed from universal to particular.

(2) **E** proposition.

The Complete inverse of E is O, and the Partial inverse of E is I.

Let us begin with obversion :

E. No S is P........................*Invertend*; ... ... (*a*)
A. ∴All S is not-P...............[by obverting (*a*)]; ... (*b*)
I. ∴Some not-P is S..........[by converting (*b*)]; ... (*c*)
O. ∴Some not-P is not not-S [by obverting (*c*)]. ... (*d*)

But an **O** proposition cannot be converted. Hence we reach no inverse by beginning with obversion.

Let us begin with conversion :

E. No S is P........................*Invertend*; ... ... (*a*)
E. ∴No P is S....................[by converting (*a*)]; ... (*b*)
A. ∴All P is not-S...............[by obverting (*b*)]; ... (*c*)
I. ∴Some not-S is P..........[by converting (*c*)]. ... (*d*)
(*Partial Inverse*);
O. ∴Some not-S is not not-P...[by obverting (*d*)]
(*Complete Inverse*).

Or, take a concrete example :

E. No men are perfect.......................*Invertend*; ... (*a*)
E. ∴No perfect beings are men...[by converting (*a*)]; ... (*b*)

A. ∴All perfect beings are not-men...[by obverting (*b*)]; ... (*c*)

I. ∴Some not-men are perfect...[by converting (*c*)] ... (*d*) (*Partial Inverse*);

O. ∴Some not-men are not not-perfect...[by obverting (*d*)] (*Complete Inverse*).

Thus we see that in the case of the inversion of **E**, we are to begin with conversion and the quantity is changed from universal to particular.

(3) **I** proposition. An **I** proposition (Some S is P) cannot be inverted, because neither can it be contraposed, nor can its converse be contraposed. In other words, if we begin with obversion, we get an **O** proposition (Some S is not not-P), which cannot be converted. So we cannot proceed further. Again, if we begin with conversion, we get an **I** proposition (Some P is S), which, when obverted, gives an **O** proposition (Some P is not not-S), which cannot be converted. So here also we cannot proceed further. Thus we see that there is no inverse of **I**.

I cannot be inverted.

(4) **O** proposition. An **O** proposition (Some S is not P) cannot be inverted, because it has no converse, nor can its contrapositive be contraposed. In other words, we cannot begin with conversion, for an **O** proposition cannot be converted. Again, if we begin with obversion, we get an **I** proposition (Some S is not-P), which, when converted, gives an **I** proposition (Some not-P is S), and the obverse of **I** is **O** (some not-P is not not-S), which cannot be converted. So we cannot proceed further. Thus we see that there is no inverse of **O**.

O cannot be inverted.

The above results may also be proved by means of diagrams.

Hence we may sum up the results: Only universal propositions can be inverted. Particular propositions have no inverse. In order to invert **A** we must begin with obversion, and in order to invert **E** we must begin with conversion. In all other cases we meet with the impossibility of converting an **O** proposition. The rules for Inversion may thus be laid down: (1) *Convert the obverted contrapositive of* **A** and (2) *Convert the obverted converse of* **E**. It is also to be noticed that the complete inverse in cases of both **A** and **E** is of the *same quality* as the invertend, and that all inversion, whether complete or partial, involves *a change in quantity* from universal to particular.

Inversion of Hypothetical propositions.

Inversion of Hypothetical Propositions. We have already seen how hypothetical propositions can be converted, obverted and contraposed. On the same ground and in the same way hypotheticals may also be inverted.

Examples

Illustrations.

Invertend		*Inverse*
A. If A is B, C is D ...	**I.**	In some cases if A is not-B, C is not-D. (*Complete*).
		Or
	O.	In some cases if A is not-B, C is not D (*Partial*).
E. If A is B, C is not D ...	**I.**	In some cases if A is not-B, C is D (*Partial*).
		Or
	O.	In some cases if A is not-B, C is not not-D (*Complete*).

A. If any man is honest, he is trusted............(*Invertend*);

O. ∴ If a man is not honest, he is sometimes not trusted.... (*Inverse*),

Or

I. If a man is not honest, he is sometimes distrusted...... (*Inverse*).

E. If any man is happy, he is not vicious......(*Invertend*);

I. ∴ If a man is not happy, he is sometimes vicious......... (*Inverse*).

Or

O. If a man is not happy, he is sometimes not non-vicious (*Inverse*).

The validity of the process of Inversion vindicated.

Note. The validity of the process of Inversion has often been doubted. From 'All S is P' we infer by the process of Inversion 'Some not-S is not P'. In the conclusion the term P is distributed while it is not distributed in the premise. But how can we validly pass from 'All S is P', in which P is undistributed, to 'Some not-S is not P' in which P is distributed? The conclusion, though apparently incorrect, is, however, arrived at by the valid inferential processes of conversion and obversion. How then can we explain this anomaly and vindicate the validity of Inversion? The answer is to be found in the assumption to which we

have already referred in the *Note* added to the previous section, *viz.*, that every term, whether subject or predicate of a proposition, has a contradictory. We tacitly assume in all the different forms of Eductions that the terms S and P—the subject and predicate of the original proposition—do not exhaust the whole universe of discourse, that there also exist 'not-S' and 'not-P' in the universe of discourse. Thus when the original proposition 'All S is P' is given, we not only think of 'P' as existing, but also of things that are not P, and this *latter* idea may be logically expressed in the form, *some things are not P*. Hence, in the case of Inversion of 'All S is P', the conclusion 'Some not-S is not P' follows not merely from the given premise 'All S is P' taken by itself, but from this given premise combined with the assumed premise 'Some things are not P', and in this assumed premise the term 'P' is distributed.

Summary of Educations.

§ 5. **Summary of Eductions.** The Converse of **A** is **I**, of **E** is **E**, of **I** is **I**. The Obverse of **A** is **E**, of **E** is **A**, of **I** is **O**, and of **O** is **I**. The Contrapositive of **A** is **E**, of **E** is **I**, of **O** is **I**. The Inverse (Partial Inverse) of **A** is **O**, of **E** is **I**. There are no Converse of **O**, no Contrapositive of **I**, and no Inverse of **I** and **O**.

In obversion the subject of the conclusion is the same as the original subject; and in inversion the subject of the conclusion is the contradictory of the original subject. In conversion the subject of the conclusion is the predicate of the original proposition; and in contraposition the subject of the conclusion is the contradictory of the original predicate.

It may further be noted that the converse, contrapositive and inverse of a proposition have positive terms for their predicates. So, if the converse, contrapositive and inverse be obverted, we get conclusions having negative terms for their predicates. These may be respectively called obverted converse, obverted contrapositive and obverted inverse. The

Eductions from a given proposition have been summarized symbolically in the following table

Complete Table of Eductions

	A	E	I	O
Original Proposition	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
1. Converse	$P\,i\,S$	$P\,e\,S$	$P\,i\,S$	
2. Obverse	$S\,e\,\bar{P}$	$S\,a\,\bar{P}$	$S\,o\,\bar{P}$	$S\,i\,\bar{P}$
3. Obverted Converse	$P\,o\,\bar{S}$	$P\,a\,\bar{S}$	$P\,o\,\bar{S}$	
4. Contrapositive	$\bar{P}\,e\,S$	$\bar{P}\,i\,S$		$\bar{P}\,i\,S$
5. Obverted Contrapositive	$\bar{P}\,a\,\bar{S}$	$\bar{P}\,o\,\bar{S}$		$\bar{P}\,o\,\bar{S}$
6. Inverse (Partial)	$\bar{S}\,o\,P$	$\bar{S}\,i\,P$		
7. Obverted Inverse (Full Inverse)	$\bar{S}\,i\,\bar{P}$	$\bar{S}\,o\,\bar{P}$		

In this table S and P indicate the positive terms of a proposition, $\bar{S}$ and $\bar{P}$ the corresponding negative; the letter *a*, *e*, *i*, or *o* between them indicates the quality and quantity of the proposition Thus *a* indicates universal affirmative and *o* particular negative. For instance, $Sa\bar{P}$ means 'All S is not-P', $\bar{S}i\bar{P}$ means 'Some not-S is not-P', etc. It may be noted that the converse and obverted converse of **A** are the same as those of **I**, and that the contrapositive and obverted contrapositive of **E** are the same as those of **O.**

Note. **Eductions of Disjunctive Propositions.** "Eductions can only be drawn from disjunctive propositions in which alternative predicates are affirmed of one subject. The derived propositions, however, are not themselves disjunctive.

(*i*) The symbolic expressions of the eductions from a universal disjunctive are:

Orig. Prop............	Every S is either P or Q.
Obverse................	No S is both not-P and not-Q.
Converse.............	Some things that are either P or Q are S.
Obv. Converse......	Some things that are either P or Q are not not-S.
Contrap...............	Nothing that is both not-P and not-Q is S.
Obv. Contrap........	Everything that is both not-P and not-Q is not-S.
Inverse.................	Some not-S's are neither P nor Q.
Obv. Inv..............	Some not-S's are both not-P and not-Q.

Eductions from a universal disjunctive.

Eductions from a particular disjunctive.

(*ii*) The Symbolic form of the obverse of the particular disjunctive 'Some S's are either P or Q' is 'Some S's are not both not-P and not-Q' The forms of the converses are the same as those from the universal disjunctive" (Welton, *Manual of Logic*, I, p. 274).

'By opposition we infer from the truth or falsity of one proposition the truth or falsity of the opposed propositions.' *There are four Laws of Inference by Opposition.*

§ **6. Opposition.** Opposition is a form of immediate inference in which from the truth or falsity of one proposition we infer either the truth or falsity of another, the inferred proposition having the same subject and predicate as the original, but differing from it in quality or quantity or both. Corresponding to the four forms of Opposition (Ch. XII, § 3), there are *four Laws of Inference by Opposition*, which are ultimately based on the three Laws of Thought.

1. In Subaltern Opposition, the truth of the universal implies the truth of the particular, but not conversely; the falsity of the particular implies the falsity of the universal, but not

1. Law of Subaltern Opposition. We know that subaltern opposition exists between a universal and a particular proposition of the same quality, *i.e.*, between **A** and **I** and between **E** and **O**. The question here is: Given either the universal or the particular as true, or as false, what can we infer about the truth or falsity of the other? The answer is given in the following two laws:

(*a*) *The truth of the universal implies the truth of the particular, but not conversely.*

(*b*) *The falsity of the particular implies the falsity of the universal, but not conversely.*

conversely. Proof of the rules.

These rules follow from the Principle of Identity, for the truth of these rules follows from the very meaning of universal and particular propositions. Thus, if 'All men are mortal' (**A**) be true, then 'Some men are mortal' (**I**) is true. If 'No men are perfect' (**E**) be true, then 'Some men are not perfect' (**O**) is true. But if 'Some men are wise' (**I**) be true then 'All men are wise' (**A**) is doubtful, *i.e.*, it may be true or false; if 'Some men are not rich' (**O**) be true, then 'No men are rich' (**E**) is doubtful. Again, if 'Some men are perfect' (**I**) be false, then 'All men are perfect' (**A**) is false; if 'Some men are not mortal' (**O**) be false, then 'No men are mortal' (**E**) is false. But if 'All men are wise' (**A**) be false, then 'Some men are wise' (**I**) is doubtful; if 'No men are happy' (**E**) be false, then 'Some men are not happy' (**O**) is doubtful. Hence we see that in the case of subaltern opposition, from the truth of the universals follows necessarily the truth of the particulars; from the truth of the particulars nothing can be inferred respecting the truth of the universals. The falsity of the universals does not justify any conclusion respecting the particulars; but from the falsity of the particulars must follow the falsity of the universals.

2. In Contradictory Opposition, the truth of the one implies the falsity of the other, and conversely.

2. **Law of Contradictory Opposition.** Contradictory opposition exists between **A** and **O,** and between **E** and **I.** In contradictory opposition, *the truth of the one implies the falsity of the other, and conversely*. According to the Principle of Contradiction, *contradictories cannot be true together;* according to the Principle of Excluded Middle,

contradictories cannot be false together. Thus it follows that of two contradictory propositions, if one be true, the other must be false ; and if one be false, the other must be true. For example, if 'All men are mortal' (**A**) be true, then 'Some men are not mortal' (**O**) must be false ; if 'All men are wise' (**A**) be false, then 'Some men are not wise' (**O**) must be true. Again, if 'Some men are not intelligent' (**O**) be true, then 'All men are intelligent' (**A**) must be false ; if 'Some men are not mortal' (**O**) be false, then 'All men are mortal' (**A**) must be true. The law in the case of **E** and **I** may be similarly illustrated.

3. In Contrary Opposition, the truth of the one implies the falsity of the other, but not conversely.

3. **Law of Contrary Opposition.** Contrary opposition exists between **A** and **E**. In contrary opposition, *the truth of the one implies the falsity of the other, but not conversely.* This Law involves both *Contradiction* and *Subalternation,* hence it is based on the Principles of Contradiction and Identity. Thus, if **A** be true, **I** is true (*by Subalternation*), but if **I** be true, **E** must be false (*by Contradiction*) ; therefore, if **A** be true, *E* is false. Similarly we
therefore, if **A** be true, **E** is false. Similarly we
see that *contraries cannot be true together.* Again, if **A** be false, **O** is true (*by Contradiction*), but if **O** be true, **E** is doubtful (*by Subalternation*) ; therefore, if **A** be false, **E** is doubtful. Similarly we may prove that if **E** be false, **A** is doubtful. Hence we see that *Contraries may be false together.* Thus it is clear that of two contrary propositions, if one be true, the other must be false ; but if one be false, the other is doubtful, *i.e.,* it may be true or false. For example, if 'All men are mortal' (**A**) be

true, then 'No men are mortal' (**E**) must be false ; if 'No men are perfect' (**E**) be true, then 'All men are perfect' (**A**) must be false. But if 'All men are happy' (**A**) be false, then 'No men are happy' (**E**) is doubtful ; if 'No men are wise' (**E**) be false, then 'All men are wise' (**A**) is doubtful.

4. In Sub-contrary Opposition, the falsity of the one implies the truth of the other, but not conversely.

4. **Law of Sub-contrary Opposition.** Sub-contrary opposition exists between **I** and **O**. In sub-contrary opposition, *the falsity of the one implies the truth of the other, but not conversely*. This law involving both Contradiction and Subalternation is also based on the Principles of Contradiction and Identity. Thus, if **I** be false, **E** is true (*by Contradiction*), but if **E** be true, **O** is true (*by Subalternation*) ; therefore, if **I** be false, **O** is true. Similarly we may prove that if **O** be false, **I** is true. Hence we see that *sub-contraries cannot be false together*. Again, if **I** be true, **E** is false (*by Contradiction*), but if **E** be false, **O** is doubtful (*by Subalternation*) ; therefore, if **I** be true, **O** is doubtful. Similarly we can prove that if **O** be true, **I** is doubtful. Hence we see that *Sub-contraries may be true together*. Thus it is clear that of two sub-contrary propositions, if one be false, the other must be true ; but if one be true, the other is doubtful, *i.e.*, it may be true or false. For example, if 'Some men are wise' (**I**) be true, then 'Some men are not wise' (**O**) is doubtful, *i.e.*, it may be true or false. But if 'Some men are perfect' (**I**) be false, then 'Some men are not perfect' (**O**) must be true. It may be noted that the Law of Sub-contrary Opposition is just the reverse of the Law of Contrary Opposition.

The results of Inference by Opposition may be summarized in a tabular form thus :—

Given		A	E	I	O
1	A true		false	true	false
2	A false		doubtful	doubtful	true
3	E true	false		false	true
4	E false	doubtful		true	doubtful
5	I true	doubtful	false		doubtful
6	I false	false	true		true
7	O true	false	doubtful	doubtful	
8	O false	true	false	true	

The following tables may help the student to remember the results :—

No. 1

Truth of		A	E	I	O
	A	T	F	T	F
	E	F	T	F	T
	I	D	F	T	D
	O	F	D	D	T

No. 2

Falsity of		A	E	I	O
	A	F	D	D	T
	E	D	F	T	D
	I	F	T	F	T
	O	T	F	T	F

Hints to fill up the tables :

No. 1.

(1) Draw a square and divide it into 16 equal parts;

(2) Fill up the diagonals by T T T T and F F F F; thus—

T			F
	T	F	
	F	T	
F			T

(3) Fill up the remaining parts by F T, F T and D D, D D.

No. 2.

Proceed conversely.

Comparison between Contrary and Contradictory Opposition

Note. In a *Contrary Opposition*, a universal negative, *e.g.*, 'No men are perfect', is set against a universal affirmative, *e.g.*, 'All men are perfect'—the opposed propositions *differing only in quality*. Here we have a complete contrast between two universal statements. But this form of opposition is not effective in debate, as it lacks *sufficiency for disproof*. Both the propositions 'No men are perfect' and 'All men are perfect' cannot be true, but *both may be false*. "It may be, in point of fact, that only some men are perfect. A complete negation leaves room for a middle assertion." Moreover, in opposing a universal proposition by its contrary, we have to face the most difficult task of establishing a universal proposition by examining a whole field of things where one exception is sufficient to destroy our universal proof.

In a *Contradictory Opposition*, on the other hand, a particular negative, *e.g.*, 'Some men are not perfect', is set against a universal affirmative, *e.g.*, 'All men are perfect', or a particular affirmative, *e.g.*, 'Some men are perfect', is set against a universal negative, *e.g.*, 'No men are perfect'—the opposed propositions *differing both in quality and quantity*. It is obvious that the contradictory of a proposition denies much less than its contrary, 'it asserts the *minimum* of denial, excluding the possibility of a smaller denial'. For this reason, there is no mean between a proposition and its contradictory, as together they exhaust all possible alternatives. This form of opposition is thus so very effective in debate, for here we meet our opponent's general statement by speaking of 'some', which logically may mean 'at least one'. 'Opponent's task is a difficult one, as he has to *secure every individual case*; our task is very easy, for we have only to *destroy one*'. Hence this form of opposition has been regarded as *sufficient and effective for purposes of disproof and denial*. (*Vide* Bain, *Deductive*

Logic, Bk. I, Ch. III, § 10 and § 11.) Again, of two contradictories, both cannot be true, both cannot be false, one must be true and the other must be false; whereas of two contraries, both may be false; and of two sub-contraries as well as of two subalterns, both may be true. In other words, contradictory propositions are incompatible with regard to both truth and falsity, while contrary propositions are incompatible with regard to truth only, and sub-contrary propositions are incompatible with regard to falsity only. In no other forms of opposition than contradiction, the opposed propositions are mutually inferable. For these reasons, it has been said that *Contradiction is the most perfect form of Opposition.*

Contradiction is the most perfect form of opposition.

§ 7. **Change of Relation.** Change of Relation is a form of immediate inference in which from a proposition of one relation we infer another proposition of a different kind of relation. Thus, we can infer (1) a hypothetical from a categorical, (2) a categorical from a hypothetical, (3) hypotheticals from a disjunctive, and (4) a disjunctive from hypotheticals.

Change of Relation consists in inferring from a proposition of one relation another proposition of a different kind of relation.

(1) **Inferring a hypothetical from a categorical.** From the categorical 'All S is P' follows the hypothetical 'If S is, P is' and from the categorical 'No S is P' follows the hypothetical 'In all cases if S is, P is not.' Similarly, from the categoricals 'Some S is P' and 'Some S is not P' follow respectively the hypotheticals 'In some cases if S is, P is' and 'In some cases if S is, P is not'.

Inferring a hypothetical from a categorical

(2) **Inferring a categorical from a hypothetical.** From the hypothetical 'If A is B, C is D' follows the categorical 'All cases of A being B are cases of C being D', and from the hypothetical 'If A is B, C is not D' follows the categorical 'No case of A being B is a case of C being D'. Similarly, from the hypotheticals 'In some cases if A is B, C is D' and 'In some cases if A is B, C is not D' follow respectively the categoricals 'Some cases of A being B are cases

Inferring a categorical from a hypothetical.

of C being D' and 'Some cases of A being B are not cases of C being D'.

Inferring hypotheticals from a disjunctive.

(3) **Inferring hypotheticals from a disjunctive.** We have already seen (Ch. XII, § 3), according to Ueberweg and other writers, that four hypotheticals may follow from a disjunctive proposition. Thus from the disjunctive proposition 'A is either B or C' we get the following hypotheticals:

(a) If A is not B, A is C ;
(b) If A is not C, A is B ;
(c) If A is B, A is not C ;
(d) If A is C, A is not B.

According to Mill and others, on the other hand, from the above disjunctive we get only two hypotheticals, namely—

(a) If A is not B, A is C ;
(b) If A is not C, A is B.

As these hypotheticals may be reduced to their corresponding categoricals, it further follows that categorical propositions may also be inferred from a disjunctive.

Inferring a disjunctive from hypotheticals

(4) **Inferring a disjunctive from hypotheticals.** When hypothetical propositions are given, we can infer a disjunctive proposition from them. Suppose we are given four hypotheticals, namely

(a) If A is B, A is not C ;
(b) If A is C, A is not B ;
(c) If A is not B, A is C ;
(d) If A is not C, A is B.

Let us first prove that from (c) and (d) we may infer the disjunctive 'A is either B or C', in Mill's sense. We know, according to Mill, that the alternatives of a disjunctive proposition are so related that the falsity of the one implies the truth of the other. Now, if 'A is B' be false, then 'A is not-B' is true (by *the Law of Excluded Middle*); if 'A is not-B' be true, then 'A is not B' is true (by *obversion*); if 'A is not B' be true, then from (c) 'A is C' is true (*vide* Hypothetical-categorical Syllogisms, Ch. XVIII, § 1). Similarly, it can be proved that if 'A is C' be false, then

'A' is 'B' is true. Therefore, of the two propositions 'A is B' and 'A is C,' if one be false the other is true,—they are thus the two alternatives of the disjunctive proposition 'Either A is B or A is C' or 'A is either B or C,' in Mill's sense.

Let us next prove that from (*a*), (*b*), (*c*) and (*d*) we may infer the disjunctive 'A is either B or C', in Ueberweg's sense. We know, according to Ueberweg, that the alternatives of a disjunctive proposition are so related that the truth of the one implies the falsity of the other, and the falsity of the one implies the truth of the other. That the falsity of the one implies the truth of the other has been proved above from (*c*) and (*d*). It remains to prove from (*a*) and (*b*) that the truth of the one implies the falsity of the other. If the proposition 'A is B' be true, then from (*a*) the proposition 'A is not C' is true (Ch. XVIII, § 1); if 'A is not C' be true, then 'A is not-C' is true (*by obversion*), if 'A is not-C' be true, then 'A is C' is false (*by the Law Contradiction*). Similarly, from (*b*) it can be proved that if 'A is C' be true, then 'A is B' is false. Thus it is proved that of the two propositions 'A is B' and 'A is C', if one be true, the other is false, and *vice versa*, *i.e.*, they are the two alternatives of the disjunctive proposition 'A is either B or C,' in Ueberweg's sense.

We are next to consider whether a disjunctive proposition can be inferred from a single hypothetical. Let the given hypothetical be

(*a*) 'If A is B, A is C.'

By contraposing this, we get

(*b*) 'If A is not C, A is not B.'

If 'A is C' be false, then 'A is not C' is true (as above), and therefore, from (*b*) 'A is not B' is true (as above). Again, if 'A is not B' be false, then 'A is B' is true (as above), and therefore, from (*a*) 'A is C' is true (as above). Thus, of the two propositions 'A is C' and 'A is not B', the falsity of the one implies the truth of the other, *i.e.*, they are the two alternatives of the disjunctive proposition 'Either A is not B or A is C', in Mill's sense. Hence it is evident that from a single hypothetical we can infer a disjunctive in Mill's sense of it.

But of the above two propositions (*viz.*, 'A is C' and 'A is not B'), if also the truth of the one imply the falsity of the other, then they may be the alternatives of a disjunctive proposition in Ueberweg's sense. But this does not hold good in the case of the above propositions. For, if 'A is C' in (*a*) be true, then 'A is not C' is false (as above), but from the falsity of 'A is not C' we cannot infer anything regarding 'A is not B' in (*b*) (*vide* Ch. XVIII, § 1). Similarly, from the truth of 'A is not B' we cannot infer anything regarding 'A is C' in (*a*). Thus of the two propositions 'A is not B' and 'A is C,' the truth of the one

does not imply the falsity of the other. Hence it is evident that from a single hypothetical we cannot infer a disjunctive in Ueberweg's sense of it.

Modal consequence consists in drawing an inference from a given proposition by changing its modality

§ 8. Modal Consequence. This is a form of immediate inference in which from a given proposition we infer another proposition of a different kind of modality. We have already divided propositions, according to different kinds of modality, into necessary, assertory and probable. There are *two rules* for this kind of inference:—

Two rules of inference.

(1) *From the truth of a greater degree of certainty, we can infer the truth of a lower degree; but not conversely.* Thus, from the truth of a necessary proposition, we can infer the truth of the corresponding assertory or problematic proposition; and from the truth of an assertory proposition, we can infer the truth of the corresponding problematic proposition; but not conversely.

(2) *From the falsity of a lower degree of certainty, we can infer the falsity of a greater degree; but not conversely.* Thus, from the falsity of a problematic proposition, we can infer the falsity of the corresponding assertory or necessary proposition; and from the falsity of an assertory proposition, we can infer the falsity of the corresponding necessary proposition; but not conversely.

Application of the rules.

By applying the above two rules we get the following results:—

If true	*True*	*Doubtful*
Necessary—A must be B	A is B A may be B	———
Assertory—A is B	A may be B	A must be B
Problematic—A may be B	———	A is B A must be B

If false	*False*	*Doubtful*
Necessary—A must be B	———	A is B A may be B
Assertory—A is B	A must be B	A may be B
Problematic—A may be B	A is B A must be B	———

§ 9. **Inference by Complex Conception.** This is a kind of immediate inference in which from a given proposition we infer another proposition by joining a complex idea both to the original subject and predicate under the same limitation. For example, from 'The rose is a flower' we may infer 'The smell of the rose is the smell of a flower'; from 'Poverty is a social evil' we may infer 'The removal of poverty is the removal of a social evil.' It is to be noted, however, that the complex conception (which limits the meaning of the subject and the predicate) must bear the *same sense* in subject and in predicate, otherwise the inference will be erroneous. We cannot always guard ourselves against such an error by using the same word in each case, for the meaning of terms is so often determined by the context. Thus from 'Students are men' we cannot infer 'A large number of students is a large number of men', for what is large with reference to students is not so with reference to men.

This consists in inferring from a given proposition another proposition by joining a complex idea both to the original subject and predicate under the same limitation.

The above process may be reversed, and instead of passing to a proposition of narrower extent, we may pass to a proposition of wider extent by leaving out a common conception. Thus from 'The head of a dog is the head of a quadruped' we may infer 'A dog is a quadruped.' This is known as **Inference by Simple Conception.**

§ 10. **Inference by Added Determinants.** This is a kind of immediate inference in which from a

This consists in adding the

same qualifying epithet both to the subject and the predicate of a proposition.

given proposition we infer another proposition by adding a qualifying epithet both to the original subject and predicate, in such a way as to have the same effect upon both sides. The added qualification is called a *Determinant*. For example, from 'A dog is a quadruped' we may infer 'A black dog is a black quadruped'; from 'All Indians are men' we may infer 'Every honest Indian is an honest man'. But such inferences like Inferences by Complex Conception are erroneous, when the meanings of subject and predicate are not affected in an identical way. Thus from 'A mouse is a quadruped' we cannot infer 'A big mouse is a big quadruped'.

The above process may be reversed by leaving out the qualifying epithets from the subject and the predicate of a given proposition equally. Thus from 'A white horse is a white animal' we may infer 'A horse is an animal'. This is known as **Inference by Omitted Determinants.**

From what has been said above it is clear that Inference by Added Determinants is very similar to Inference by Complex Conception. The only difference consists in this that while in Inference by Complex Conception the added element is itself limited and determined by the original subject and predicate, in Inference by Added Determinants the additional epithet limits and determines the original subject and predicate.

It should be noted that Inferences by Complex Conception and by Added Determinants, like Inference by Converse Relation (Ch. XVI, § 1, *Note 2*), depend, for their validity, not so much on the form of the propositions, as on the *meaning* we attach to the subject and the predicate. Such processes have thus been appropriately called *Material Eductions.*

§ 11. Examples Worked out.

Q. 1. Give the converse, obverse and the contradictory, when possible, of the following :—

(*a*) Few fled.
(*b*) Not to go on is to go back.
(*c*) When beggars die there are no comets seen.
(*d*) White cats with blue eyes are generally deaf.
(*e*) All swans are not white.

Ans. : —

(*a*) Few fled
= Most beings are not those that fled
= Some beings are not those that fled (**O**).
It has no converse, for it is an **O** proposition.
Its obverse = Some beings are not-those-that-fled.
Its contradictory = All beings are those that fled.

(*b*) Not to go on is to go back
= All cases of not-going on are cases of going back (**A**).
Its converse = Some cases of going back are cases of not-going on.
Its obverse = No case of not-going on is not-case-of-going-back.
Its contradictory = Some cases of not-going on are not cases of going back.

(*c*) When beggars die there are no comets seen
= No comets are seen at beggars' death (**E**).
Its converse = Nothing seen at beggars' death is a comet.
Its obverse = All comets are not-things-seen-at-beggars' death.
Its contradictory = Some comets are seen at beggars' death.

(*d*) White cats with blue eyes are generally deaf
= Some white cats with blue eyes are deaf (**I**).
Its converse = Some deaf creatures are white cats with blue eyes.
Its obverse = Some white cats with blue eyes are not not-deaf.
Its contradictory = No white cats with blue eyes are deaf.

(*e*) All swans are not white
= Some swans are not white (**O**).
It has no converse, for it is an **O** proposition.
Its obverse = Some swans are not-white.
Its contradictory = All swans are white.

Q. 2. Give the obverse, contrapositive and inverse, if possible, of the following :—

(*a*) None think the fools great but the fools themselves.
(*b*) No cultured persons are prejudiced.
(*c*) Things are not what they seem.

(*d*) One of you at least should be able to answer the question.

Ans. :—

(*a*) None think the fools great but the fools themselves = All who think the fools great are the fools themselves (**A**).

Its obverse = None who think the fools great are not-fools themselves.

Its contrap = No not-fools themselves are those who think the fools great.

Its inverse = Some who do not think the fools great are not the fools themselves.

(*b*) No cultured persons are prejudiced (**E**).

Its obverse = All cultured persons are non-prejudiced.

Its contrap = Some non-prejudiced persons are cultured.

Its inverse = Some not-cultured persons are prejudiced.

(*c*) Things are not what they seem = Some things are not what they seem (**O**).

Its obverse = Some things are what they do not seem.

Its contrap = In some cases what things do not seem to be are what they are.

It has no inverse.

(*d*) One of you at least should be able to answer the question = Some of you are persons who should be able to answer the question (**I**).

Its obverse = Some of you are not persons who should not be able to answer this question.

It has no contrapositive.

It has no inverse.

Q. 3. Give the converse of the contrapositive of the following :

(*a*) The virtuous are happy.

(*b*) Some men are not just.

Ans. :—

(*a*) The virtuous are happy (**A**).

Its contrap = No not-happy being are virtuous (**E**).

The contrap. when converted gives—No virtuous (men) are not-happy beings.

(*b*) Some men are not just (**O**).

Its contrap. = Some not-just (beings) are men (**I**).

The contrap. when converted gives—Some men are not-just.

Q. 4. Infer as many propositions as you can from the premise—Every phenomenon has a cause.

Ans :—Every phenomenon has a cause = All phenomena are caused (**A**).

(1) Converse = Some things caused are phenomena.

(2) Obverse = No phenomenon is not-caused.

(3) Obverted converse = Some things caused are not non-phenomena.

(4) Contrapositive = No not-caused things are phenomena.
(5) Obverted contrapositive = All non-caused things are non-phenomena.
(6) Inverse = Some non-phenomena are not caused.
(7) Obverted Inverse = Some non-phenomena are not-caused.
(8) Subalternate = Some phenomena are caused.
(9) Modal Consequence = All phenomena may be caused.
(10) Change of Relation = If there be a phenomenon, it has a cause.

Q. 5. Show all the logical inferences that can be drawn by Opposition from the truth of (*a*) All men are not happy, (*b*) Some men are professors; and from the falsity of (*c*) No men are rational.

Ans. :—

(*a*) All men are not happy = Some men are not happy (**O**)—given true.
∴ All men are happy (**A**)—false,
& ∴ Some men are happy (**I**)—doubtful,
& ∴ No men are happy (**E**)—doubtful.
(*b*) Some men are professors (**I**)—given true.
∴ All men are professors (**A**)—doubtful,
& ∴ No men are professors (**E**)—false,
& ∴ Some men are not professors (**O**)—doubtful.
(*c*) No men are rational (**E**)—given false.
∴ All men are rational (**A**)—doubtful,
& ∴ Some men are rational (**I**)—true,
& ∴ Some men are not rational (**O**) doubtful.

Q. 6. Discuss whether from the proposition 'All successful men are happy' we can draw the inference 'Some unhappy men are not unsuccessful.'

Ans. :—All successful men are happy (**A**).
∴ No successful men are unhappy (*By obversion*);
∴ No unhappy men are successful (*By conversion*);
∴ 'Some unhappy men are successful' is false (*By opposition*);
∴ 'Some unhappy men are not unsuccessful' is also false (*By obversion*).

Q. 7. Examine the following Immediate Inferences :—
(*a*) Uneasy lies the head that wears a crown; therefore, easy lies the head that wears no crown.
(*b*) Sweet is agreeable; therefore, bitter is painful.
(*c*) In all cases if there is the sun, there is heat; therefore, in all cases if there is heat, there is the sun.
(*d*) Absolute difference excludes all likeness; therefore, any likeness is a proof of sameness.
(*e*) Books are a source of instruction; therefore, our knowledge must come from books.

(*f*) Some Indians are not industrious; therefore, some industrious men are not Indians.

(*g*) A professor is a man; therefore, a bad professor is a bad man

Ans. —

(*a*) Uneasy lies the head that wears a crown
=All heads that wear a crown lie uneasy (**A**);
∴ No heads that wear a crown lie easy (*By obversion*);
No things that lie easy are heads that wear a crown (*By conversion*);
∴ All things that lie easy are heads that wear no crown (*By obversion*),
∴ Some heads that wear no crown lie easy (*By conversion*),
∴ The given conclusion, which is universal, cannot be drawn.

(*b*) This is a case of Material Obversion and is not a formal process of immediate inference. Such inference is often erroneous.

(*c*) In all cases if there is the sun, there is heat.

This is a universal hypothetical proposition, and so cannot be converted *simply*.

Its converse=In some cases if there is heat, there is the sun

Hence the given conclusion cannot be drawn.

(*d*) Absolute difference excludes all likeness
=No absolute difference is likeness (**E**),
∴ No likeness is absolute difference;
∴ All likeness is not-absolute difference.

Again, the given conclusion, when reduced to its logical form All likeness is sameness

But, 'sameness' is not the same thing as 'not-absolute difference', for 'not-absolute difference' may mean either 'partial difference' or 'sameness'. Therefore, the given conclusion is wrong

(*e*) The conclusion 'All sources of instruction are books'. But by converting the given proposition we get 'Some sources of instruction are books'. The given conclusion has been obtained by *simply* converting the given proposition which is an **A** proposition Hence the given conclusion is invalid.

(*f*) Some Indians are not industrious (**O**).

The given proposition is an **O** proposition and therefore cannot be converted. But the conclusion has been arrived at by converting the given proposition. Hence it is wrong.

(*g*) This is an instance of the Fallacy of Added Determinants. The subject and the predicate have not been affected in a parallel way by the addition of the epithet 'bad'.

§ 12. Exercises.

1. Explain the nature and importance of Reasoning or Inference in general. Classify different kinds of Inference with examples.

2. Explain the ambiguity of the term 'inference' and distinguish between (1) Inductive and Deductive Inference, (2) Mediate and Immediate Inference.

3. Indicate the character of Immediate Inference. Is Immediate Inference a true form of inference at all?

4. Enumerate, explain and exemplify the forms of Immediate Inference.

5. Define Conversion and state its rules. Distinguish its different forms, giving examples of each. Show that **O** cannot be converted.

6. What do you mean by Eduction? What are its main forms? Show that they are ultimately based on the Fundamental Laws of Thought. Summarize the results of Eduction.

7. Explain Obversion. What is the basis of Obversion? Mention its rule and give some examples of Obversion as applied to hypothetical propositions. Explain Material Obversion. Is it a proper form of Obversion?

8. Explain Contraposition. Give the rules and hints of Contraposition and apply them to the four forms of propositions. Show that **I** cannot be contraposed.

9. Explain Inversion and apply it to different forms of propositions. Show that **I** and **O** cannot be inverted. Distinguish between a Complete and a Partial Inversion.

10. Distinguish between Obversion, Contraposition and Inversion, giving a concrete example of each. Also distinguish between Converted Obverse and Obverted Converse of a proposition.

11. What are the rules of Immediate Inference by Modal Consequence? Explain Immediate Inferences by Complex Conception and by Added Determinants. How would you distinguish between the two?

12. State the rules of Immediate Inference by Opposition, and prove them from the Fundamental Laws of Thought. How are the Laws of Opposition applied to Hypothetical propositions?

13. Prove that two contraries may both be false, but both cannot be true; and that two sub-contraries may both be true but both cannot be false.

14. What will you infer from the truth or falsity of **A, E, I** or **O**? Why is it desirable in a controversy to refute a false statement by its Contradictory and not by its Contrary?

15. Show by means of sub-contrary propositions that contrary propositions may both be false. Prove by means of contradictory propositions that sub-contrary propositions cannot both be false. Prove that of two sub-contrary propositions, one or the other must be true.

16. Draw as many inferences as you can from the truth or falsity of each of the following propositions :—

(*a*) Not every man ought to be trusted.
(*b*) None but God is infallible.
(*c*) Some useful things are disagreeable.
(*d*) Every poet is a man of genius.
(*e*) Some plants produce no flowers.
(*f*) None but the ignorant despise knowledge.

17. Draw as many Immediate Inferences as you can from each of the following propositions :—

(*a*) None but the brave deserve the fair.
(*b*) The hand of the diligent maketh rich.
(*c*) Only what affect our senses are worthy of belief.
(*d*) Science knows nothing of ultimate origin.
(*e*) Beauty as well as virtue lies in a medium.
(*f*) Some men are not just.
(*g*) If there is rain there is cloud.
(*h*) Few men are unselfish.
(*i*) The earth is the only planet that has no atmosphere.
(*j*) Every term has not connotation.

18. Deduce Hypothetical and Categorical propositions from the following :—

(*a*) James is either honest or dishonest.
(*b*) John is either a fool or a knave.
(*c*) The Viceroy is either at Simla or at Calcutta.

19. Examine the following inferences :—

(1) All mangoes are not sweet; therefore, all sweet things are not mangoes.

(2) An ant is an animal; therefore, a big ant is a big animal.

(3) My watch was found under your pillow. So the presumption is against you, you might have stolen the watch. You are, then, a thief. You must, therefore, be punished.

(4) Those who live in society are tempted to do wrong; therefore, the easiest and surest way to be virtuous is to renounce society.

(5) Students are men; therefore, a majority of students is a majority of men.

(6) This thing is not white; therefore, it is back.

(7) Nothing is beautiful except truth; therefore, whatever is true is beautiful.

(8) All parallelograms are quadrilateral figures; therefore, no parallelogram can be a pentagon.

(9) Many are the deserving men who are unfortunate; therefore, most fortunate men are undeserving men.

(10) All wise men are modest; therefore, no immodest men are wise.

(11) All men are mortal; therefore, all mortals are men.

(12) Abstract subjects are difficult to grasp; therefore, concrete subjects are easy.

(13) Only children behave in this way; therefore, every one who behaves thus is a child.

(14) War brings ruin, therefore, people prosper during times of peace.

(15) Some Chinese are Buddhists; therefore, some men who are not Chinese are non-Buddhists.

(16) The pen is mightier than the sword; therefore, the sword avails less than the pen.

(17) Prudence is a virtue; therefore, a prudent man is a virtuous man.

(18) No one is admitted without payment, therefore, all who are admitted are persons who paid.

CHAPTER XVII

SYLLOGISMS

Syllogism—its Definition.

§ 1. Definition and Characteristics of Syllogism. We have already seen that Deductive Inference may be either Immediate or Mediate (Ch. II, § 2 and Ch. XV, § 1). Having discussed the different forms of Immediate Inference, we are now to consider the Mediate form of Deductive Inference, technically called Syllogism. A Syllogism may thus be defined as a form of mediate inference in which from two given propositions we infer another proposition, which necessarily follows from the two given propositions taken jointly, and is never more general than either of them. The word 'Syllogism' (*Gr. Syn*, together, and *Logos*, thought or reason) literally means 'computation' or 'thinking together'. A Syllogism is, therefore, the putting together in thought of two given propositions. Dr. Schiller briefly defines a Syllogism as "a combination of three propositions so arranged that the third follows with logical necessity from the first two" (*Formal Logic*, p. 170). For example,

It is the mediate form of deductive inference in which the conclusion follows with logical necessity from two given propositions taken jointly.

All men are mortal	(*Premise*),
Poets are men	(*Premise*) ;
∴ Poets are mortal	(*Conclusion*).

Its characteristics.

What, then, are the **Characteristics of a Syllogism?**

(1) The conclusion follows

First, the conclusion follows from two propositions taken jointly, and not from either of them taken separately. The two propositions are put together in thought. Hence it has rightly been said

that a Syllogistic inference is one indivisible act of thought. This characteristic distinguishes a Syllogism as mediate inference from an Immediate Inference, in which the conclusion follows from a single proposition, without the help of any other proposition. *from two propositions taken jointly.*

(2) *Secondly*, the conclusion can never be more general than either of the two given propositions from which it follows. In the above example, the conclusion 'Poets are mortal' is less general than the proposition 'All men are mortal.' This characteristic distinguishes a Syllogism as Deductive inference from an Induction, in which the conclusion is always more general than the premises. *(2) The conclusion can never be more general than the premises.*

(3) *Thirdly*, the conclusion follows necessarily from the given propositions. It is not possible to obtain a conclusion by putting together any two propositions at random. The given propositions must fulfil certain conditions and stand in certain definite relations to each other. This characteristic ensures the formal correctness of Syllogistic inference and thus distinguishes a correct or valid Syllogism from an apparent or invalid one, which consists of propositions having no connection with each other and whose conclusion does not follow with logical necessity from the given propositions. *(3) The conclusion follows with logical necessity from the premises.*

(4) *Fourthly*, the conclusion is only conditionally true (materially valid), *i.e.*, true on condition that the given propositions are true. In all Deductive or Formal reasoning the premises are taken for granted. It is not the business of Formal Logic to consider whether the premises are true in fact, it only considers whether the conclusion necessarily follows or not from the given premises. Thus the conclusion *(4) The material truth of the conclusion depends on that of the premises.*

of a Syllogism is materially true, if the given propositions be so ; and the conclusion is materially invalid, if either of the given propositions be materially wrong (*vide* Ch. II, § 3). Hence we see that so far as formal correctness is concerned, the conclusion of a Syllogism is necessary in character ; but so far as material truth is concerned, it is only *hypothetically necessary* in character that is to say, *if* the given premises or antecedent be materially true, the conclusion or consequent is materially true ; *if not*, not.

A Syllogism consists of three propositions—the two given propositions called the premises and the inferred proposition called the conclusion.

§ 2. Structure of Syllogism. A fully expressed Syllogism consists of three propositions—the two given propositions called the Premises, and the inferred proposition called the Conclusion. Take the following example :

Major Premise—All men (*Middle term*) are mortal (*Major term*),
Minor Premise—Poets (*Minor term*) are men (*Middle term*) ;
Conclusion—, Poets (*Minor term*) are mortal (*Major term*).

Or symbolically,

Major Premise—M (*Middle term*) is P (*Major term*),
Minor Premise—S (*Minor term*) is M (*Middle term*) ;
Conclusion—S (*Minor term*) is P (*Major term*).

There are three terms in a Syllogism, *viz*., the Major, the Minor and the Middle. The relation between the major and minor terms can

Here the proposition 'Poets are mortal' is the conclusion, and the two other propositions 'All men are mortal' and 'Poets are men' are the premises. Every proposition consists of two terms, and as each term occurs twice over in the argument, there are only three terms, *viz*., 'Men', 'Mortal', and 'Poets'. The subject of the conclusion, *viz*., 'Poets', is called the **Minor Term** and is commonly represented by the symbol S ; the predicate of the conclusion, *viz*., 'Mortal', is called the **Major Term** and is commonly represented by the symbol P ; and the term which is

present in both the premises but absent in the conclusion, *viz.*, 'Men', is called the **Middle Term** and is commonly represented by the symbol M. It is of great importance to note the function of the middle term. The conclusion of a Syllogism establishes a relation between the major and minor terms by comparing each of these terms with the same middle term common to the two premises. The major and minor terms are called the **'Extremes'** as opposed to the **'Mean'** or the middle term. The relation between the two extremes cannot be established independently and immediately, but through the medium or help of the middle term. Hence we distinctly see the reason why the Syllogism is called mediate reasoning. The premise containing the major term is called the **Major premise,** *e.g.*, 'All men are mortal' (in the above example) ; and the premise containing the minor term is called the **Min** *e.g.*, 'Poets are men' (in the above example). The propositions of which the Syllogism is constituted may occur in any order in actual reasoning. It is often a fact that we start from the conclusion and then state the premises by way of supporting it. Thus we often say—'Poets are mortal', because 'They are men' and 'Men are mortal.' But when the Syllogism is arranged in Logical form, the major premise must be put first, the minor next, and the conclusion last. We are, however, to remember that the distinguishing mark of the major premise is not its occurring first, but its containing the *predicate of the conclusion*. Similarly, the distinguishing mark of the minor premise is its containing the *subject of the conclusion*.

be established only through the medium of the middle term.

The premise containing the major term or the predicate of the conclusion is called the *Major premise*, and the premise containing the minor term or the subject of the conclusion is called the *Minor premise*.

Note. A question may be asked: Why should the subject of the conclusion be called the minor term and

Why the subject

of the conclusion is called the minor term and the predicate the major term.

the predicate the major term? Dr. Ray says: "It should be observed that the distinction between the major and the minor term is purely conventional. There is no reason why the subject of the conclusion should be called the minor and the predicate the major term. It is due to usage that the two names 'minor term' and 'major term', are applied to the subject and the predicate, respectively, in the conclusion" (*Deductive Logic*, p. 165). But there seems to be good reason in giving these names if we consider that Aristotelian Syllogism arose out of Dialectic, where the conclusion was the problem or thesis—the point at issue. Now, the conclusion being our starting-point, we are to interpret the proposition standing for the conclusion in the way which was regarded as the usual or natural way by Aristotle and his followers. According to them, the normal mode of interpreting a proposition was to regard the subject as being contained in the predicate, the latter being wider in extent than the former, or to regard the predicate as an attribute (a wider notion) inhering in a thing, the subject (a narrower notion)—*vide* Ch. XIV, § 2. This is the reason why the predicate of the conclusion was called the major term and the subject the minor term

Syllogisms may be pure or mixed, according as their constituent propositions are of the same or of different relations. Pure Syllogisms are of three kinds, viz., Pure Categorical, Pure Hypothetical and Pure Disjunctive. Mixed Syllogisms are of three kinds, viz., Hypothetical-categorical,

§ 3. **Kinds of Syllogism.** Syllogisms have been broadly divided into two kinds, *viz.*, **Pure** or Simple, and **Mixed** or Complex, according as the propositions of which they are composed are of the same or of different relations. Pure Syllogisms are of three kinds, *viz.*, Pure Categorical, Pure Hypothetical and Pure Disjunctive. If both the premises and the conclusion are categorical, the Syllogism is called **Pure Categorical**; if both the premises and the conclusion are hypothetical, the Syllogism is called **Pure Hypothetical**; if all the constituent propositions are disjunctive, the Syllogism may be called **Pure Disjunctive.** Mixed Syllogisms are of three kinds, *viz.*, Hypothetical-categorical, Disjunctive-categorical and the Dilemma. If the major premise is hypothetical, and the minor categorical, the Syllogism is called **Hypothetical-categorical.** If the major premise is disjunctive, and the minor categorical, the Syllogism is called **Disjunctive-**

categorical. If the major premise is hypothetical and the minor disjunctive, the Syllogism is called the **Dilemma.**

Disjunctive-categorical and the Dilemma.

The same rules are applicable to all kinds of Pure Syllogisms. We shall confine ourselves in this Chapter to the exposition of Pure Syllogisms, and the next Chapter will be devoted to the examination of the different forms of Mixed Syllogisms.

§ 4. **Axioms of Syllogism.** Some Logicians (*e.g.*, Hamilton, Mansel and Welton) consider that Syllogistic reasoning, like all other purely formal reasonings, is ultimately founded upon the Fundamental Laws of Thought. Thus the Law of Identity and the Law of Contradiction are regarded as the bases respectively of affirmative Categorical Syllogisms and negative Categorical Syllogisms, while the Law of Sufficient Reason is regarded as the basis of Pure Hypothetical Syllogisms (*vide* Welton, *Manual of Logic*, I, pp. 282-3). Thus all inferences are brought under the Law of Consistency. But Bain rightly criticizes the above view by holding that it is calculated to abolish the distinction between immediate and mediate inference, and he finds fault with it because it ignores the peculiar sort of consistency which is involved in Syllogistic reasoning. Logicians have, therefore, attempted to derive certain axioms, which, though more developed forms of the Fundamental Laws of Thought (and hence sometimes called *Axiomata media* or 'middle axioms'), are adapted to the peculiar nature of Syllogistic reasoning. Thus Whately gives the following two axioms as the Canons or Fundamental Principles of all pure Syllogism :—

Syllogistic reasoning, though ultimately based on the Fundamental Laws of Thought, directly rests on certain canons adapted to its peculiar nature.

(1) *If two terms agree with one and the same third, they agree with each other.*

Whatley's Canons of pure Syllogisms.

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(2) ***If one term agrees and another disagrees with one and the same third, these two disagree with each other.***

The first is the principle of all affirmative Syllogisms, the second is the principle of all negative Syllogisms. To the above two Jevons adds a third Canon, viz., '*Two terms both disagreeing with one and the same third may or may not agree with each other,*' which ensures that no conclusion can be drawn from two negative premises. (For various other statements of the Canon—*vide* Welton, I, p. 284.)

Aristotle's *Dictum de omni et nullo* or the Statement concerning all and none.

§ 5. **Aristotle's Dictum.** Aristotle and his followers regarded only that form of Syllogism as the 'perfect' form in which the middle term is subject of the major premise and predicate of the minor. This form may be symbolically represented thus:—

All (or No) M is P,
S is M;
∴ S is (or is not) P.

In this form the major premise lays down a general principle and the minor premise applies the general principle to some particular case or class of cases. They tested the validity of all other forms of Syllogism by reducing them to this 'perfect' form with the help of immediate inference being applied to the premises (*vide* § 17).

Hence Aristotle formulated an axiom which directly served as the Fundamental Principle of the 'perfect' form of Syllogism and indirectly as that of all other forms, the validity of which was tested by first 'reducing' them to the 'perfect' form and thus applying the axiom to them. This axiom is known

as the *'Dictum de omni et nullo'* (Statement concerning all and none), which has been variously enunciated by Logicians. Thus Mill enunciates it as follows: 'Whatever can be affirmed (or denied) of a class, may be affirmed (or denied) of everything included in the class.' Whately, Jevons, Keynes and others would prefer to enunciate it in the following way: 'Whatever is predicated, whether affirmatively or negatively, of a term distributed, may be predicated in like manner of everything contained under that term.'

Various renderings of the Dictum.

Versions of Aristotle's Dictum. It must be pointed out here that Mill's rendering of the Dictum is quite self-evident, being merely an exposition of the meaning of the term *class*. If 'class' be understood in the sense of being nothing but a definite collection of individuals contained under it, the *dictum* is reduced to an identical proposition, a definition merely, or at least a form of immediate inference, since what is true of a *class* (*e.g.*, *'man'*) must also be true of *some individuals* (*e.g.*, *'poets'*) or any particular individual (*e.g.*, *'Socrates'*) belonging to that class. For this reason some have accepted Whately's version as less open to the above charge, inasmuch as it treats of the class as the *class indefinite*, the precise range of which is never fully known to us, so that there remains a need of showing by the help of the minor premise that some individual or individuals belong to that *class*. Hence the reasoning is no longer an immediate inference. The most essential point to remember in this connection is that Aristotle's *Dictum* should not be interpreted as basing the Syllogism exclusively on the extension or denotation of the terms compared but on the extension as determined by the connotation or intension.

Mill's rendering is objectionable, inasmuch as it amounts merely to an exposition of the meaning of the term *class*. Whately's version, however, is free from the above charge.

This is the view which Bain seems to entertain when he proposes to amend Mill's Statement thus: "Whatever is true of a whole class (class indefinite, fixed by connotation), is true of whatever thing can be affirmed to come under or belong to the class (as ascertained by connotation)." Thus amended, the Aristotelian *Dictum* is the most exact and suitable representation of the essential feature of Syllogistic reasoning.

Bain's amendment of Mill's version expresses the real signification of the Dictum.

Some Logicians, who object to Aristotle's *Dictum* as referring only to the extension of terms, propose to substitute for it statements having exclusive reference to the connotation or intension of terms. J. S. Mill, for instance, proposes to restate the dictum thus: *Nota notæ, nota rei ipsius*, and interpretes it as follows: 'whatever (S) has any

Connotative statements of the Dictum, as given by Mill, and

mark (M) has that (P or not P) which it is mark of;' or simply, 'the mark of a mark is a mark of the thing itself.'*

Thus in the following Syllogism—

All ruminants (M) are herbivorous (P),
Camels (S) are ruminants (M);
∴ Camels are herbivorous.

C. Read.

'A camel is a mark of ruminating; and ruminating is a mark of feeding upon herbage: therefore a camel is a mark of feeding upon herbage.' Again, C. Read gives a connotative interpretation of the *Dictum* by stating it thus: 'Whatever we have reason to regard as constantly connected with the nature or connotation of class or class-name, we may expect to be similarly connected with whatever can be shown to have that nature or connotation. Thus (in the above example) the feeding upon herbage, being connected with the nature of ruminants, is connected with camels, because they ruminate' (*vide* C. Read, *Logic*, p. 160).

The true interpretation of the Dictum is Denotative-Connotative

Of these connotative interpretations it may be said that they are no doubt good substitutes for Aristotle's *Dictum*, and bring to prominence the aspect of intension which the *Dictum* leaves in the background. But they are clearly one-sided and far from representing the true Aristotelian view, which obviously bases the Syllogism on the denotation of its terms. The true significance of the *Dictum* seems to have been expressed in the following words of Mercier: "The logical connexion between premises and conclusion demands that the terms should be simultaneously considered from both points of view—the extensive and the comprehensive. The predicate of the conclusion, forming part of the *comprehension* of an abstract term which contains *in its extension* the subject of the conclusion, may be definitely predicated of this subject" (quoted by Coffey, *Science of Logic*, I, p. 302). Thus in the Syllogism 'All men are mortal', 'Socrates is a man'; therefore, 'Socrates is mortal'—'mortality' is a part of 'humanity' (*i.e.*, 'man' taken in intension), and 'Socrates' is contained in, *i.e.*, forms a part of, 'humanity' considered in its extension (*i.e.*, 'all men'); therefore, 'mortality' may be predicated of 'Socrates'.

The Dictum is really a double axiom.

The *Dictum* of Aristotle is really a double axiom and has an affirmative and a negative side. The affirmative part—*Dictum de omni*—states that what may be affirmed

* This is the formula given by Mill for a Syllogism when the minor premise is a singular proposition; when both the premises are universal, he formulates it thus: 'Whatever is a mark of any mark, is a mark of that which this last is a mark of' (Mill, *System of Logic*, Bk. II, Ch. II, § 4).

of a term distributed, may also be affirmed of anything contained under it. The negative part—*Dictum de nullo*—states that what may be denied of a term distributively, may also be denied of anything contained under it. The first is the basis of all affirmative Syllogisms, and the second is the basis of all negative Syllogisms.

The *Dictum*, however, is not to be regarded as more fundamental than the general axioms of Syllogism considered in § 4. Like them, the *Dictum* also is a more developed form of the Fundamental Laws of Thought, and is necessary, self-consistent and self-evident in character. This can easily be understood from the fact that its denial involves a contradiction in terms. Hence the *Dictum* can be safely acknowledged to be the axiom of all Syllogistic reasoning, and consequently a Syllogistic reasoning, which does not conform to it, is to be regarded as erroneous. The *Dictum* being thus the basis and test of all Syllogistic reasoning, the General Rules governing such reasoning must be derivable from it.

The Dictum is necessary and self-evident in character, and may, therefore, be accepted as the axiom of all Syllogistic reasoning.

Derivation of Rules from the Dictum. Let us state the *Dictum* itself in the form of a Syllogism :

Whatever (P) is predicated of a term distributed (M) [*i.e.*, All (or No) M is P]
Under which term (M) something else (S) is contained [*i.e.*, S is M]
May be predicated of that (S) which is so contained [*i.e.*, S is P].

The *Dictum* is thus split up into three parts, the first of which is the major premise, the second the minor, and the third the conclusion.

As the Dictum is the basis of all Syllogistic reasoning, the general rules of such reasoning may be deduced from it.

(1) From the above analysis it appears that the *Dictum* requires three and only three terms : 'Whatever is predicated' being the major term, 'a term distributed', *i.e.*, 'a class', being the middle term, and 'something else belonging to the class' being the minor term. Hence we have the rule : *A Syllogism must have three and only three terms.*

(2) Similarly, the *Dictum* requires three and only three propositions. Hence we have the rule : *A Syllogism must have three and only three propositions.*

(3) According to the *Dictum*, the middle term must be distributed in the major premise. From this we get the general rule : *The middle term must be distributed at least once.*

(4) The *Dictum* says that in the conclusion the original predication can be made of something else shown to belong to the class. Hence it follows that if the original predication be made by an undistributed term, the same term must remain undistributed in the conclusion, and if the 'something else' (minor term) be undistributed in the

premise, it must remain undistributed in the conclusion. Hence we have the rule: *No term must be distributed in the conclusion which was not distributed in a premise.*

(5) The *Dictum* requires that the minor premise must be affirmative, for it asserts that something belongs to a certain class. Hence we have the general rule: *One, at least, of the premises must be affirmative.* In other words, *if both the premises be negative, no conclusion follows.*

(6) The *Dictum* further says that the predication in the conclusion must be made *in like manner, i.e.*, if the major premise be affirmative (the minor in the 'perfect' form being always affirmative), the conclusion must be affirmative; and if the major premise be negative, the conclusion must be negative. Hence we get the general rule: *If one premise be negative, the conclusion must be negative, and vice versa.*

To these six rules which are regarded as fundamental and sufficient in themselves to enable us to test, and to detect errors in, Syllogistic reasoning, four more rules are added; though, in reality, these four are only corollaries, derived from the last four rules taken together. These are: (7) *If both the premises be affirmative, the conclusion must be affirmative, and vice versa;* (8) *From two particular premises nothing can be inferred;* (9) *If one premise be particular the conclusion must be particular;* (10) *From a particular major and a negative minor nothing can be inferred.* We shall examine these ten rules in § 8. Of the six fundamental rules, the first two relate to the structure of the Syllogism, the third and the fourth relate to the distribution of terms, and the fifth and the sixth relate to the quality of the constituent propositions. Students may remember them with the help of the following verses of Messrs. Holman and Irvine (*Questions on Logic*, p. 65):

"Of terms have but three; proposition as term;
Distribute the middle term—in this be most firm;
Distribute no term in conclusion, beside,
Unless in a premise 'tis equally wide;
One premise affirmative, this you must learn,
For negative premises nothing affirm;
A negative head has a negative tail,
And the converse of this is of equal avail."

Definition of 'Figure.'

§ 6. Figures of Syllogism. Figure is the form of a syllogism as determined by the position of the middle term in relation to the extremes in the premises. As there are two premises, and the middle term occurs in both the premises, it may occupy four different positions, and accordingly there are *four possible figures*. If M be the middle term, P the

There are four possible figures.

major term, and S the minor term, then the four figures may be represented thus :—

I.	II.	III.	IV.
M P,	P M,	M P,	P M,
S M ;	S M ;	M S ;	M S ;
∴ S P.	∴ S P.	∴ S P.	∴ S P.

In the first form, *i.e.*, in the **First Figure,** the middle term is subject of the major and predicate of the minor premise. In the **Second Figure,** the middle term is predicate of both the premises. In the **Third Figure,** the middle term is subject of both the premises. In the **Fourth Figure,** the middle term is predicate of the major and subject of the minor premise.

Concrete Examples.

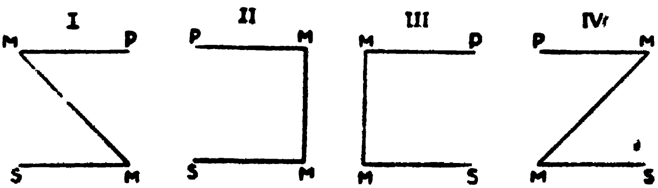
Concrete Examples

Figure I	Figure II
All *men* are mortal,	No angels are *mortal*,
All kings are *men* ;	All men are *mortal* ;
∴ All kings are mortal.	∴ No men are angels.

Figure III	Figure IV
All *men* are rational,	All philosophers are *men*,
All *men* are imperfect ;	All *men* are rational ;
∴ Some imperfect beings are rational.	∴ Some rational beings are philosophers.

C. Read's scheme for remembering the

The following scheme, as given by C. Read, may well help one in remembering the peculiar nature of the different figures.



peculiar nature of the different Figures.

The horizontal lines represent the premises ; and at the angles formed with them by the slanting or by the perpendicular lines, the middle term occurs.

Note further that the scheme of the fourth figure resembles Z, the last letter of the alphabet: this helps one to remember it in contrast with the first, which is thereby also remembered. Figures II and III seem to stand back to back.

There would be only three possible Figures, if we have no means of distinguishing between major and minor terms.

Note. As we know, the subject of the conclusion is the minor term, and the predicate, the major; so if the conclusion be not given and the premises only are given, we have no means of distinguishing between major and minor terms and consequently the distinction between major and minor premises vanishes. Thus either of the premises may come before, or after, the other. In this case, there can be *three posible ways* in which the middle term can be arranged in relation to the other terms of the syllogism, and accordingly there can be *three possible figures*. In the first figure, according to this view, the middle term is subject of one premise and predicate of the other—the first and fourth figures of the above four-fold classification being thus blended into one. In the second figure, the middle term is predicate of both the premises. In the third figure, the middle term is subject of both the premises.

Definition of 'Mood.' Three senses in which the term 'mood' has been used:

§ V. **Moods of Syllogism.** Mood is the form of a syllogism as determined by the quality and quantity of its constituent propositions. Thus we have the moods **AAA, EAA, EIO,** etc. There are *three different senses* in which the term 'mood' has been used:—

The widest sense,

(1) In the *widest sense*, the term 'mood' implies the form of a syllogism as determined by the quality and quantity of all the constituent propositions—the premises and the conclusion.

the wider sense, and

(2) In the *wider sense*, the term 'mood' implies the form of a syllogism as determined by the quality and quantity of the premises only. In our ordinary use of the term we shall stick to this sense.

the narrow sense.

(3) In the *narrow sense*, the term 'mood' implies only the valid forms of syllogism.

In the wider sense, there are 16 possible moods in each figure, and 64 in all the figures

Now, what are the possible moods or arrangements of propositions in a syllogism? In the wider sense given above, *i.e.*, if we take into account the premises only, there are sixteen possible combinations of propositions, for there are four kinds of propositions (**A**, **E**, **I** and **O**), and in each figure, every one of the four may be the major premise, and each of the four majors may have four different minors. The sixteen possible moods in each figure are indicated below, the first letter of each pair standing for the major premise and the second for the minor :—

AA	**EA**	**IA**	**OA**
AE	**EE**	**IE**	**OE**
AI	**EI**	**II**	**OI**
AO	**EO**	**IO**	**OO**

And as there are four figures, the number of possible moods is sixty-four in all the figures.

In the widest sense, there are 64 possible moods in each figure, and 256 in all the figures

In the widest sense, however, that is, if we take the premises and the conclusion into account, there are 64 possible moods in each figure, for each of the sixteen pairs of premises may have four different conclusions; as, for instance, from the combination of premises **AA** we get four different moods, *viz.*, **AAA, AAE, AAI, AAO.** Hence the number of possible moods is 4×16, *i.e.*, 64 in each figure, and 256 in all.

In the narrow sense, there are 19 moods in all

Again, all the possible combinations do not form valid syllogisms. As we shall presently see, only nineteen of the above possible combinations are regarded as *valid* or *legitimate*, since they do not violate any of the rules of Syllogism. Hence, in the narrow sense, the possible number of moods is only *nineteen*. Those combinations of propositions in which the conclusions do not follow necessarily from

the premises, inasmuch as they violate certain rules of Syllogism, are called *invalid* moods.

Different Tests of Syllogism.

8. Tests and General Rules of Syllogism.

A syllogism, as mentioned above, consists of three propositions arranged in the form of an argument. It has also been remarked that any random putting together of three propositions does not necessarily constitute a valid argument, *i.e.*, an argument in which the conclusion follows necessarily from the premises. Hence Logicians have laid down different kinds of Test for distinguishing a valid from an invalid syllogism. These are: (1) Test by General Rules, (2) Test by Special Rules, (3) Test by Aristotle's Dictum and Reduction, and (4) Diagrammatic Test. We shall now deal with the exposition and application of these Tests. First of all, let us take up the most important method, *viz.*, that of testing syllogisms by certain General Rules (already derived from Aristotle's Dictum—§ 5), to which every syllogistic inference must conform in order to be valid.

General Rules of Syllogism.

I. A Syllogism must have three and only three terms.

Rule I. A syllogism must have three and only three terms. This rule follows from the meaning of Syllogism. A syllogism, we know, establishes a relation between the two extremes (*i.e.*, the major and minor terms) through the mean (*i.e.*, the middle term). Hence it must necessarily contain three terms only. If there are two terms in an argument, it is an immediate inference; if there are four terms (or more), the argument consists of more than one syllogism or is no syllogism at all. For example, from the premises 'All men are mortal' and 'All philosophers are wise' no conclusion follows, for

there is no middle term, which serves as the medium of comparison. Again, take the following example :—

All men are mortal,
All philosophers are men,
Socrates is a philosopher,
∴ Socrates is mortal.

Here we have, no doubt, more than three terms, but the argument is clearly analysable into two distinct syllogisms :—

(1) All men are mortal,
All philosophers are men;
∴ All philosophers are mortal.

(2) All philosophers are mortal,
Socrates is a philosopher;
∴ Socrates is mortal.

The violation of this rule gives rise to the *Fallacy of Four Terms.*

The violation of this rule gives rise to a fallacy, technically called the *Fallacy of Four Terms.* It must also be noted here that an ambiguous term, having two distinct meanings, is really equivalent to two terms; and if such a term occurs in an argument, then the argument really contains four terms instead of three. Such an argument may involve either (*a*) Fallacy of Ambiguous Middle or (*b*) Fallacy of Ambiguous Major or (*c*) Fallacy of Ambiguous Minor. All these, again, are but different forms of the Fallacy of Four Terms. For example :—

Ambiguous Middle

Dates are fruits,
The first of January is a *date;*
∴ The first of January is a fruit.

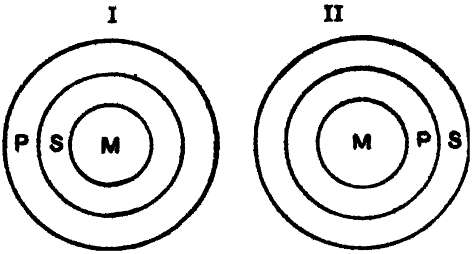
Ambiguous Major

No courageous creature *flies* (*i.e.*, runs away through fear),
The eagle is a courageous creature;
∴ The eagle does not *fly* (*i.e.*, fly with wings).

In example (1), the major term 'animals' is distributed in the conclusion, being the predicate of an **E** proposition, while it is not distributed in the major premise, being the predicate of an **A** proposition. Hence it is an instance of the *Illicit Process of the Major*, or briefly, *Illicit Major*. In example (2) the minor term 'animals' is distributed in the conclusion, being the subject of an **E** proposition, while it is not distributed in the minor premise, being the predicate of an **A** proposition. Hence it is an instance of *Illicit Minor*.

The rule can be illustrated by diagrams thus:—

All M is P,
All M is S,
∴ All S is P



The conclusion follows from diagram I, but it does not follow from diagram II. Hence the conclusion cannot be true generally. The common conclusion, which is correct, is 'Some S is P'.

The converse of this rule is not true.

It must be noted, however, that *the converse of the rule is not true*. It is an error to suppose that because a term is undistributed in the conclusion, it ought to be undistributed in its premise. A term may be undistributed in the conclusion, though it is distributed in its premise, for in Deductive reasoning we are allowed to take in the conclusion less, but never more, than what is given in the premise.

V. One, at least, of the

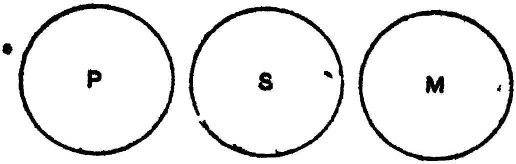
Rule V. One, at least, of the premises must be affirmative. In other words, *if both the premises be*

negative, no conclusion follows. If both the premises be negative, neither of the extremes is shown to be connected with the middle term, so the middle term fails to establish any possible relation between the two extremes, and consequently no conclusion can be drawn. Thus no conclusion follows from 'No men are brutes' and 'No angels are men'. The violation of this rule gives rise to what has been called the *Fallacy of Two Negative Premises*.

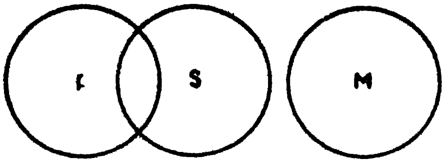
premises must be affirmative.

The violation of this rule gives rise to the *Fallacy of Two Negative Premises*

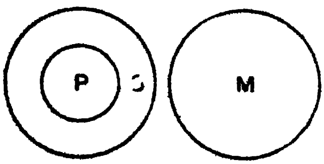
I



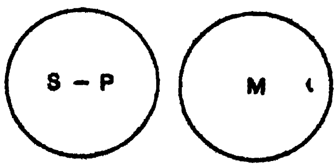
II



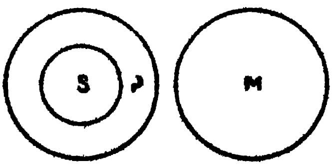
III



IV



V



Representing the negative premises by 'No M is P' and 'No S is M', all the possible relations between S and P may be exhibited by the above five diagrams. Thus S may lie outside P (I), S may intersect P (II), S may contain P (III), S may coincide with P (IV), or S may be contained in P (V). From the different positions as shown in the different diagrams, no common relation can be established between S and P. Thus we conclude that one, at least, of the premises must be affirmative.

Do two negative premises ever justify any conclusion?

Note 1. In this connection Jevons remarks: "It must not however be supposed that the mere occurrence of a negative particle (*not* or *no*) in a proposition renders it negative in the manner contemplated by this rule. Thus the argument

'What is not compound is an element,
Gold is not compound,
Gold is an element.'

contains negatives in both premises, but is nevertheless valid, because the negative in both cases affects the middle term, which is really the negative term *not-compound*" (*Elementary Lessons in Logic*, p. 134)

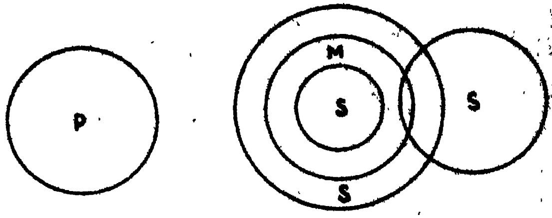
Note 2. The rule must not be construed as implying that from two *propositions* which are negative in form, nothing can be inferred. What the rule enjoins is that when these negative propositions stand as actual *premises* of a syllogism, nothing can be inferred as to the relation between the two extremes from comparing them with a single third term. Before we can use these negative propositions as real premises, at least one of them must be given the affirmative form. In fact, when the premises are of the form 'No not-M is P' and 'No S is M', we have not a syllogism at all, for there are *four* terms—not-M, P, S and M. In order to use the given propositions as real premises of a syllogism with a common middle term, the minor premise must be obverted, thus making it an affirmative premise of the form 'All S is not-M'. Hence we see that before we can legitimately draw a conclusion from premises, they must be in accordance with this rule.

VI. If one premise be negative, the conclusion must be negative, and *vice versa*.

Rule VI. If one premise be negative, the conclusion must be negative, and vice versa. If one premise be negative, the other must be affirmative (Rule V). Now, the affirmative premise affirms a connection between the middle term and one of the extremes, and the negative premise denies any connection between the middle term and the other extreme. From the affirmation of connection in one case and the denial of connection in the other, we

cannot affirm in the conclusion any connection between the extremes, *i.e.*, we can draw a negative conclusion only. The rule may be illustrated by diagrams thus :—

No P is M,
Some M is S;
∴ Some S is not P.



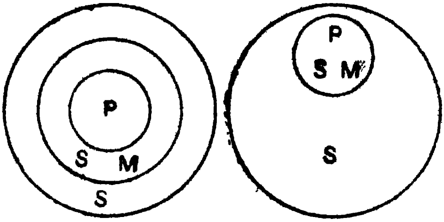
The negative premise is represented by two circles P and M lying outside each other, and the affirmative one is represented by the circles M and S either containing each other, or intersecting, or coinciding with each other. In all these different positions, a part of S must lie within M, which always lies outside P. Hence we may conclude that a part of S lies outside P, *i.e.*, Some S is not P.

Conversely, if the conclusion be negative, one of the premises must be negative. This can be proved by the fact that in the conclusion we cannot deny any connection between the extremes, unless we have previously denied any connection between the middle term and one of the extremes. In other words, the absence of connection between the two extremes can only be inferred from a presence of connection between the middle term and one extreme (*i.e.*, an affirmative premise) and an absence of connection between the middle term and the other extreme (*i.e.*, a negative premise). Thus we see that a negative conclusion requires a negative premise.

VII. If both the premises are affirmative, the conclusion must be affirmative, and *vice versa*.

Rule VII. If both the premises be affirmative, the conclusion must be affirmative, and vice versa. An affirmative proposition establishes a connection between the subject and the predicate. If both the premises be affirmative, it means that there is a connection between the middle term and each of the extremes. Hence it follows that the extremes themselves are connected with each other, that is to say, the conclusion must be affirmative. This rule may be illustrated by diagrams thus:—

All P is M,
All M is S;
∴ Some S is P,
or All P is S.



Conversely, if the conclusion be affirmative, both the premises must be affirmative. For, if one of the premises be negative, the conclusion must be negative (Rule VI). Hence it follows that if the conclusion be affirmative, neither premise can be negative, *i.e.*, both the premises must be affirmative.

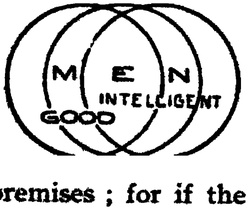
VIII. From two particular premises nothing can be inferred.

Rule VIII. From two particular premises nothing can be inferred. The possible combinations of two particular premises are **OO**, **II** and **IO** *in any order*. Of these, **OO** is rejected at once, for two negative premises yield no conclusion (Rule V). **II** is also rejected, as no term is distributed in any of the premises, so that the middle term has not a chance of being distributed even once (Rule III). In the combination **IO** (in any order), we find that only one term is distributed, *viz.*, the predicate of **O**, which must be the middle term. One of the pre-

mises being negative, the conclusion must be negative (Rule VI), and as the predicate of a negative proposition is always distributed, the predicate of the conclusion, the major term, becomes distributed. But the major term was not distributed in the premise. Hence there arises the fallacy of *Illicit Major*, if we attempt to draw any conclusion from the combination **IO**, in any order (Rule IV). Thus it follows universally that from two particular premises nothing can be inferred.

A conclusion may, however, be drawn from two particular premises when the argument is in the Third Figure and the premises are plurative propositions.

There is one case, however, in which a conclusion may be drawn from two particular premises. If two different predicates can separately be affirmed of most of the individuals denoted by some common term (*i.e.*, when the argument is in the third figure and the premises are plurative propositions), there must be some individuals in common of which they can both be affirmed together. Consequently, in such cases, the predicates themselves may be affirmed of each other. Thus, from the premises 'Most men are good' and 'Most men are intelligent', we may infer 'Some men are both good and intelligent', and hence we can legitimately draw the conclusions 'Some intelligent men are good men' and 'Some good men are intelligent men'. This may be understood with the help of the following diagram :—



It is to be remembered, however, that such a conclusion is possible only in the third figure, in which the same term (the middle term) is subject in both the premises ; for if the middle term be predicate in any case, it is not possible to determine, from the ordinary form of a logical proposition, what the

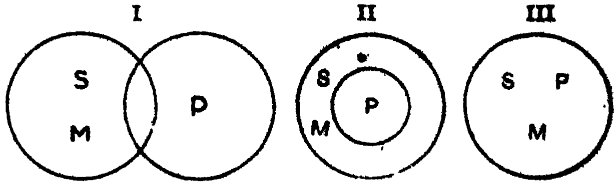
quantity of the predicate is, *i.e.*, whether more than half of its denotation is taken in the case under consideration, and consequently there is no certainty as to whether there is any common ground of comparison.

IX. If one premise be particular, the conclusion must be particular.

Rule IX. If one premise be particular, the conclusion must be particular. As two negative and two particular propositions yield no conclusion (Rules V & VIII), so, when one premise is particular, the possible combinations of premises are **AI, AO** and **EI,** *in any order.* In the combination **AI,** in any order, only one term is distributed, *viz.*, the subject of **A,** and that must be the middle term, for the middle term must be distributed at least once (Rule III) ; and as no other term is distributed in the premise, so no term can be distributed in the conclusion (Rule IV), *i.e.*, the conclusion must be an **I** proposition, which is the only proposition in which no term is distributed. In each of the combinations **AO** and **EI,** in any order, only two terms are distributed, *viz.*, the subject of **A** and the predicate of **O** in the combination **AO** in any order, and both the subject and predicate of **E** in the combination **EI** in any order. One of these two distributed terms must be the middle term (Rule III). And, in each case, the conclusion must be negative, for one of the premises is negative (Rule VI). The negative conclusion distributes its predicate, which is the major term. Therefore, the other term distributed in the premises must be the major term, in order to avoid the fallacy of *Illicit Major* (Rule IV). Hence the minor term which is not distributed in the premise cannot be distributed in the conclusion, where the minor term occupies the position of subject. Thus the subject of the conclusion being undistributed, the conclusion must

be particular. Hence it follows universally that if one premise be particular, the conclusion must be particular. This can be illustrated by diagrams. Thus take the combination **IA** :—

Some P is M,
All M is S;
∴ Some S is P.



From diagram III a universal conclusion may follow, but this does not follow from the other diagrams. Hence the common conclusion is 'Some S is P.'

It should be observed that *the converse of this rule, viz., that a particular conclusion necessitates a particular premise, is not true*, for a term may be undistributed in the conclusion though it is previously distributed in a premise. We have already discussed that we are allowed to take less in the conclusion than what is given in the premise. Thus from the premises 'All material bodies gravitate' and 'All stones are material bodies', we can legitimately draw the conclusion 'Some stones gravitate'. In such cases the premises assume more than what is required to establish the conclusion.

The converse of this rule is not true.

Note. From the above rule it further follows that *if the conclusion be universal, both the premises must be universal*. For, if one premise be particular, the conclusion is particular (Rule IX), and if both the premises be particular, no conclusion follows (Rule VIII). Rules VI and IX may be combined into one rule, *viz., the conclusion follows the weaker premise*. A negative proposition is considered weaker than an affirmative one, and a particular proposition, weaker than a universal one.

If the conclusion be universal, both the premises must be universal.

The conclusion follows [illegible]

X. From a particular major and a negative minor nothing can be inferred.

Rule X. From a particular major and a negative minor nothing can be inferred. If the minor premise be negative, the major must be affirmative (Rule V). Hence the major premise is particular affirmative, *i.e.*, an I proposition, which distributes no term. Thus the major term is not distributed in its premise. Now, one premise being negative, the conclusion, if possible at all, must be negative (Rule VI). As the negative conclusion distributes its predicate, the major term, there arises the fallacy of *Illicit Major*, for the major term is distributed in the conclusion without being distributed in its premise (Rule IV). Hence no conclusion can be drawn from a particular major and a negative minor.

Determination of general valid moods.

We have already seen (§ 7) that there are sixteen possible combinations of premises. It has also been remarked that all of them do not yield valid conclusions. By applying the above ten rules we can at once reject those combinations of premises which do not lead to valid conclusions. Thus the combinations **EE, EO, OE** and **OO** are rejected, because both the premises are negative (Rule V). The combinations **II, IO** and **OI** are rejected, because both the premises are particular (Rule VIII). The combination **IE** is rejected, because the major premise is particular and the minor, negative (Rule X). Hence we are left with *eight* combinations of premises, *viz.*, **AA, AE, AI, AO, EA, EI, IA** and **OA**. All these eight combinations, again, are not valid in all the figures. Some of these are valid in the first figure, some in the second, some in the third and some in the fourth. Some that are valid in one figure may not be valid in another figure. Hence we shall now proceed to determine which of these combinations

form valid moods, and which of these do not, in each figure ; and shall also explain the *Special Rules*, that are deducible from the General Rules and that may guide us in testing arguments in the different figures.

§ 9. **Valid Moods and Special Rules of the First Figure.** Let us examine here the remaining *eight* combinations of premises, in order to determine which of them lead to valid conclusions in the first figure, in which the middle term is subject of the major premise and predicate of the minor.

Moods of the First Figure.

(1) **AA.** All M is P, All men are mortal,
All S is M ; All kings are men ;
∴All S is P. ∴All kings are mortal.

(1) **AA** valid (*Barbara*).

Here the middle term M is distributed in the major premise, being the subject of an **A** proposition. The minor term S is also distributed in the minor premise. Both the premises being affirmative, the conclusion must be affirmative (Rule VII), *i.e.*, **A** or **I.** There is no fallacy of *Illicit Minor*, if we draw the **A** conclusion, for the minor term, which is the subject of the conclusion, has previously been distributed in its premise. Hence the combination **AA** yields the valid conclusion **A** in the first figure. This valid mood **AAA** in the first figure is technically called **Barbara.** In this and in all other technical names which follow, the first vowel represents the major premise, the second the minor, and the third the conclusion.

(2) **AE.** All M is P,
No S is M ;
No conclusion.

(2) **AE** invalid.

Here one of the premises is negative; so, if any conclusion be drawn, it must be negative (Rule VI), distributing its predicate, the major term. But the major term is not distributed in the major premise, being the predicate of an affirmative proposition. Thus if any conclusion be drawn, it will involve the fallacy of *Illicit Major* (Rule IV). Hence we see that the mood **AE** is invalid in the first figure.

(3) **AI.** All M is P, All men are mortal,
Some S in M ; Some wise beings are men ;
∴Some S is P. ∴Some wise beings are mortal.

(3) **AI** valid (*Darii*).

Here both the premises being affirmative, the conclusion must be affirmative (Rule VII). And one of the premises

being particular, the conclusion must be particular (Rule IX). Thus we see that the conclusion must be particular and affirmative, *i.e.*, **I**. And as **I** distributes no term, there is no possibility of *Illicit Process*. Moreover, the middle term has been distributed in the major premise. Hence we see that the combination **AI** leads to the valid conclusion **I** in the first figure. This valid mood **AII** in the first figure is technically called **Darii**.

(4) AO invalid.

(4) **AO.** All M is P,
Some S is not M;
No conclusion.

Here, if any conclusion be drawn, it must be negative, for one of the premises is negative (Rule VI); and the predicate of the negative conclusion, the major term, shall be distributed in the conclusion without being distributed in the major premise, where it stands as predicate of an affirmative proposition. Thus there arises the fallacy of *Illicit Major* (Rule IV). Hence we see that the mood **AO** is invalid in the first figure.

(5) EA valid (*Celarent*).

(5) **EA.**	No M is P,	No men are perfect,
	All S is M;	All philosophers are men;
	∴ No S is P.	∴ No philosophers are perfect.

Here the middle term is distributed in the major premise. One of the premises being negative, the conclusion must be negative (Rule VI). No rule is violated by inferring **E** in this case, for the subject and predicate of the conclusion, the minor and major terms, are distributed in the conclusion as also in the respective premises. Thus there is no *Illicit Process*. Hence we see that the combination **EA** leads to the valid conclusion **E** in the first figure. This valid mood **EAE** in the first figure is technically called **Celarent**.

(6) EI valid (*Ferio*).

(6) **EI.**	No M is P,	No men are perfect,
	Some S is M;	Some created beings are men;
	∴ Some S is not P.	∴ Some created beings are not perfect.

The middle term is distributed in the major premise. One premise being particular and one premise negative, the conclusion must be particular and negative, *i.e.*, **O** (Rules IX and VI). The **O** conclusion distributes only its predicate, the major term, which has previously been distributed in the major premise. Thus there is no *Illicit Process*. Hence we see that the combination **EI** leads to the valid conclusion **O** in the first figure. This valid mood **EIO** in the first figure is technically called **Ferio**.

(7) **IA.** Some M is P,
All S is M;
No conclusion.

Here the middle term is not distributed in either premise. Hence the combination involves the fallacy of Undistributed

buted Middle (Rule III), and does not yield a valid conclusion. Hence we see that the mood **IA** is invalid in the first figure.

(8) **OA.** Some M is not P, **(8) OA invalid.**
All S is M;
No conclusion.

This combination, like the preceding one, does not yield a valid conclusion, for it involves the fallacy of *Undistributed Middle* (Rule III). Hence we see that the mood **OA** is invalid in the first figure.

Thus we get four valid moods in the first figure, *viz.*, **AAA, EAE, AII** and **EIO,** which are technically called **Barbara, Celarent, Darii** and **Ferio.** From a comparison of these valid moods, we can generalize the following two **Special Rules** of the first figure :—

Special Rules of the First Figure.

(1) *The major premise must be universal.*

(2) *The minor premise must be affirmative.*

These two special rules may be proved by the general syllogistic rules thus :—

Proof by the general syllogistic rules.

(1) If the major premise be not universal, then, if possible, let it be particular. The middle term, being thus the subject of a particular proposition, is not distributed in the major premise. Therefore it must be distributed in the minor premise, where it is predicate. And as only negative propositions distribute their predicates, it follows that the minor premise must be negative in order to distribute the middle term. The minor premise being negative, the major premise must be affirmative (Rule V), and the conclusion must be negative (Rule VI). The negative conclusion will thus distribute its predicate, the major term, which has not been distributed in the major premise, being the predicate of an affirmative proposition. Thus the supposition of a particular major in the first figure involves the fallacy of

Illicit Major. Hence in the first figure the major premise cannot be particular, *i.e.*, it must be universal.

(2) If the minor premise be not affirmative, then, if possible, let it be negative. The major premise must, in that case, be affirmative (Rule V), and the conclusion must be negative (Rule VI). The negative conclusion will distribute its predicate, the major term, which has not been distributed in the major premise, being the predicate of an affirmative proposition. Thus the supposition of a negative minor in the first figure gives rise to the fallacy of *Illicit Major*. Hence in the first figure the minor premise cannot be negative, *i.e.*, it must be affirmative.

As all the valid moods in the first figure show these two characteristics, any argument in the first figure may as well be tested by the special rules as by the general rules. As the special rules themselves constitute an independent Test of Syllogisms, we may determine the valid moods of the first figure by means of its special rules. Of the sixteen possible moods, **IA, IE, II, IO, OA, OE, OI** and **OO** are rejected by the first special rule, for the major premise in all of them is particular. And **AE, EE, AO** and **EO** are rejected by the second special rule, for the minor premise in all of them is negative. Thus we are left with four valid moods, *viz.*, **AA, EA, AI** and **EI**.

Moods of the Second Figure.

§ 10. **Valid Moods and Special Rules of the Second Figure.** Let us now examine the same eight moods in the second figure, in order to determine which of them are valid in the second figure, in which the middle term is predicate of both the premises.

(1) **AA.** All P is M,
All S is M;
No conclusion.

(1) AA invalid.

Here the middle term is not distributed in either premise. Thus there arises the fallacy of *Undistributed Middle* (Rule III). Hence we see that the mood **AA** is invalid in the second figure.

(2) **AE.**	All P is M,	All men are mortal,
	No S is M;	No gods are mortal;
	∴No S is P.	∴No gods are men.

(2) AE valid (Camestres).

Here the middle term is distributed in the minor premise, being the predicate of a negative proposition. One of the premises being negative, the conclusion must be negative. No rule is violated by inferring **E** in this case, for the subject and predicate of the conclusion, the minor and major terms, are distributed in the conclusion as also in the respective premises. Thus there is no *Illicit Process*. Hence we see that the combination **AE** leads to the valid conclusion **E** in the second figure. This valid mood **AEE** in the second figure is technically called **Camestres.**

(3) **AI.** All P is M,
Some S is M;
No conclusion.

(3) AI invalid.

This combination yields no conclusion, as the middle term has not been distributed in either premise. It involves the fallacy of *Undistributed Middle*. Hence the mood **AI** is invalid in the second figure.

(4) **AO.**	All P is M,	All men are mortal,
	Some S is not M;	Some beings are not mortal;
	∴Some S is not P.	∴Some beings are not men.

(4) AO valid (Baroco).

Here the middle term has been distributed in the minor premise. One of the premises being particular and negative, the conclusion must be particular and negative, *i.e.*, **O,** which distributes only its predicate, the major term. But the major term has also been distributed in the major premise, being the subject of an **A** proposition. Thus there is no *Illicit Process*. Hence we see that the combination **AO** leads to the valid conclusion **O** in the second figure. This valid mood **AOO** in the second figure is technically called **Baroco.**

(5) **EA.**	No P is M,	No men are birds,
	All S is M;	All swans are birds;
	∴No S is P.	∴No swans are men.

(5) EA valid (Cesare).

Here the middle term is distributed in the major premise, being the predicate of a negative proposition. One of the premises being negative, the conclusion must be negative. No rule is violated by inferring **E** in this case, for the

subject and predicate of the conclusion, the minor and major terms, are distributed in the conclusion as also in the respective premises. Thus there is no *Illicit Process*. Hence we see that the combination **EA** leads to the valid conclusion **E** in the second figure. This valid mood **EAE** in the second figure is technically called **Cesare.**

(6) EI valid *(Festino)*

(6) **EI.**

No P is M,	No men are quadrupeds,
Some S is M,	Some animals are quadrupeds,
Some S is not P.	Some animals are not men.

Here the middle term is distributed in the major premise. One of the premises being negative, the conclusion must be negative, and one of the premises being particular, the conclusion must be particular. Thus the conclusion must be particular and negative, *i.e.* **O,** which distributes only its predicate, the major term. But the major term being previously distributed in the major premise, there is no *Illicit Process*. Hence we see that the combination **EI** leads to the valid conclusion **O** in the second figure. This valid mood **EIO** in the second figure is technically called **Festino.**

(7) IA invalid

(7) **IA.** Some P is M,
All S is M,
No conclusion.

This combination yields no conclusion, as the middle term has not been distributed in either premise. It involves the fallacy of *Undistributed Middle*. Hence the mood **IA** is invalid in the second figure.

(8) OA invalid

(8) **OA.** Some P is not M,
All S is M,
No conclusoin.

Here, if any conclusion be drawn, it must be negative, for one of the premises is negative. The negative conclusion will distribute its predicate, the major term, which has not been distributed in its premise. Thus there arises the fallacy of *Illicit Major*. Hence we see that the mood **OA** is invalid in the second figure.

Thus we get four valid moods in the second figure, *viz.*, **EAE, AEE, EIO** and **AOO,** which are technically called **Cesare, Camestres, Festino** and **Baroco.** From a comparison of these valid moods, we can generalize the following three **Special Rules** of the second figure:—

Special Rules of the Second Figure.

(i) The major premise must be universal.

(ii) One of the premises must be negative.

(iii) The conclusion must be negative.

These three special rules may be proved by the general rules thus:—

Proof by the general syllogistic rules.

(1) If the major premise be not universal, then, if possible, let it be particular. In the second figure, the major term is subject of the major premise. So if the major premise be particular, the major term in the major premise remains undistributed; to avoid *Illicit Process* of the major, the major term must remain undistributed in the conclusion (Rule IV), *i.e.*, the conclusion must be affirmative. If the conclusion be affirmative, then both the premises must be affirmative (Rule VII). In that case, the middle term, occupying the position of predicate in both the premises, has not the chance of being distributed even once, for affirmative premises do not distribute their predicate. Thus the supposition of a particular major in the second figure gives rise to the fallacy of *Undistributed Middle* (Rule III). Hence the major premise in the second figure cannot be particular, *i.e.*, it must be universal.

(2) If one of the premises be not negative, then, if possible, let both the premises be affirmative. In that case, the middle term, occupying the position of predicate in both the premises, has not the chance of being distributed at least once, for affirmative premises do not distribute their predicates. Thus the supposition of two affirmative premises in the second figure involves the fallacy of *Undistributed Middle*. Hence both the premises in the second figure cannot be affirmative, *i.e.*, one of the premises must be negative.

(3) If the conclusion be not negative, then, if possible, let it be affirmative. In that case both the premises must be affirmative, giving rise to the fallacy of *Undistributed Middle*, as shown above.

Hence the conclusion in the second figure cannot be affirmative, *i.e.*, it must be negative.

As all the valid moods in the second figure show the above three characteristics, any argument in the second figure may as well be tested by the special rules as by the general rules. As the special rules themselves constitute an independent Test of Syllogisms, we may determine the valid moods of the second figure by means of its special rules. Of the sixteen possible moods, **IA, IE, II, IO, OA, OE, OI** and **OO** are rejected by the first special rule, for the major premise in all of them is particular. And **AA, AI, EE** and **EO** are rejected by the second special rule, for in none of them is *one* premise negative. Thus we are left with four valid moods, *viz.*, **AE, AO, EA** and **EI.**

Moods of the Third Figure.

§ 11. **Valid Moods and Special Rules of the Third Figure.** Let us next proceed to determine the valid moods in the third figure, in which the middle term is subject of both the premises.

(1) AA valid (*Darapti*).

(1) **AA.** All M is P, All men are mortal,
All M is S; All men are rational;
∴Some S is P. ∴Some rational beings are mortal.

Here the middle term is distributed in both the premises. Both the premises being affirmative, the conclusion must be affirmative, *i.e.*, either **A** or **I.** But the conclusion cannot be **A,** for in that case there would arise the fallacy of *Illicit Minor.* No rule, however, is violated by inferring **I.** Hence we see that the combination **AA** leads to the valid conclusion **I** in the third figure. This valid mood **AAI** in the third figure is technically called Darapti.

(2) AE invalid.

(2) **AE.** All M is P,
No M is S;
No conclusion.

Here, if any conclusion be drawn, it shall be negative, for one of the premises is negative; and the negative conclusion shall distribute its predicate, the major term, which has not previously been distributed in its premise. Thus there arises the fallacy of *Illicit Major.* Hence we see that the mood **AE** is invalid in the third figure.

(3) **AI.** All M is P, All men are mortal, (3) **AI** valid (*Datisi*).
Some M is S; Some men are wise;
∴Some S is P. ∴Some wise beings are mortal.

Here the middle term is distributed in the major premise. Both the premises being affirmative, the conclusion must be affirmative; and one of the premises being particular, the conclusion must be particular. Thus the conclusion must be particular and affirmative, *i.e.*, **I**. And as **I** distributes no term, there is no possibility of *Illicit Process*. Hence we see that the combination **AI** leads to the valid conclusion **I** in the third figure. This valid mood **AII** in the third figure is technically called **Datisi.**

(4) **AO.** All M is P, (4) **AO** invalid.
Some M is not S;
No conclusion.

Here, if any conclusion is drawn, it shall be negative, for one of the premises is negative; and the negative conclusion shall distribute its predicate, the major term, which has not been previously distributed in its premise. Thus there arises the fallacy of *Illicit Major*. Hence we see that the mood **AO** is invalid in the third figure.

(5) **EA.** No M is P, No men are perfect, (5) **EA** valid (*Felapton*).
All M is S; All men are rational;
∴Some S is not P. ∴Some rational beings are not perfect.

Here the middle term is distributed in both the premises. The major term is distributed in the major premise, but the minor term is not distributed in the minor premise. One of the premises being negative, the conclusion must be negative, *i.e.*, either **E** or **O**. But the conclusion cannot be **E**, for in that case there would arise the fallacy of *Illicit Minor*. There is, however, no possibility of *Illicit Process*, if we infer **O**, for **O** distributes only its predicate, the major term, which has been distributed in the major premise. Hence we see that the combination **EA** leads to the valid conclusion **O** in the third figure. This valid mood **EAO** in the third figure is technically called **Felapton.**

(6) **EI.** No M is P, No men are perfect, (6) **EI** valid (*Ferison*).
Some M is S; Some men are wise;
∴Some S is not P. ∴Some wise beings are not perfect.

Here the middle term is distributed in the major premise. One of the premises being negative, the conclusion must be negative; and one of the premises being particular, the conclusion must be particular. Thus the conclusion must be particular and negative, *i.e.*, **O**, which distributes only its predicate, the major term. But the major term being previously distributed in the major premise, there is no *Illicit Process*. Hence we see that the combination **EI** leads to the valid conclusion **O** in the third figure.

This valid mood **EIO** in the third figure is technically called **Ferison.**

(7) IA valid (*Disamis*).

(7) **IA.** Some M is P, All M is S; ∴Some S is P. — Some men are intelligent, All men are mortal; ∴Some mortal beings are intelligent.

Here the middle term is distributed in the minor premise. Both the premises being affirmative, the conclusion must be affirmative; and one premise being particular, the conclusion must be particular. Thus the conclusion must be particular and affirmative, *i.e.*, **I.** And as **I** distributes no term, there is no possibility of *Illicit Process*. Hence we see that the combination **IA** leads to the valid conclusion **I** in the third figure. This valid mood **IAI** in the third figure is technically called **Disamis.**

(8) OA valid (*Bocardo*).

(8) **OA.** Some M is not P, All M is S; ∴Some S is not P. — Some men are not honest, All men are rational; ∴Some rational beings are not honest.

Here the middle term is distributed in the minor premise. The major premise being particular and negative, the conclusion must be particular and negative, *i.e.*, **O,** which distributes only its predicate, the major term. As the major term has been previously distributed in the major premise, there is no *Illicit Process*. Hence the combination **OA** leads to the valid conclusion **O** in the third figure. This valid mood **OAO** in the third figure is technically called **Bocardo.**

Thus we get six valid moods in the third figure, *viz.*, **AAI, IAI, AII, EAO, OAO** and **EIO,** which are technically called **Darapti, Disamis, Datisi, Felapton, Bocardo** and **Ferison.** From a comparison of these valid moods, we can generalize the following three **Special Rules** of the third figure :—

Special Rules of the Third Figure.

(1) *The minor premise must be affirmative.*

(2) *At least one of the premises must be universal.*

(3) *The conclusion must be particular.*

Proof by the general syllogistic rules.

These three special rules may be proved by the general rules thus :—

(1) If the minor premise be not affirmative, then, if possible, let it be negative. Then the major

premise must be affirmative (Rule V), and the conclusion, negative (Rule VI). The negative conclusion will distribute its predicate, the major term, which, however, could not be distributed in the affirmative major premise, where it stands as predicate. Thus the supposition of a negative minor in the third figure will involve the fallacy of *Illicit Major*. Hence the minor premise in the third figure cannot be negative, *i.e.*, it must be affirmative.

(2) In the third figure the middle term is subject of both the premises, so if one of the premises be not universal, the middle term has not the chance of being distributed even once. Thus to avoid the fallacy of *Undistributed Middle*, one of the premises must be universal.

(3) In the third figure the minor term is predicate in the minor premise, and the minor premise is affirmative (Sp. Rule 1). The minor term is not thus distributed in the minor premise, being the predicate of an affirmative proposition. Therefore the minor term cannot be distributed in the conclusion (Rule IV). As the undistributed minor term is subject of the conclusion, the conclusion must be particular. The rule may also be proved independently thus: If the conclusion be not particular, then, if possible, let it be universal. Then the universal conclusion will distribute its subject, the minor term, which, to avoid the fallacy of *Illicit Minor*, must also be distributed in the minor premise (Rule VI). As the minor term is predicate of the minor premise, the minor premise must be negative in order to distribute its predicate, the minor term. If the minor premise be negative, the major premise must be affirmative (Rule V), and the conclusion, negative (Rule IV). The negative conclusion will distribute its predicate, the major

term, which, however, could not be distributed in the affirmative major premise, where it stands as predicate. Thus the supposition of a universal conclusion in the third figure gives rise to the fallacy of *Illicit Major*. Hence the conclusion in the third figure cannot be universal, *i.e.*, it must be particular.

As all the valid moods in the third figure show the above three characteristics, any argument in the third figure may as well be tested by the special rules as by the general rules. It may be noted in this connection that the second special rule of this figure is one of the general rules, and so it has been omitted by many writers. But as we have taken the special rules to be an independent method of testing the validity of Syllogisms, the second rule should deserve a separate treatment here as a characteristic mark of the valid moods of the third figure. Let us see how we can determine the valid moods of the third figure by the special rules. Of the sixteen possible moods, **AE, AO, EE, EO, IE, IO, OE** and **OO** are rejected by the first special rule, for the minor premise in all of them is negative. And the moods **II** and **OI** are rejected by the second special rule, for both the premises in them are particular. Therefore the remaining six moods **AA, IA, AI, EA, OA** and **EI** are valid in the third figure. If, however, we omit the second special rule, we have no means of rejecting the moods **II** and **OI**, and the purpose of testing the validity of Syllogisms independently by the special rules fails.

Moods of the Fourth Figure.

§ 12. **Valid Moods and Special Rules of the Fourth Figure.** Lastly, let us determine the valid moods of the fourth figure, in which the middle term is predicate of the major premise and subject of the minor.

(1) **AA.** All P is M, All kings are men,
All M is S; All men are mortal;
∴Some S is P. ∴Some mortal beings are kings.

(1) AA valid (*Bramantip*).

Here the middle term is distributed in the minor premise. Both the premises being affirmative, the conclusion must be affirmative, *i.e.*, either **A** or **I.** But the conclusion cannot be **A,** for in that case the subject of the conclusion, the minor term, will be distributed in the conclusion without being distributed in the minor premise. No rule, however, is violated by inferring **I.** Hence we see that the combination **AA** leads to the valid conclusion **I** in the fourth figure. This valid mood **AAI** in the fourth figure is technically called **Bramantip.**

(2) **AE.** All P is M, All crows are birds,
No M is S; No birds are men;
∴No S is P. ∴No men are crows.

(2) AE valid (*Camenes*).

Here the middle term is distributed in the minor premise. One of the premises being negative, the conclusion must be negative, *i.e.*, either **E** or **O.** No rule is violated if we infer **E,** for an **E** conclusion distributes both the subject and predicate, the minor and major terms, which have been distributed in the respective premises as well. Thus there is no possibility of *Illicit Process*. Hence we see that the combination **AE** leads to the valid conclusion **E** in the fourth figure. This valid mood **AEE** in the fourth figure is technically called **Camenes.**

(3) **AI.** All P is M,
Some M is S;
No conclusion.

(3) AI invalid.

Here the middle term has not been distributed in either premise. The combination **AI** is, therefore, invalid in the fourth figure, for it involves the fallacy of *Undistributed Middle*.

(4) **AO.** All P is M,
Some M is not S;
No conclusion.

(4) AO invalid.

Here the middle term has not been distributed in one, at least, of the premises. The mood **AO** is, therefore, invalid in the fourth figure, for it involves the fallacy of *Undistributed Middle*.

(5) **EA.** No P is M, No birds are men,
All M is S; All men are rational;
∴Some S is not P. ∴Some rational beings are not birds.

(5) EA valid (*Fesapo*).

Here the middle term is distributed in both the premises. One of the premises being negative, the conclusion must be negative, *i.e.*, either **E** or **O.** But the conclusion cannot be **E,** for, in that case the subject of the conclusion, the minor term, will be distributed in the con-

clusion without being distributed in the minor premise, where it is predicate of an affirmative proposition. No rule, however, is violated by inferring **O**, for an **O** conclusion distributes only its predicate, the major term, which has also been distributed in the major premise. Thus there is no *Illicit Process*. Hence the combination **EA** leads to the valid conclusion **O** in the fourth figure. This valid mood **EAO** in the fourth figure is technically called **Fesapo.**

(6) **EI** valid (*Fresison*).

(6) **EI.** No P is M. / Some M is S; / ∴Some S is not P.

No men are perfect,
Some perfect beings are happy;
∴Some happy beings are not men.

Here the middle term is distributed in the major premise. One premise being negative, the conclusion must be negative, and one premise being particular, the conclusion must be particular. Thus the conclusion must be particular and negative, *i.e.*, **O**, which distributes only its predicate, the major term. But the major term having been previously distributed in the major premise, there is no possibility of *Illicit Process*. Hence we see that the combination **EI** leads to the valid conclusion **O** in the fourth figure. This valid mood **EIO** in the fourth figure is technically called **Fresison.**

(7) **IA** valid (*Dimaris*).

(7) **IA.** Some P is M, / All M is S; / ∴Some S is P.

Some men are honest,
All honest men are trustworthy;
∴Some trustworthy beings are men.

Here the middle term is distributed in the minor premise. Both the premises being affirmative, the conclusion must be affirmative; and one of the premises being particular, the conclusion must be particular. Thus the conclusion must be **I**, which distributes no term; thus there is no possibility of *Illicit Process*. Hence we see that the combination **IA** leads to the valid conclusion **I** in the fourth figure. This valid mood **IAI** in the fourth figure is technically called **Dimaris.**

(8) **OA** invalid.

(8) **OA.** Some P is not M,
All M is S;
No conclusion.

Here, if any conclusion be drawn, it will be negative, for one of the premises is negative. The negative conclusion will distribute its predicate, the major term, which could not be distributed in the major premise, where the major term is subject of a particular proposition. Thus there arises the fallacy of *Illicit Major*. Hence we see that the mood **OA** is invalid in the fourth figure.

Thus we get five valid moods in the fourth figure, viz., AAI, AEE, IAI, EAO and EIO, which are

technically called **Bramantip, Camenes, Dimaris, Fesapo** and **Fresison.** From a comparison of these valid moods, we can generalize the following six **Special Rules** of the fourth figure:—

Special Rules of the Fourth Figure.

(1) *If the major premise be affirmative, the minor must be universal.*

(2) *If the minor premise be affirmative, the conclusion must be particular.*

(3) *If either premise be negative, the major premise must be universal.*

(4) *No premise can be particular and negative.*

(5) *At least one of the premises must be affirmative.*

(6) *The conclusion can never be universal and affirmative.*

These six special rules may be proved by the general rules thus:—

Proof by the general syllogistic rules.

(1) If the major premise be affirmative, it does not distribute its predicate, the middle term. So, to avoid the fallacy of *Undistributed Middle*, it must be distributed in the minor premise, where it occupies the position of subject. Hence the subject of the minor premise must be distributed, *i.e.*, the minor premise must be universal.

(2) If the minor premise be affirmative, it does not distribute its predicate, the minor term, which cannot, therefore, be distributed in the conclusion, where it stands as subject. In other words, the conclusion must be particular

(3) If either premise be negative, the conclusion must be negative. A negative conclusion will distribute its predicate, the major term. To avoid *Illicit Process*, the major term must be distributed in the major premise, where it stands as subject. In other words, the major premise must have a distributed

term for its subject, *i.e.*, the major premise must be universal.

(4) If one premise be particular and negative, *i.e.*, **O**, the conclusion must be **O** (Rules IX and VI), and the other premise must be **A** (Rules VIII and V). Now, if the major premise be **O**, the minor must be **A**, and the conclusion **O**. This will give rise to the fallacy of *Illicit Major*, for the conclusion being **O**, the major term is distributed in the conclusion without being distributed in the particular major premise, where it stands as subject. Again, if the minor premise be **O**, the major must be **A**.. This will give rise to the fallacy of *Undistributed Middle*, the middle term being the predicate of **A** and the subject of **O**. Hence, in any case, **O** cannot be a premise in the fourth figure.

(5) This is one of the general rules (Rule V), and has already been proved. The reason for including it in the special rules of the fourth figure has already been explained in connection with the second special rule of the third figure (*vide* § 11).

(6) If possible, let the conclusion be universal and affirmative, *i.e.*, an **A** proposition. In that case, both the premises must be **A** (Rule IX, *Note*). Consequently, the minor term will be distributed in the supposed universal conclusion, without being distributed in the minor premise, where it stands as predicate of an **A** proposition. Thus there arises the fallacy of *Illicit Minor*. Hence in the fourth figure the conclusion cannot be **A**.

As all the valid moods in the fourth figure are characterized by these six marks, any argument in the fourth figure may as well be tested by these special rules as by the general rules. As the special

rules themselves form an independent Test of Syllogisms, we may determine the valid moods of the fourth figure by means of its special rules. Of the sixteen possible moods, **AI, AO, II** and **IO** are rejected by the first special rule ; **IE, OA, OE, OI** and **OO** are rejected by the third special rule ; **EO** is rejected by the fourth special rule, and **EE** is rejected by the fifth special rule. Thus we are left with five valid moods, *viz.*, **AA, AE, IA, EA** and **EI.**

Valid Moods in the different Figures.

§ 13. **Summary of Results.** The following are the valid moods in the different figures :—

AA, EA, AI and **EI** are valid in the First Figure.

EA, AE, EI and **AO** are valid in the Second Figure.

AA, IA, AI, EA, OA and **EI** are valid in the Third Figure.

AA, AE, IA, 'EA and **EI** are valid in the Fourth Figure.

Mnemonic verses.

The technical names of these valid moods have been given in the following mnemonic lines :—

Barbara, Celarent, Darii, Ferioque prioris :
Cesare, Camestres, Festino, Baroco, secundæ :
Tertia, **Darapti, Disamis, Datisi, Felapton,**
Bocardo, Ferison, habet : Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

In each name there are three vowels. The first vowel stands for the major premise, the second for the minor premise, and the third for the conclusion. The significance of the consonants will be explained later on in connection with Reduction (§ 17).

Convenient summing up of the above results.

Thus it is clear that :—

AA is valid in Figs. I, III and IV.
EA " " " all the Figures.
AI " " " Figs. I and III.
EI " " " all the Figures.
AE " " " Figs. II and IV.
AO " " " Fig. II.
IA " " " Figs. III and IV.
OA " " " Fig. III.

We also find that four moods are valid in the first figure, four in the second, six in the third and five in the fourth figure. Hence there are *nineteen* valid moods in all the four figures taken together.

We may further observe that :—

A conclusion can be proved in only *one* mood **(AAA)** and in Fig. I only.

E conclusion can be proved in *four* moods—one in Fig. I **(EAE)**, two in Fig. II **(EAE, AEE)**, and one in Fig. IV **(AEE)**.

I conclusion can be proved in *six* moods—one in Fig. I **(AII)**, three in Fig. III **(AAI, IAI, AII)**, and two in Fig. IV. **(AAI, IAI)**.

O conclusion can be proved in *eight* moods—one in Fig. I **(EIO)**, two in Fig. II **(EIO, AOO)**, three in Fig. III **(EAO, OAO, EIO)**, and two in Fig. IV **(EAO, EIO)**.

The above results can further be summed up thus :—

(1) **A** can be proved in *one*, **E** in *four*, **I** in *six* and **O** in *eight* ways; thus making *nineteen valid moods* in all.

(2) **A** can be proved in Fig. I only; **E** in Figs. I, II and IV; **I** in Figs. I, III and IV; **O** in all the four figures.

As we see that **A** can be proved only in one mood and in the first figure only, the remark holds good that it is the most difficult proposition to prove, but if once proved, it has the greatest scientific value, for science is concerned with general truths. But as **O**, the contradictory of **A**, can be proved in all the figures and in eight moods, it is the easiest proposition to prove, and thus its contradictory, **A**, is the easiest to disprove (for if **O** be true, its contradictory **A** must be false; *vide* Ch. XVI, § 6, *Note*).

In a Fundamental Syllogism, the distribution of terms in the premises is just what

§ 14. Fundamental and Strengthened Syllogisms: Weakened Syllogisms or Subaltern Moods: Perfect and Imperfect Moods: Indirect Moods.

A **Fundamental Syllogism** is one in which neither premise is stronger than is necessary to justify its conclusion, or, in other words, in which neither of

the extreme terms is distributed in the premises without being distributed in the conclusion, and in which the middle term is distributed only once.. A **Strengthened Syllogism,** on the other hand, is one in which one of the premises is unnecessarily stronger than what is just required to prove its conclusion, or, in other words, in which either of the extreme terms is distributed in the premises without being distributed in the conclusion, or the middle term is twice distributed. Thus of the nineteen valid moods considered above, only four are instances of strengthened syllogism. These are—*Darapti*, *Felapton*, *Fesapo* (in which moods the middle term is twice distributed), and *Bramantip* (in which mood the major term is distributed in the major premise without being distributed in the conclusion). In such a case, the same conclusion can be drawn even if for one of the premises the corresponding particular proposition is substituted. Thus take *Felapton* in the third figure—'No M is P', 'All M is S'; therefore 'Some S is not P.' Here the same conclusion 'Some S is not P' may follow from the premises 'Some M is not P' and 'All M is S', or from the premises 'No M is P' and 'Some M is S'. The remaining fifteen moods are instances of fundamental syllogism, for, in them, there is no unnecessary distribution of terms in the premises, and so neither premise is stronger than what is just required to establish the conclusion.

is necessary to justify its conclusions; while in a Strengthened Syllogism, the distribution of terms in the premises is unnecessarily stronger.

Again, a **Weakened Syllogism** is one in which the conclusion is unnecessarily weaker than what is justified by the premises, or, in other words, in which a particular conclusion is drawn when the premises warrant the corresponding universal conclusion. When in a syllogism the conclusion is universal, *i.e.*,

In a Weakened Syllogism, the conclusion is unnecessarily weaker than what is justified

by the premises.

A or **E**, we may as well draw the corresponding particular conclusion, *i.e.*, **I** or **O**. For example, in the mood **AAA** in the first figure, if we draw the particular conclusion **I** instead of the universal conclusion **A**, the mood **AAI** will be an instance of weakened syllogism. It is thus evident that the moods **AAI** and **EAO** in the first figure (corresponding to the universal moods *Barbara* and *Celarent*), **EAO** and **AEO** in the second figure (corresponding to the universal moods *Cesare* and *Camestres*), and **AEO** in the fourth figure (corresponding to the universal mood *Camenes*) are the possible instances of weakened syllogism. These have been named *Barbari*, *Celaront*, *Cesaro*, *Camestrop*, and *Camenop*, after the universal moods to which they respectively correspond. There can be no such instance in the third figure, in which the conclusion is always particular. Such a syllogism is also called a **Subaltern Mood**, because its conclusion might be obtained by *subalternation* from the universal conclusion of the corresponding syllogism. When the universal conclusion is allowable, it is of no use to draw the particular one. Hence these subaltern moods, though valid, are superfluous and practically useless, and consequently they have been omitted from the list of the nineteen useful and independent moods.

A Weakened Syllogism is also called a Subaltern Mood.

Note. It may be noted here that all the subaltern moods are strengthened syllogisms, except *Camenop* (**AEO** in Fig. IV); for in each of the other four moods the minor premise is unnecessarily strengthened and may be weakened without affecting the conclusion. Thus, when the subaltern moods are taken into consideration, there are *eight* strengthened syllogisms in all, *viz.*, the four independent moods (*Darapti*, *Felapton*, *Fesapo*, *Bramantip*) named above, and the four subaltern moods (*Barbari*, *Celaront*, *Cesaro* and *Camestrop*). Hence it follows that the difference between a strengthened syllogism and a weakened syllogism is one of standpoint. In the former a syllogism is considered from the standpoint of the *premise* and in

the latter from the standpoint of the *conclusion*. In a strengthened syllogism one of the premises is unnecessarily strengthened and may be weaker, while in a weakened syllogism the conclusion is weakened and should be wider than it is.

Perfect Moods are the valid moods in the first figure. They are so called because the first figure, according to Aristotle, is the perfect figure, as his *Dictum*, which is the basis of all syllogistic reasoning, is directly applicable to this figure alone (see §§ 5 and 17). **Imperfect Moods** are the valid moods in the second, third and fourth figures. They are so called because the second, third and fourth figures, according to Aristotle, are imperfect, as his *Dictum* is not directly applicable to them. We shall see below (§ 17) how the validity of a mood in an imperfect figure was tested by Aristotle by reducing or bringing back the mood to the corresponding mood in the perfect figure.

Perfect Moods are the valid moods in the first figure, while Imperfect Moods are the valid moods in the second, third and fourth figures.

Note. **Indirect Moods** are those moods 'in which the extreme employed in the first premise becomes the subject, and the extreme employed in the second premise, the predicate of the conclusion.' Thus the scheme for the indirect moods of the first figure will be

M———P
S———M
———————
∴ P———S (and not 'S——P', the usual form of the direct moods).

The valid moods with the conclusion of the form 'P—S' in each figure are called its Indirect Moods.

In this arrangement, out of the sixteen possible moods, only **AAI, EAE, AII, AEO** and **IEO** in the first figure may be proved to be valid by the general rules. The above, then, are the five valid indirect moods of the first figure. If we transfer the premises of these moods, they become respectively **AAI** (*Bramantip*), **AEE** (*Camenes*), **IAI** (*Dimaris*), **EAO** (*Fesapo*), and **EIO** (*Fresison*) *i.e.*, the moods proved to be valid in the fourth figure. Hence the moods of the fourth figure are regarded as the indirect moods of the first. In the same way we may find out the indirect moods of the second figure, *viz.*, **AEE, EAE, IEO** and **OAO**; and also of the third figure, *viz.*, **AAI, AEO, AII, AOO, IAI** and **IEO.** It is evident that these indirect moods of the second and third figures are nothing but the ordinary valid moods of these figures with their premises transposed. Hence we see that the distinction

between direct and indirect moods disappears in the case of the second and third figures and holds good only in the case of the first and fourth figures.

Aristotelian distinction between perfect and imperfect figures has been set aside by Lambert, who holds that all figures are equally perfect, and formulates a special dictum for each figure.

§ 15. Axioms of the Different Figures. Aristotle regarded the first figure only as perfect and all other figures as imperfect, inasmuch as his *Dictum* is directly applicable to the first figure alone. (*Vide* § 17.) It is, however, extremely doubtful as to how far Aristotle was right in making such a distinction between the first and other figures, seeing that each figure with its distinctive characteristics is best suited to a certain kind of arguments, and that Syllogisms in all the four figures are equally conclusive. In fact, syllogistic arguments are valid, not because they conform to the *Dictum*, but because they are ultimately based on the Fundamental Laws of Thought. For this reason, Lambert, an eminent German Logician, has held that all figures are equally perfect, and has formulated a special *dictum* for each figure, thus ignoring the Aristotelian distinction between the perfect and imperfect figures. Lambert accepts as usual the *Dictum de omni et nullo* as the canon for the first figure, and enunciates *Dictum de diverso*, *Dictum de exemplo* and *Dictum de reciproco* as the canons for the second, third and fourth figures respectively.

Dictum de omni et nullo is the canon for Fig. I. *Dictum de diverso* is the canon for Fig. II.

Dictum de diverso is thus enunciated: "If, one term be contained in, and another excluded from, a third term, they are mutually excluded." For illustration, take the mood *Cesare*:

No P is M,
All S is M;
∴ No S is P.

Here one term S is contained in, and another term P is excluded from, a third term M; hence the terms S and P are mutually excluded.

Dictum de exemplo is the canon for Fig. III.

Dictum de exemplo is thus enunciated: "Two terms which contain a common part, partly agree, or if one term contains a part which the other does not, they partly differ." For illustration, take the mood *Darapti*:

All M is P,
All M is S;
∴ Some S is P.

Here M is a part of P, and also a part of S; hence they partly agree, *i.e.*, Some S is P. Similarly the *dictum* may be illustrated by any negative mood of the figure.

Dictum de reciproco is the canon for Fig. IV.

Dictum de reciproco is thus enunciated: "If no M is B, no B is this or that M; if C is or is not this or that B, there are B's which are or are not C." Dr. Ray restates it more clearly thus: "If a term be included in a second term which is excluded from a third, then the third is excluded from the first; if a term be included in (or excluded from) a second term which is included in a

third, then a part of the third is included in (or excluded from) the first." The first part of the *dictum* may be illustrated by the mood *Camenes*, while the second part, by the moods *Bramantip*, *Dimaris*, *Fesapo*, and *Fresison*. Take, for example, the mood *Fresison* :

No P is M,
Some M is S;
∴ Some S is not P.

Here P (the first term) is excluded from M (the second term) which is included in S (the third term); hence a part of S (the third term) is excluded from P (the first term), *i.e.*, Some S is not P.

Lambert's *dicta*, like Aristotle's *dictum*, are only developed forms of the Fundamental Laws of Thought and are, therefore, as much self-evident as the *dictum* of Aristotle (*vide* § 5) and Whately's canons (*vide* § 4). They directly enable us to determine the validity of the moods of the several figures. For this reason, Lambert claims for each of the different figures an equally fundamental and independent nature and further supports his position by pointing to certain special purposes to which each one of the figures is best suited (see below).

§ **16. Characteristics and Uses of the Different Figures.** Having discussed the equality and independence of the different figures, let us now indicate their characteristics and uses :—

The *First Figure*.

Figure I. (1) The common and natural mode of interpreting a proposition, as explained before (Ch. XIV, § 2), is to take the subject in denotation and the predicate in connotation. Accordingly, the major term in the first figure is to be understood as taken in connotation, since it stands as the predicate of the major premise ; and the minor term is to be understood as taken in denotation, since it stands as the subject of the minor premise. Now, in the conclusion the minor term is the subject as is also the case in the premise, and the major term is the predicate as is also the case in the premise. Thus it is evident that in passing from premises to conclusion, we pass from the denotative use of a term in the premise to its denotative use in the conclusion, and

(1) It may appropriately be termed Perfect Figure, since it is the *only* figure which represents the natural and consistent form of reasoning.

from the connotative use of a term in the premise to its connotative use in the conclusion. Hence, in the first figure, the order of thought in the conclusion is the same as in the premises. This natural mode of argument is not, however, observed in any other figure. Thus the first figure is the only figure which represents the most natural and consistent form of reasoning. Herein consists the real superiority of the first figure to all others, and in this sense it may very appropriately be termed the **Perfect Figure.**

(2) It proves all the four forms of propositions.

(2) The first figure is the only figure which proves all the four forms of logical propositions, *viz.*, **A, E, I** and **O.**

(3) It is the *only* figure which proves A propositions.

(3) This is the only figure which proves **A** propositions. According to some Logicians, this is the most important characteristic of the first figure, for all sciences are concerned with general truths and thus aim at establishing **A** propositions. For this reason, the proofs of all demonstrative sciences like Mathematics run in the mood *Barbara*. Hence it is also called the **Scientific Figure.**

The *Second Figure*.

(1) In this figure there is a partial change in the use of the extremes.

Figure II. (1) In this figure, the subject of the conclusion (*i.e.*, the minor term) is also the subject of the minor premise, but the predicate of the conclusion (*i.e.*, the major term) is the subject of the major premise. So the major term is taken in denotation in the premise and in connotation in the conclusion. Thus the passage from premises to conclusion involves a change from the denotative to the connotative use of the major term. Hence, in this figure, there is a partial change in the use of the extremes in the conclusion, and so an argument in this figure is not so natural and consistent as one in the first figure.

(2) It proves *only* negative propositions, and so it is very useful

(2) The second figure proves only negative conclusions, and so it is very useful for proving distinctions between things and for disproving statements of an adversary.

for proving distinctions.

(3) By a series of syllogisms in this figure, we may go on excluding all possible predicates of a subject, until we happen to find the correct one. Hence this figure has also been called the **Exclusive Figure.**

(3) It helps us to find the correct predicate by excluding all posible predicates of a subject.

Figure III. (1) In this figure, the predicate of the conclusion (*i.e.*, the major term) is also the predicate of the major premise, but the subject of the conclusion (*i.e.*, the minor term) is the predicate of the minor premise. Thus the passage from premises to conclusion involves a change from the connotative to the denotative use of the minor term. Hence in this figure there is also a partial change in the use of the extremes in the conclusion, and consequently an argument in this figure is not so natural and consistent as one in the first figure.

The *Third Figure.* (1) Here also there is a partial change in the use of the extremes.

(2) The third figure proves only particular conclusions, and so it is very useful for proving instances or exceptions and thus for disproving a general rule by establishing some exceptions to it. This figure has accordingly been called the **Enstatic Figure** (from 'Enstasis', meaning objection), for it is best adapted to contradict universal propositions (*i.e.*, **A** or **E**) of an adversary by establishing an objection in the form of a particular conclusion (*i.e.*, **O** or **I**).

(2) It proves *only* particular propositions and so it is very useful for proving instances or exceptions.

(3) When the middle term is singular or definite in quantity, an argument is most naturally expressed in this figure. This is so, because in a proposition consisting of singular and general terms, the singular term naturally becomes the subject, and in the third figure the middle term is subject in both the premises.

(3) When the middle term is singular, an argument is most naturally expressed in this form.

The Fourth Figure. (1) In this figure there is a complete change in the use of the extremes.

Figure IV. (1) In the fourth figure, the subject of the conclusion (*i.e.*, the minor term) is the predicate of the minor premise, and the predicate of the conclusion(*i.e.*, the major term) is the subject of the major premise. Thus the passage from premises to conclusion involves a change from the denotative to the connotative use of the major term as well as from the connotative to the denotative use of the minor term. Hence in this figure there is a complete change in the use of the extremes. There being thus a complete inversion of the order of thought, the fourth figure has been rejected by ancient and modern Logicians as an unnatural expression of syllogistic reasoning. Aristotle did not recognize this figure, it was added by Galen, a Roman physician of the second century, and is thus called the **Galenian Figure.** We cannot, however, ignore this figure altogether, inasmuch as its consideration is of theoretical importance in Formal Logic, since it represents one of the possible ways in which the middle term can be arranged in relation to the other terms of the Syllogism.

(2) It is suited to prove exclusion of the different species of a genus. Lambert on the uses of the four figures.

(2) This figure proves all kinds of propositions except **A,** and is suited to prove exclusion of the different species of a genus.

Note I. Lambert has expressed the uses of the four figures thus: "The first figure is suited to the discovery or proof of the properties of a thing; the second to the discovery or proof of the distinctions between things; the third to the discovery or proof of instances and exceptions; the fourth to the discovery or exclusion of the different species of a genus." Welton makes the following observations on the use of the fourth figure mentioned by Lambert: "The relation of species and genus would be much more satisfactorily established by a syllogism in the first figure, in which the name of the species is the minor, and that of the genus the major term than by one in the fourth figure, in which the major term denotes the species and the minor term the genus."

Note 2. We have said above that the fourth figure was not recognized by Aristotle and that it was introduced by the celebrated Roman physician, Galen (on the testimoney of the Arabian philosopher Averroës). The value and use of the fourth figure have been much discussed by Logicians. Some modern Logicians have rejected the figure as an unnatural mode of reasoning and some have thought it quite useless. Thus Whately thinks that the fourth figure expresses in a clumsy and unnatural way what would much more naturally and consistently be expressed in the first figure. Creighton makes this point clear with the help of the following example:

Discussion as to the cogency and utility of the Fourth Figure.

"The whale is a mammal,
All mammals are vertebrates;
. Some vertebrates are whales"

"It is plain," he says, "that the conclusion of this argument is somewhat strained. That is, it would be more natural to obtain the conclusion 'whales are vertebrates,' than to infer that 'some vertebrates are whales;' for this statement seems to make the species, or less inclusive term, the predicate of the genus, or wider term. It was for this reason, apparently, that Aristotle omitted this figure, as improperly making the real major term a minor, and the real minor a major, and so stating in a less adequate way an argument which could have been better formulated in the first figure" (*Introductory Logic*, p. 130). Thomson, again, condemns it on a different ground to which we have already referred. Thus he says: "In this fourth figure, the order of thought is wholly inverted, the subject of the conclusion had only been a predicate, whilst the predicate had been the leading subject in the premise Against this the mind rebels" (*Laws of Thought*, p. 178) Some Logicians have, again, condemned it as useless, for, according to them, it is only the first figure with the conclusion converted. Thus Bowen writes: "What is called the fourth figure is only the first with a converted conclusion; that is, we do not actually reason in the fourth, but only in the first, and then if occasion requires, convert the conclusion of the first" (*Logic*, p. 192). But this view is not true so far as it goes. It is true that the conclusions of *Bramantip*, *Camenes* and *Dimaris* are the converses of the conclusions of *Barbara*, *Celarent* and *Darii*. But it is not true that the conclusions of *Fesapo* and *Fresison* may be obtained by converting the conclusion of *Ferio*. And it is apparent that the conclusion of *Ferio* being an **O** proposition, cannot, indeed, be converted. As to whether the fourth figure represents an independent type, we have seen before (§ 14) that the moods of the fourth figure may be regarded as indirect moods of the first figure. And Joseph seems to supply an important information in this respect, when he says that it "is not an independent type, its first three moods are merely moods

of the first figure with the conclusion converted, as the process of reducing them assumes; its last two moods draw conclusions which are shown to be valid most naturally by reduction to the third (figure)" (*Introduction to Logic*, p. 290).

From what has been said above it follows that the practical utility of the fourth figure may be much less in comparison with other figures, as no argument, when expressed consistently and naturally, will assume this form. But it is beyond doubt that its theoretical importance in Formal Logic can never be ignored. Thus Welton remarks: "The chief value of the fourth figure, indeed, is theoretical; as it is a possible arrangement of terms, its recognition as such is necessary to complete the formal doctrine of figure." So also C. Read says "The truth is that, if distinction of figure be recognised at all, the fourth figure is scientifically necessary, because it is inevitably generated by an analysis of the possible positions of the middle term."

Aristotle's Dictum is the basis of all syllogistic reasoning, inasmuch as the special rules of the first figure and the validity of the perfect moods (i.e., the valid moods in the first figure) directly follow from it, while the imperfect moods (i.e., the valid moods in the second, third and fourth figures) may be shown to

§ 17. Reduction. We have already seen (§ 5) that Aristotle and his followers regarded the *Dictum de omni et nullo* as the one supreme principle of all syllogistic reasoning, and taught that all forms of syllogistic arguments, in order to be valid, must ultimately conform to it. If the *Dictum* itself be stated in the form of a Syllogism thus—

1. Whatever is predicated of a term distributed (*major premise*),
2. Under which something else is contained (*minor premise*),
3. May be predicated of that which is so contained (*conclusion*)—

it is evident that the *Dictum* implies two things, *viz.*, (1) the major premise must be universal, and (2) the minor premise must be affirmative. These, we know (§ 9), are the special rules of the first figure. The *Dictum* is thus directly applicable to the first figure alone, which is, therefore, called by Aristotle the *Perfect Figure*. It is also clear that by the help of the *Dictum* we can directly determine which of the sixteen possible moods are valid in the first figure.

The valid moods thus discovered are *Barbara*, *Celarent*, *Darii* and *Ferio*, which Aristotle called *perfect* moods. Aristotle regarded other figures than the first, and also the valid moods under them, as *imperfect*, inasmuch as his *Dictum* is not directly applicable to them and their validity cannot directly be tested by it. But if, as Aristotle thinks, the *Dictum* be regarded as the basis of all syllogistic reasoning, then the valid moods in the imperfect figures must also be shown to be conformable to it. In Aristotle's time this was done by 'reducing' or bringing back an imperfect mood to a perfect mood. In other words, Aristotle regarded a mood in the second, third or fourth figure as valid only if it could be reduced or brought back to a valid mood in the first figure. Thus, by the process of **Reduction** the imperfect moods are ultimately referred to, and tested by, the *Dictum*, and the *Dictum* itself is established as the basis of all Syllogisms—directly of Syllogisms in the first figure, and indirectly of Syllogisms in other figures.

be conformable to it by the process of Reduction.

There is a dispute among Logicians as to the necessity and importance of Reduction. From the Aristotelian standpoint as explained above, Reduction is absolutely necessary and forms an important part of the syllogistic doctrine; because, in Aristotle's time, there was no other means of determining the validity of a mood in an imperfect figure than by reducing it to an equivalent mood in the first figure. But in modern times, when the validity of imperfect moods can be tested by General Rules or Special Rules or Diagrams, Reduction is not, indeed, absolutely necessary. But it cannot be denied that it has some important uses. Thus unity is the goal of the scientific study of a subject, and Reduction shows the essential unity underlying all Syllogisms, inasmuch

***Use and Importance of Reduction.* In Aristotle's time Reduction was absolutely necessary, for there was no other means of testing the validity of an imperfect mood.**

Though not absolutely necessary in modern times, yet it has some important uses. It shows the essential unity underlying all syllogisms and is a good logical exercise

as Reduction enables us to transform the moods of the imperfect figures into those of the perfect figure, which undoubtedly represents the most natural and consistent mode of syllogistic reasoning. If there were no intrinsic unity among the different moods of the different figures, they could not be reduced to one another. It further reveals that syllogistic arguments in all the four figures are directly or indirectly based upon *one* Fundamental Principle, *viz.*, the *Dictum de omni et nullo*. Thus we have *one* Fundamental Principle of syllogistic reasoning; *one* perfect or natural mode of reasoning (*i.e.*, the first figure); essential *unity* of the different figures, in spite of specific differences; and *unity* among the valid moods, transformable into one another. Besides, Reduction supplies a good intellectual exercise for a student of Logic. For these reasons, we hold that Reduction forms a necessary part of the doctrine of Syllogism.

Meaning of the term 'Reduction'

In a wide sense, the term 'Reduction' means the transformation of a mood in any figure into a mood in any other figure In a narrow sense, it means the transformation of a mood in the second, third, or fourth figure into one in the first figure, in order to test the validity of the imperfect mood by Aristotle's *Dictum*. The term is generally understood in the narrow sense.

Kinds of Reduction.

Reduction is of two kinds, *viz.*, (I) Direct or Ostensive Reduction, and (II) Indirect Reduction or *Reductio ad absurdum* or *Reductio per impossible*.

(I) Direct Reduction.

(I) Direct Reduction is the process of transforming an *imperfect* mood into an equivalent *perfect* mood. In other words, it consists in changing the premises of a given mood in the second, third or fourth figure by the help of one or more of the

processes of conversion, obversion and transposition, so as to form with them a mood in the first figure, having a conclusion which is the same as the given one, or from which the given conclusion can be deduced by some process of immediate inference. (II) **Indirect Reduction** consists in showing by the help of the first figure and the Laws of Opposition that the contradictory of the conclusion of a given mood in the second, third, or fourth figure is false, and, therefore, the given conclusion is true. We shall explain and illustrate these two methods of Reduction below.

(II) Indirect Reduction.

Direct Reduction. In order to understand the method of Direct Reduction, we are to refer once more to the mnemonic lines which we have already quoted (§ 13), and which run thus:—

Mnemonic lines.

> **Barbara, Celarent, Darii, Ferio**que prioris:
> **Cesare, Camestres, Festino, Baroco,** secundæ:
> Tertia, **Darapti, Disamis, Datisi, Felapton,**
> **Bocardo, Ferison,** habet: Quarta insuper addit
> **Bramantip, Camenes, Dimaris, Fesapo, Fresison.**

Significance of the letters of the mnemonic lines.

These mnemonic lines (brought into common use by Petrus Hispanus, afterwards Pope John XXI, nearly six centuries ago) are an artificial aid to memory and serve two purposes. In the first place, they help us to remember the *nineteen* valid moods together with the different kinds of propositions of which they are composed. In the next place, they indicate how an *imperfect* mood is to be reduced to an equivalent *perfect* mood.

Each of the names stands for a valid mood. Thus there are *four* valid moods in the first figure ('Prioris'), *four* in the second ('Secundæ'), *six* in the third ('Tertia'), and *five* in the fourth ('Quarta'). The

three vowels in each name indicate the three propositions of which a valid mood is composed. Thus, the name '*Cesare*' indicates that it is a mood in the second figure ('Secundæ') and it has **E** for its major premise, **A** for its minor, and **E** for its conclusion. The initial capital consonants—**B, C, D, F**—indicate that the imperfect moods beginning with them are to be reduced to the perfect moods having the same initial consonants. Thus *Bramantip* is to be reduced to *Barbara, Cesare* to *Celarent, Darapti* to *Darii, Fesapo* to *Ferio,* and so forth. The small consonant '**s**' indicates that the proposition represented by the vowel preceding it is to be converted *simply*. Thus '**s**' in *Festino* means that the major premise **E** (which precedes '**s**') is to be converted *simply*. The small consonant '**p**' signifies that the proposition represented by the vowel preceding it is to be converted *per accidens*. Thus '**p**' in *Felapton* means that the minor premise **A** (which precedes '**p**') is to be converted *per accidens*. When '**s**' or '**p**' occurs after the third vowel (conclusion) of a name, it implies that the conclusion of the new syllogism formed in the first figure (and not the conclusion of the given syllogism) is to be converted *simply* or *per accidens* as the case may be. Thus '**p**' in *Bramantip* implies that the **A** conclusion of the new syllogism in *Barbara,* to which *Bramantip* is to be reduced, should be converted *per accidens* to obtain the **I** conclusion of the given syllogism in *Bramantip.* The small letter '**m**' indicates that the premises are to be transposed, *i.e.,* the major of the given syllogism is to become the minor of the new one, and the minor of the given one is to become the major of the new one. The small consonant '**c**' in *Baroco*

and *Bocardo* implies that these moods cannot be reduced *directly* but are to be reduced *indirectly*. The position of the letter 'c' further implies that the premise indicated by the vowel immediately preceding it is to be omitted in forming the new syllogism in *Barbara*, whose premises are to be supplied by the other premise of the given syllogism and the contradictory of the given conclusion. This will be illustrated under Indirect Reduction to be presently explained. Other small consonants—'l,' 'n,' 't,' 'b,' 'd,' 'r'—are meaningless.

Direct Reduction of *Baroco* and *Bocardo*

The two moods, *Baroco* and *Bocardo*, having an **O** proposition as a premise, could not be reduced *directly* by the ancient Logicians. The reason was that an **O** proposition cannot be converted *simply* or *per accidens*, and obversion (by which such a proposition may be converted—*vide* Conversion by Negation, Ch. XVI, § 1) was not found out or recognized by them as a form of immediate inference. Merely for the sake of these two moods they had to invent the Indirect Method of Reduction. Now that obversion is reckoned as a form of immediate inference, we may reduce these two moods also by the Direct Method. It is to be remembered, however, that in order to reduce them directly we are to change the names *Baroco* and *Bocardo* respectively into **Faksoko** and **Doksamosk**. The initial capital consonants 'F' and 'D' signify, as before, that *Faksoko* and *Doksamosk* are to be reduced respectively to '*Ferio*' and '*Darii*.' The small consonant 'k' means obversion of the premise indicated by the vowel preceding it, and consequently 'ks' together mean obversion followed by simple conversion, and 'sk' together mean simple conversion followed by obversion. Let

Illustrations.

us now illustrate Direct Reduction by the following examples :—

(1) *Cesare* reduced to *Celarent*.

(1) *Cesare* (Fig II) reduced to *Celarent*.

No men are birds, →	No birds are men (converse of the original major),
All swans are birds, →	All swans are birds (original minor);
No swans are men	No swans are men

(2) *Camestres* reduced to *Celarent*

(2) *Camestres* (Fig. II) reduced to *Celarent*

All men are mortal,	No mortals are gods (converse of the original minor)
No gods are mortal;	All men are mortal (original major),
No gods are men	No men are gods No gods are men (converse of the new conclusion)

(3) *Disamis* reduced to *Darii*

(3) *Disamis* (Fig. III) reduced to *Darii*

Some men are honest,	All men are mortal (original minor),
All men are mortal;	Some honest beings are men (converse of the original major);
Some mortals are honest	Some honest beings are mortal · Some mortals are honest (converse of the new conclusion)

(4) *Felapton* reduced to *Ferio*

(4) *Felapton* (Fig. III) reduced to *Ferio*.

No men are beasts, →	No men are beasts,
All men are animals, →	Some animals are men (converse of the original minor);
Some animals are not beasts	Some animals are not beasts.

(5) *Bramantip* (Fig. IV) reduced to *Barbara*.

(5) *Bramantip* reduced to *Barbara*.

All kings are men, → All men are mortal (original minor),

All men are mortal; → All kings are men (original major);

∴ Some mortals are kings. ∴ All kings are mortal: ∴ Some mortals are kings (converse of the new conclusion).

(6) *Camenes* (Fig. IV) reduced to *Celarent*.

(6) *Camenes* reduced to *Celarent*

All men are mortal, → No mortals are angels,

No mortals are angels; → All men are mortal:

No angels are men. No men are angels: No angels are men

(7) *Fresison* (Fig. IV) reduced to *Ferio*.

(7) *Fresison* reduced to *Ferio*.

No beasts are men, → No men are beasts,

Some men are cooking animals; → Some cooking animals are men;

∴ Some cooking animals are not beasts. ∴ Some cooking animals are not beasts

(8) *Fesapo* (Fig. IV) reduced to *Ferio*.

(8) *Fesapo* reduced to *Ferio*.

No horses are men, → No men are horses,

All men are rational, → Some rational beings are men;

∴ Some rational beings are not horses. Some rational beings are not horses.

(9) *Baroco* (Fig. II) named *Faksoko* reduced to *Ferio*.

(9) *Baroco* named *Faksoko* reduced to *Ferio*

All horses are quadrupeds, → No not-quadrupeds are horses (converted obverse or contrapositive of the original major),

Some animals are not quadrupeds; Some animals are not-quadrupeds (obverse of the original minor);

Some animals are not horses. ∴ Some animals are not horses.

(10) *Bocardo* named *Doksamosk* reduced to *Darii.*

(10) *Bocardo* (Fig. III) named *Doksamosk* reduced to *Darii.*

Some men are not wise,	All men are mortal (original minor),
All men are mortal;	Some not-wise beings are men (converted obverse of the original major);
Some mortals are not wise	Some not-wise beings are mortal
	Some mortals are not-wise (converse of the new conclusion):
	Some mortals are not wise (obverse of the converse)

The older Logicians invented Indirect Reduction for testing the validity of *Baroco* and *Bocardo*. But the other imperfect moods may also be reduced by the same method.

(II) **Indirect Reduction.** Although Indirect Reduction was resorted to by the older Logicians in order to test the validity of the imperfect moods *Baroco* and *Bocardo*, yet other imperfect moods also may be reduced by the same method. The Indirect Method of Reduction consists, as shown below, in proving the validity of an imperfect mood by showing, with the help of an argument in the first figure, that the supposition of the falsity of its conclusion involves an absurdity, inasmuch as such a supposition contradicts one of its premises, which are assumed to be true. Let us reduce indirectly some of the imperfect moods.

(1) Indirect Reduction of *Baroco.*

(1) *Baroco* (Fig. II) is indirectly reduced to *Barbara* thus:—Take a concrete example of *Baroco*,—

All horses are quadrupeds,
Some animals are not quadrupeds;
∴ Some animals are not horses.

The premises in deductive reasoning are assumed to be true. So we may question the validity of the conclusion only. If the conclusion 'Some animals are not horses' be not true, suppose the conclusion to be false; then its contradictory 'All animals are horses' must be true by the Laws of Opposition. Taking this as the minor premise and

combining it with the original major premise 'All horses are quadrupeds' as the major, we obtain the following syllogism in *Barbara*:—

All horses are quadrupeds,
All animals are horses;
∴ All animals are quadrupeds.

Now, this new conclusion 'All animals are quadrupeds' is the contradictory of the original minor premise 'Some animals are not quadrupeds,' which is assumed to be true. But two contradictory propositions cannot both be true. Hence the new conclusion in Barbara must be false, since it contradicts the original minor premise, the truth of which is undisputed. The falsity of the new conclusion must be due either to the *form* (*i.e.*, the process) of reasoning or to its *matter* (*i.e.*, the premises). But the falsity cannot be due to the *form* or process of reasoning, for the new syllogism is in the perfect mood *Barbara* which is directly deducible from the self-evident *Dictum*. Therefore the falsity must be due to the falsity of the *matter* or premises of reasoning. But the major premise of the new syllogism, being the major premise of the original syllogism, is taken to be true. Hence the minor premise 'All animals are horses,' which has been supposed to be true for the sake of argument, must be false. Therefore, its contradictory 'Some animals are not horses,' which is the conclusion of the given syllogism in *Baroco*, must be true, and not false, as supposed before. Thus the validity of the imperfect mood *Baroco* is proved by the help of the perfect mood *Barbara*.

(2) Indirect Reduction of *Bocardo*.

(2) *Bocardo* (Fig. III) is reduced indirectly to *Barbara*. Take a concrete example of *Bocardo*,—

Some men are not wise,
All men are mortal;
∴ Some mortals are not wise.

The premises in deductive reasoning are assumed to be true. So we may question the truth of the conclusion only. If the conclusion 'Some mortals are not wise' be not true, suppose it to be false. Then its contradictory 'All mortals are wise' must be true by the Laws of Opposition. With this proposition as the major premise, and the original minor premise 'All men are mortal' as the minor, we have the following syllogism in *Barbara*:—

All mortals are wise,
All men are mortal;
∴ All men are wise.

The new conclusion 'All men are wise' is the contradictory of the original major premise 'Some men are not wise.' One of these propositions, therefore, must be false by the Laws of Opposition. But the original major premise is given as true, the new conclusion in *Barbara*, therefore, must be false. The falsity of the new conclusion must be due either to the process of reasoning or to the premises.

But there can be no error in the process of reasoning, for the argument is in the perfect mood *Barbara*, which strictly conforms to the *Dictum*. The falsity, therefore, must be due to the falsity of the premises. But the minor premise of the new syllogism in *Barbara*, being the minor premise of the original syllogism in *Bocardo*, is given as true. Therefore the major premise 'All mortals are wise', which is supposed to be true for the sake of argument, must be false. Therefore its contradictory 'Some mortals are not wise'—the conclusion of the original syllogism in *Bocardo*—is true, and not false, as supposed before. Thus the validity of the imperfect mood *Bocardo* is proved with the help of the perfect mood *Barbara*.

(3) Indirect Reduction of *Cesare*.

(3) *Cesare* (Fig. II) is reduced indirectly to *Ferio*. Let us take a symbolical example of *Cesare*,—

No P is M,
All S is M;
∴ No S is P.

The premises are given as true, the validity of the conclusion alone may be disputed. If the conclusion 'No S is P' be not true, suppose it to be false. Then its contradictory 'Some S is P' must be true by the Laws of Opposition. Combining this as the minor premise with the original major 'No P is M' as the major premise, we obtain the following syllogism in *Ferio* :—

No P is M,
Some S is P;
∴ Some S is not M.

The new conclusion 'Some S is not M' and the original minor premise 'All S is M' are contradictory propositions, both of which cannot be true; one of these propositions, therefore, must be false by the Laws of Opposition. But the original minor premise is given as true, the new conclusion in *Ferio*, therefore, must be false. The falsity of the new conclusion, however, is not due to the process of reasoning, for the argument is in the perfect mood *Ferio*. Therefore the falsity must be due to the falsity of the premises. But it cannot be due to the major premise 'No P is M,' which is assumed as true in the given syllogism in *Cesare*. Therefore, the minor premise 'Some S is P,' which has been supposed to be true for the sake of argument, must be false, and consequently, its contradictory 'No S is P,' the conclusion of the given syllogism in *Cesare*, is true. Thus the validity of the imperfect mood *Cesare* is established by means of the perfect mood *Ferio*.

(4) Indirect Reduction of *Ferison*.

(4) *Ferison* (Fig. III) is reduced indirectly to *Darii*. Let us take a symbolical example of *Ferison*,—

No M is P,
Some M is S;
Some S is not P.

The premises in deductive reasoning are assumed as true. So we may question the validity of the conclusion only. If the conclusion 'Some S is not P' be not true, suppose it to be false. Then its contradictory 'All S is P' must be true by the Laws of Opposition. With this proposition as the major premise, and the original minor premise 'Some M is S' as the minor, we get the following syllogism in *Darii*:-

All S is P,
Some M is S;
∴ Some M is P.

The new conclusion 'Some M is P' is contradictory to the original major premise 'No M is P.' One of these propositions, therefore, must be false by the Laws of Opposition. But the original major premise is given as true, the new conclusion in *Darii*, therefore, must be false. The falsity is not due to the process of reasoning, for the argument is in the perfect mood *Darii*, which strictly conforms to the *Dictum*. The error must, therefore, be in the premises. But the minor premise of the new syllogism in *Darii*, being the minor premise of the given syllogism in *Ferison*, is assumed to be true. Therefore, the major premise 'All S is P,' which is supposed to be true for the sake of argument, must be false. Therefore its contradictory 'Some S is not P' the conclusion of the original syllogism in *Ferison* is true, and not false. Thus the validity of the imperfect mood *Ferison* is proved with the help of the perfect mood *Darii*.

(5) *Bramantip* (Fig. IV) is reduced indirectly to *Celarent*. Let us take a concrete example of *Bramantip*,— (5) Indirect Reduction of *Bramantip*.

All kings are men,
All men are mortal;
∴ Some mortals are kings.

The premises in deductive reasoning are assumed to be true. So we may question the validity of the conclusion only. If the conclusion 'Some mortals are kings' be not true, suppose the conclusion to be false. Then its contradictory 'No mortals are kings' must be true by the Laws of Opposition. Taking this as the major premise and combining it with the minor premise of the given syllogism as the minor, we obtain the following syllogism in *Celarent*:-

No mortals are kings,
All men are mortal,
∴ No men are kings;
∴ No kings are men (converse).

The converse of the new conclusion is contrary to the original major, which is given as true. But two contrary propositions cannot both be true. Hence the converse and so the new conclusion in *Celarent* must be false. The falsity of the new conclusion cannot be due to the process of

reasoning, for the argument is in the perfect mood *Celarent*. Therefore, the falsity must be due to the falsity of the premises. But the minor premise of the new syllogism in *Celarent*, being the minor premise of the given syllogism in *Bramantip*, is given as true. Hence the major premise 'No mortals are kings,' which has been supposed to be true for the sake of argument, must be false. Therefore, its contradictory 'Some mortals are kings,' which is the conclusion of the given syllogism in *Bramantip*, must be true. Thus the validity of the imperfect mood *Bramantip* is established by means of the perfect mood *Celarent*.

(6) Indirect Reduction of *Camenes*.

(6) *Camenes* (Fig. IV) is reduced indirectly to *Darii*. Let us take a symbolical example of *Camenes*,—

All P is M,
No M is S;
∴ No S is P.

If the conclusion 'No S is P' be not true, suppose it to be false. Then its contradictory 'Some S is P' must be true. Combining this as the minor premise with the original major premise 'All P is M' as the major, we have the following syllogism in *Darii*:—

All P is M,
Some S is P;
∴ Some S is M:
∴ Some M is S (converse).

The converse of the new conclusion is the contradictory of the original minor 'No M is S,' which is given as true. But two contradictory propositions cannot both be true. Hence the converse 'Some M is S,' and so the new conclusion in *Darii*, must be false. The falsity of the new conclusion is not due to the process of reasoning, which is in the perfect mood *Darii*. It must, therefore, be due to the falsity of the premises. But the major premise of the new syllogism in *Darii* is the same as the major premise of the syllogism in *Camenes*, and is therefore assumed to be true. Hence the minor premise 'Some S is P,' which is supposed to be true for the sake of argument, must be false. Therefore, its contradictory 'No S is P'—the conclusion of the given syllogism in Camenes—is true. Thus the validity of the imperfect mood *Camenes* is proved with the help of the perfect mood *Darii*

Syllogistic inferences may be tested by means of diagrams.

§ 18. Diagrammatic Test of Syllogism. We have already seen (Ch. XIII, § 6) how different kinds of propositions (*viz.*, A, I, E and O) may be represented by means of diagrams. It has also been shown how different kinds of immediate inference may be tested by diagrams. We are now to explain how syllogistic inferences also may be tested by the scheme of diagrams (*viz.*, Euler's Circles) adopted in

this treatise. In order to test a syllogism by diagrams, we are to draw all the possible diagrams representing the two premises; and then to combine with each of the diagrams representing the major premise, every diagram representing the minor premise, and lastly to see if a common conclusion follows from all the possible combinations of diagrams. If a common conclusion follows, the syllogism is valid; if not, the syllogism is invalid. This method is based on the following two axioms :—

The method is based on two axioms :

(1) Two circles coinciding with a third by any the same part coincide with each other by that part.

(1) Axiom of affirmative syllogisms.

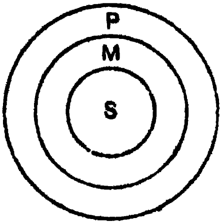
(2) Two circles of which one coincides and the other does not with a third by any the same part do not coincide with each other by that part.

(2) Axiom of negative syllogisms.

The first axiom is applicable when the premises are affirmative, and consequently, the conclusion affirmative. It may be illustrated by the following diagram :--

The first axiom illustrated.

All M is P,
All S is M;
All S is P.



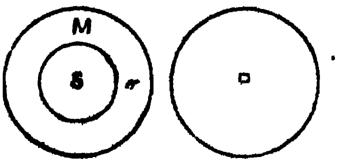
or
All S is M,
All M is P;
Some P is S

Here two circles S and P coincide with M, a third circle, by any the same part, viz., the whole of S or a part of P; therefore they coincide with each other by that part, *i.e.*, All S is P or Some P is S.

The second axiom is applicable when one of the premises is negative, and consequently the conclusion negative. It may be illustrated by the following diagram :-

The second axiom illustrated.

No M is P,
All S is M;
∴ No S is P



All S is M,
No P is M;
∴ No P is S

Here, of the two circles S and P, S coincides with a third circle M by a part, *viz.*, the whole of S, and the other P does not coincide with M by the same part, *viz.*, the whole of S; therefore they do not coincide with each other by that part, *i.e.*, No S is P or No P is S.

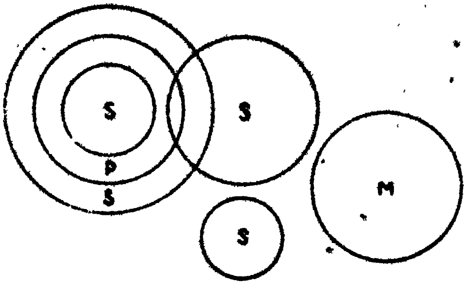
It may be observed that when neither axiom is applicable to a syllogism, there is no valid conclusion. Thus the

When neither

axiom is applicable to a syllogism, there is no valid conclusion.

fact that no conclusion follows from two negative propositions may be illustrated by the following diagram :—

No M is P,
No S is M ;
No conclusion.



Here, no conclusion can be drawn, because neither of the two axioms is applicable to this case. (In this connection, see the diagrammatic proof of General Rule V, § 8.)

Barbara tested by means of diagrams.

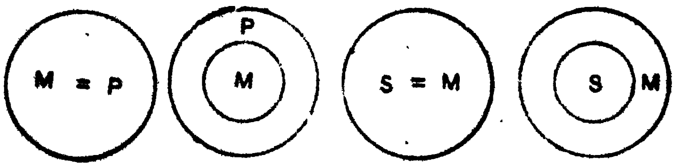
We shall further illustrate how the above method may be applied to test the validity of a mood. Let us take the valid mood *Barbara* in the first figure and see how it can be tested by diagrams :—

Barbara

All M is P,
All S is M ;
∴ All S is P.

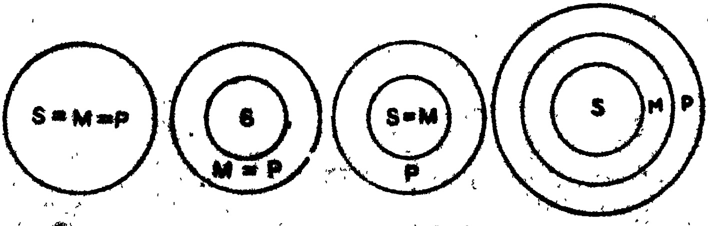
The major premise is represented by diagrams (1) and (2), and the minor by diagrams (*a*) and (*b*).

(1) (2) (*a*) (*b*)



Thus we can have the following four possible combinations :—

(1) and (*a*) (1) and (*b*) (2) and (*a*) (2) and (*b*).



The common conclusion which follows from all of these cases is 'All S is P.' Thus *Barbara* is tested by diagrams.

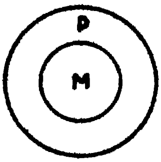
Next, take the mood *Darii*, which, owing to the presence of an I proposition (represented by the largest number of circles), requires a more complex process for being tested by diagrams.

Darii tested by means of diagrams.

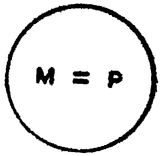
Darii

All M is P, \
Some S is M; \
Some S is P. } The major premise is represented by (1) and (2), and the minor by (*a*), (*b*), (*c*) and (*d*).

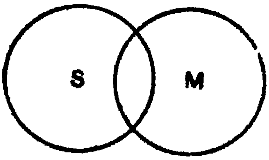
(1)



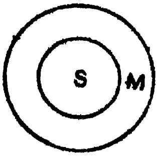
(2)



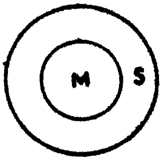
(*a*)



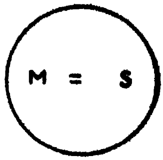
(*b*)



(*c*)

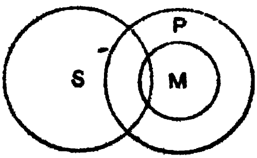


(*d*)

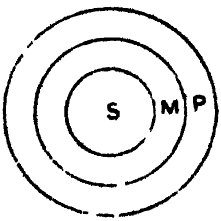


Thus we can have the following eight possible combinations. —

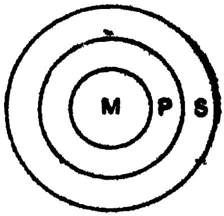
(1) and (*a*)



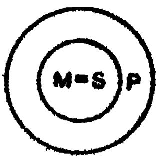
(1) and (*b*)

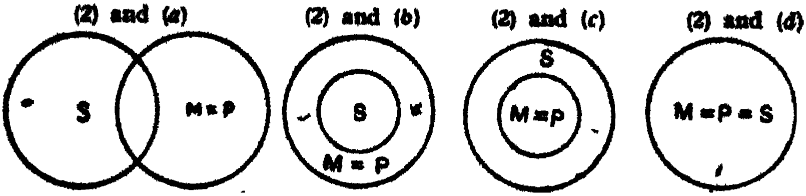


(1) and (*c*)

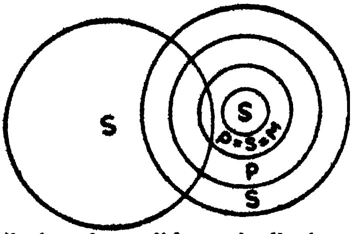


(1) and (*d*)





The common conclusion which follows from all of these cases is 'Some S is P.' Thus *Darii* is tested by diagrams. The above eight diagrams, again, may be combined in the big diagram drawn below :—



Similarly, the validity of all the other moods may be tested by means of diagrams.

The general and special rules of Pure Categorical Syllogisms are applicable to Pure Hypothetical and Pure Disjunctive Syllogisms. The nature of a hypothetical proposition analysed.

§ 19. **Pure Conditional Syllogisms.** In stating the different kinds of Syllogism (§ 3), we have noted that the same rules apply to all kinds of Pure Syllogisms. In other words, the general and special Rules of Pure Categorical Syllogism are applicable to Pure Hypothetical and Pure Disjunctive Syllogisms as well. In applying the rules to **Pure Hypothetical Syllogisms,** we are to remember the following points regarding the nature of hypothetical proposition: (1) that the antecedent and the consequent of a hypothetical proposition correspond respectively to the subject and the predicate in the categorical form ; (2) that the consequent of the conclusion is the major term, the antecedent of the conclusion is the minor term and the middle term occurs in the premises only ; (3) that its quantity is the quantity of its antecedent, expressed or implied ; (4) that its quality is the quality of its consequent ; (5) that the antecedent

is to be distributed in **A** and **E** propositions, and the consequent is to be distributed in **E** and **O** propositions. As we can thus determine the position of the middle term in relation to the major and minor terms, there will be no difficulty in ascertaining the figure of a Pure Hypothetical Syllogism ; and as from the above hints, we can easily ascertain the quality and quantity of a hypothetical proposition, there will be no difficulty in determining its mood. Moreover, the validity of Hypothetical Syllogisms may be tested by reducing them to the corresponding Categorical forms. Let us now illustrate how a Pure Hypothetical Syllogism may be tested by reducing it to the Categorical form :—

The validity of hypothetical syllogisms may be tested by reducing them to the corresponding forms.

Illustrations.

(1) In all cases if A is B, C is D,
In all cases if E is F, A is B ;
∴ In all cases if E is F, C is D

Reduced to the corresponding categorical form :—

All cases of A being B are cases of C being D,
All cases of E being F are cases of A being B ;
∴ All cases of E being F are cases of C being D.

This is *Barbara,* and hence it is valid.

(2) In no cases if C is D, A is B,
In all cases if E is F, A is B;
∴ In no cases if E is F, C is D.

Reduced to the corresponding categorical form :—

No cases of C being D are cases of A being B,
All cases of E being F are cases of A being B ;
∴ No cases of E being F are cases of C being D.

This is *Cesare,* and hence it is valid.

(3) In all cases if A is B, C is D,
In all cases if E is F, C is D;
∴ In all cases if E is F, A is B.

Reduced to the corresponding categorical form :—

All cases of A being B are cases of C being D,
All cases of E being F are cases of C being D ;
∴ All cases of E being F are cases of A being B.

This involves the fallacy of *Undistributed Middle*, and hence it is invalid.

Pure Disjunctive Syllogisms are very rare, though they are theoretically possible. In such a Syllogism, (1) there can be no distinction of figures; (2) only affirmative moods are possible; and (3) one of the alternatives in the minor premise must negative one of those in the major.

Of Pure Disjunctive Syllogisms, it may be said that such syllogisms are very rare, though they are theoretically possible, inasmuch as Pure Hypothetical Syllogisms are possible, and their constituent propositions may be reduced to corresponding disjunctive forms (*vide* Ch. XVI, § 7). But, as Welton points out (*Manual of Logic*, I, p. 350), they are possible in a very limited number of cases. (1) As the alternative members of a disjunctive proposition may be arranged in any order, there can be no distinction of figures in cases of Pure Disjunctive Syllogisms. (2) As disjunctive propositions are always affirmative, only affirmative moods are possible here. (3) Further, "we only secure a middle term when one of the alternatives in the minor premise negatives one of those in the major premise. From

S is either P or Q
S is either P or R

no conclusion can be drawn, except that *S is either P or Q or R* which simply sums up the premises. But from

S is either P or Q
S is either not-P or R

we can draw the conclusion *S is either Q or R*. This will, perhaps, be more clearly seen if each premise is expressed as a hypothetical proposition. We can write the premises in the form.

If S is not-P it is Q
If S is not-R it is not-P

whence it follows that *If S is not-R it is Q*, which expresses the disjunctive *S is either Q or R*." (*Vide* Chap. XVI, § 7.)

Besides Syllogism, there are some forms of the Non-Syllogistic Mediate Inference.

§ 20. Non-Syllogistic Mediate Inferences. This and the remaining Chapter of Book II have been devoted to the exposition of the nature of Syllogism, which has generally been accepted as the type of Mediate Deductive Inference. Logicians have, however, mentioned some other forms of mediate deductive inference, which are not truly syllogistic, but which nevertheless yield valid conclusions and are often employed in science or common life. Before passing on to the consideration of Mixed Syllogisms, let us

consider in this section certain varieties of Non-Syllogistic Mediate Inference :—

Reasonings of Quantity (Equality and Inequality) or *Mathematical Reasonings* may be illustrated as follows :— **Reasonings of quantity.**

(*a*) A is equal to B; B is equal to C : ∴ A is equal to C.	(*b*) A is greater than B; B is greater than C : ∴ A is still greater than C.
(*c*) A is less than B; B is less than C : ∴ A is less than C.	(*d*) A is a part of B; B is a part of C : ∴ A is a part of C.
(*e*) A is equal to B; C is equal to D : ∴ A+C=B+D. or, A−C=B−D. or, A×C=B×D. or, A÷C=B÷D.	(*f*) X coincides with Y; Y is equal to Z : ∴ X is equal to Z.

The above arguments are all valid, but they are non-syllogistic, inasmuch as they contain more than three terms. Thus in (*a*) there are four terms, *viz.*, A, equal to B, B, and equal to C; in (*b*) also there are four terms, *viz.*, A, greater than B, B, and greater than C; and so forth. They may, however, be reduced to the syllogistic form by taking the appropriate mathematical axioms as major premises. Thus the argument (*a*) given above may be syllogistically expressed thus : Things which are equal to the same thing are equal to one another; the two things A and C are equal to the same thing B; therefore, the two things A and C are equal to one another. Similarly, the argument (*b*), which is the argument *á fortiori*, may be expressed thus : A thing which is greater than the second, which is greater than the third, is greater than the third; A is a thing which is greater than the second B, which is greater than the third C : therefore, A is greater than the third C.

In answer to the contention whether the syllogism is the only form of mediate inference, it has rightly been held "that the syllogism deals only with propositions which express the relation of subject and attribute, and that inferences from other relations, though they may be perfectly valid, not only are not made syllogistically but, cannot be satisfactorily expressed in that form. Such, for example, is the *argumentum á fortiori—A is greater than B, B is greater than C; therefore, A is greater than C*" (Welton, I, p. 409). It is evdent that besides the subject-predicate relation which may be adequately expressed by the logical copula 'is,' there is an indefinite number of other relations (*e.g.*, those of equality, difference, causation, succession, simultaneity, magnitude, brotherhood, etc.), which cannot be so expressed by the logical copula. An attempt to express them syllogistically by means of the logical copula would involve us, as shown **Are there other Mediate Inferences besides Syllogisms?**

above, in the *fallacy of four terms*. In such cases, as C. Read remarks, we ought to read:

B—is before—C; (and not B is—before C);
A—is before—B:
∴ A—is before—C.

The Logic of Relatives.

From a consideration of these various kinds of relations, there has arisen a new department of study called **'the Logic of Relatives'**, which treats of relations in general, and not merely the subject-predicate relation expressed by the Logical copula 'is'. But, as Dr. Venn says, "the attempt to construct a Logic of Relatives seems . altogether hopeless owing to the extreme vagueness and generality of this conception of a Relation" (*Symbolic Logic*, p. 403) Of the various attempts to classify the various relations, that of Bradley has been widely accepted. According to him, there are altogether five relations:—

Mr. Bradley's classification of relations.

(1) *Synthesis of Subject and Attribute*. This forms the contents of the syllogistic inferences.

(2) *Synthesis of Identity*. Where one term has one and the same point in common with two or more terms, there these others have the same point in common......as 'If A is the brother of B, and B of C, and C is the sister of D, then A is the brother of D.'

(3) *Synthesis of Degree*. When one term does, by virtue of one and the same point in it, stand in a relation of degree with two or more other terms, then these others are also related in degree . as 'A is hotter than B and B than C, therefore A than C.'

(4 & 5) *Synthesis of Time and Space*. When one and the same term stands to two or more other terms in any relation of time or space, there we must have a relation of time or space between these others. *Examples*: 'A is north of B and B west of C, therefore C south-east of A'; 'A is a day before B, B contemporary with C, therefore C a day after A' (*Logic*, pp. 243-4). It is apparent that the validity of these non-syllogistic arguments depends on the material knowledge of the relations expressed in them and not merely on their form. (On this point read Welton, I, p 411, and Coffey, I, p. 392.)

Traductions or 'Syllogisms of Singulars.'

Of the several other kinds of reasonings which have been regarded as non-syllogistic, one that deserves mention is what is known as **Traductions** or **'Syllogisms of Singulars'**. A Traduction has been described as a kind of reasoning in which all the propositions are *singular*, and simply assert the identity of singular terms. As, for example, 'Socrates was the master of Plato, Socrates fought at Delium; therefore, the master of Plato fought at Delium.' Bain holds that such a piece of reasoning is not syllogistic, and he argues as follows: "The proposition, 'Socrates was the master of Plato, and fought at Delium,' compounded out of the two premises, is obviously nothing more than a grammatical abbreviation. No one can say that there is

here any change of meaning, or anything beyond a verbal modification of the original form. The next step is, 'the master of Plato fought at Delium,' which is the previous statement cut down by the omission of 'Socrates'.........Now, we never consider that we have made a real inference, a step in advance, when we repeat *less* than we are entitled to say, or drop from a complex statement some portion not desired at the moment...........In no way, therefore, can a syllogism with two singular premises be viewed as a genuine syllogistic or deductive inference." (*Deduction*, p. 159). But, as Keynes points out, Bain's objection is not merely applicable to "Syllogisms of Singulars," it is an objection to the syllogism itself. "In the following syllogisms the premises may be similarly compounded together :

All men are mortal, } All men are mortal and
All men are rational; } rational;
∴ Some rational beings are mortal.
All men are mortal, } All men including kings are
All kings are men; } mortal;
∴ All kings are mortal.

Do not Bain's criticisms apply to these syllogisms as much as to the syllogisms with singular premises? The method of treatment adopted is indeed particularly applicable to syllogisms in which the middle term is subject in both premises. But we may always combine the two premises of a syllogism in a single statement, and it is always true that the conclusion of a syllogism contains a part of, and only a part of, the information contained in the two premises taken together; hence we may always get Bain's result. In other words, in the conclusion of every syllogism 'we repeat less than we are entitled to say,' or, if we care to put it so, 'drop from a complex statement some portion not desired at the moment'" (*Formal Logic*, 4th Ed., p. 299).

§ 21. Application of General and Special Rules of Syllogism: Syllogistic problems worked out and arguments tested.

1. *Which of the moods are valid in all the figures? Which is the faultiest mood?* The moods **EIO** and **EAO** alone are valid in all the figures. In the case of **EIO**, in any of the four figures, the major premise **E** gives the chance of distributing the middle term once at least and also of distributing the major term; the minor term in the minor premise **I** has not been distributed and so also it remains undistributed in the conclusion, being the subject of an **O** proposition. The **O** conclusion distributes its predicate, the major term, which has also the chance of being distributed in the major premise. Thus the fallacies of *undistributed middle* and *illicit process* are obviated. Hence we see that the mood **EIO** is valid in all the figures. Similarly, it may be shown that the mood

EAO is also valid in all the figures. But it is useless in Figs. I and II, being subaltern to EAE (*Celarent* and *Cesare*). The only mood, valid in all the figures and not useless in any, is, therefore, EIO.

The mood IIE is the faultiest mood, as it violates the greatest number of syllogistic rules. It involves the fallacies of (*a*) *undistributed middle*, (*b*) *illicit process* of both major and minor terms, (*c*) *two particular premises*, (*d*) *a negative conclusion without a negative premise*.

2. *Prove that if the conclusion be universal, the middle term can be distributed but once in the premises.*

The conclusion is either **A** or **E**. (1) If the conclusion be **A**, then both the premises must be **A** (Rules VII and IX—*Note*). As **A** propositions distribute their subjects only, only two terms are distributed in the premises. The conclusion being **A**, its subject, the minor term is distributed in it, therefore, it must be distributed in the minor premise Hence, to avoid the fallacy of *undistributed middle*, the other distributed term must be the middle term. Thus the middle term is distributed but once in the premises. (2) Again, if the conclusion be **E**, then one premise must be **E**, and the other **A** (Rules VI, V and IX—*Note*). Thus the premises distribute only three terms, of which the two must be the major and minor terms which are distributed in the conclusion. Therefore the remaining distributed term must be the middle term, which is thus distributed but once in the premises.

3. *Show that if the minor premise be negative, the middle term is but once distributed.*

If the minor premise be negative, the major must be universal (Rule X) and affirmative (Rule V), and the conclusion must be negative (Rule VI). The negative conclusion distributes its predicate, the major term, which must previously be distributed in the major premise. But, the major premise, being an **A** proposition, distributes its subject only, and this must be the major term. Hence the middle term can be distributed only in the minor premise, *i.e.*, only once.

4. *If the major term of a syllogism be the predicate of the major premise, what do you know about the minor premise?*

The major term is the predicate of the major premise. It may be either distributed or undistributed. If it is distributed, then the major premise is negative and consequently the minor premise must be affirmative (Rule V). If the major term is undistributed in the premise, it must also be undistributed in the conclusion, *i.e.*, the conclusion must be affirmative. An affirmative conclusion requires both the premises to be affirmative (Rule VIII). Hence, here also, the minor premise must be affirmative.

5. *Show that when the middle term is distributed twice, the conclusion is particular.*

If the conclusion be not particular, suppose it to be universal, *i.e.*, either **A** or **E**. If the conclusion be **A**, then both the premises must be **A** (Rules VII and IX—*Note*). Now, as an **A** conclusion distributes its subject, the minor term, it must also be distributed in the minor premise. Therefore, the middle term can be distributed only once and not twice as is required by our data. Hence the conclusion cannot be **A**. Again, if the conclusion be **E**, the premises must be **E** and **A** in any order (Rules VI, V and IX), and thus altogether three terms are distributed in the premises. Now an **E** conclusion distributes both the major and minor terms, which must, therefore, be distributed in the respective premises. Thus the middle term has the chance of being distributed but once, and not twice, as is required by our data. Hence the conclusion cannot be **E**. In other words, if the middle term is distributed twice, the conclusion cannot be universal, *i.e.*, it must be particular.

6. *Prove that when the minor term is predicate in its premise, the conclusion cannot be* **A**.

Suppose, if possible, the conclusion to be **A**. In that case both the premises must be **A**. An **A** conclusion distributes its subject, the minor term, which must, therefore, be distributed in the minor premise. But this is not possible if the minor term is the predicate, because an **A** proposition does not distribute its predicate. Hence when the minor term is predicate, the conclusion cannot be **A**.

7. *Prove that whenever the minor term of a syllogism is distributed, the major premise must be universal.*

As the minor term is distributed, the minor premise must be either **A**, **E**, or **O**. If it is **E** or **O**, the major must be universal (Rule X). If it is **A**, the middle term not being distributed in the minor premise must be distributed in the major premise, *i.e.*, the major premise must be **O** or **A** or **E** (and not **I**, for **I** distributes no term). If the major premise be **O**, the conclusion will be **O**, which will distribute the predicate, the major term. But the major term remains undistributed in the major premise, for the major premise **O** distributes only one term and this must be the middle term. Thus there will arise the fallacy of *illicit major*. The major premise must, therefore, be either **A** or **E**, *i.e.*, universal.

8. *Prove that when the minor premise is negative, the major must be universal.*

If the minor premise be negative, the major must be affirmative, and the conclusion negative. A negative conclusion distributes its predicate, the major term, which must, therefore, be distributed in the affirmative major premise. Hence, the major premise must be **A** and not **I**, for **I** distributes no term. In other words, the major premise must be universal.

9. Prove that O cannot be—
(1) a premise in the first figure,
(2) a premise in the fourth figure,
(3) the major premise anywhere but in the third figure,
(4) the minor premise anywhere but in the second figure.

(1) If one premise be O, the other premise must be A (Rules V and VIII). Now, if O be the major premise, A must be the minor premise, and the mood OA in the first figure will give rise to the fallacy of *undistributed middle*. Again, if O be the minor premise, A must be the major premise, and the mood AO in the first figure will give rise to the fallacy of *illicit major*. Hence O cannot be a premise in the first figure.

(2) See special rule (4) of the fourth figure (§ 12).

(3) If the major premise be O, the minor must be A, and the conclusion O. The O conclusion will distribute its predicate, the major term, which, to avoid the fallacy of *illicit major*, must be distributed in the major premise. In other words, the major term in the major premise will occupy the position of the predicate and the middle term that of the subject. As the middle term has not the chance of being distributed in the major premise, it must be distributed in the minor premise, *i.e.*, the middle term will occupy the position of the subject in the minor premise. Thus the middle term is the subject of both the premises, and this is the condition of the third figure alone. Hence O can be the major premise only in the third figure.

(4) If the minor premise be O, the major must be A, and the conclusion O. The O conclusion will distribute its predicate, the major term, which must, therefore, be previously distributed in the major premise. In other words, the major term will stand as subject of the major premise, and the middle term as its predicate. As the middle term has not the chance of being distributed in the major premise, it must be distributed in the minor premise. In other words, the middle term will stand as predicate of the minor premise. Thus the middle term is predicate of both the premises, and this is the condition of the second figure alone. Hence O can be the minor premise only in the second figure.

10. *Which of the Syllogistic figures is alone capable of proving an* A *proposition, and in what mood or moods alone?*

An A proposition can be proved only in the first figure and the mood must be AAA (*Barbara*). If the conclusion is A, both the premises must be A (Rules VII and IX—*Note*). Since the minor term is distributed in the conclusion, it must be distributed in the minor premise (Rule IV), and so it must be the subject

of the minor premise, for it is only then that it can be distributed in the minor premise. The middle term, not being distributed in the minor premise, must be distributed in the major premise (Rule III). In other words, the middle term must be the subject of the major premise, for an **A** proposition distributes its subject only. Thus the middle term is subject of the major premise and predicate of the minor. This is the condition of the first figure alone. Hence an **A** proposition can be proved only in the first figure. The conclusion being **A**, both the premises must be **A**. The mood is, therefore, **AAA**, technically called *Barbara*. As already shown, the mood **AAA** gives rise to the fallacy of *undistributed middle* in Fig. II, and of the *illicit process of the minor* in Figs. III and IV.

11. *In the first figure, the conclusion must have the quality of the major premise, and may have the quantity of the minor premise.*

In the first figure the major term is predicate of the major premise. Now, if the major premise be negative, the conclusion must evidently be negative. If, again, the major premise be affirmative, the major term is not distributed in it, and so it cannot be distributed in the conclusion, where it stands as predicate. In other words, the conclusion must be affirmative. Thus in the first figure, the conclusion must have the quality of the major premise.

If the minor premise be particular, then the minor term, being the subject of the minor premise, is not distributed in it, and so it cannot be distributed in the conclusion, where it stands as subject. In other words, the conclusion must be particular. Again, if the minor premise be universal, then the minor term is distributed in it, and consequently it may rightly be distributed in the conclusion. In other words, the conclusion may be universal. Thus in the first figure the conclusion may have the quantity of the minor premise.

12. *Given that the major premise of a valid syllogism is affirmative and that the major term is distributed both in the premise and conclusion, while the minor term is undistributed in both. Determine the syllogism.*

As the subject of the conclusion (the minor term) is undistributed, and its predicate (the major term) is distributed, *the conclusion is* **O**. As the major premise is affirmative, the minor must be negative; and as the minor term is not distributed in the minor premise, the minor term must be subject of the minor premise. Thus *the minor premise is* **O**. The minor being **O**, *the major premise* must be **A**, and in it the major term, which is distributed, must stand as the subject. Thus the middle term is predicate in both the premises. Hence the syllogism is in *Baroco* in the second figure.

13. *Give either a particular major or a particular minor in the fourth figure, to find in each case the mood*

As **O** cannot be a premise in the fourth figure (special rule 4 of the fourth figure, § 12), the given particular premise must also be affirmative, *i.e.*, an **I** proposition. (*a*) If the major premise be **I**, the minor must be universal (Rule VIII); and also affirmative (Rule X), *i.e.*, **A**. The conclusion must, therefore, be **I** (Rules VII and IX) Thus the mood is **IAI**. (*b*) If the minor premise be **I**, the major must be universal (Rule VIII) and also negative, *i.e.*, **E**; for if it be affirmative, the middle term will not be distributed even once The conclusion must, therefore, be **O** (Rules VI and IX) Thus the *mood is* **EIO**.

14 *Given* **I** *as the major premise, to determine the mood and figure*

I as a major premise cannot be combined either with an **I** or with an **O** proposition (Rule VIII). Nor can it be combined with an **E** proposition (Rule X). **I** can, therefore, be combined only with **A**. But the combination **IA** will give no conclusion in Fig. I, for in Fig I the major premise must be universal (Special Rule 1 of Fig. I); nor in Fig II, for the middle term, being predicate of both the premises, will not be distributed even once (Rule III) In Figs III and IV, **IA** gives the valid conclusion **I** -the valid moods are *Disamis* (Fig. III) and *Dimaris* (Fig IV)

15 *Given* **O** *as the minor premise of a syllogism, determine the figure and mood*

If the minor premise be **O**, the major premise must be **A**, and the conclusion must be **O**. In the first and third figures, the mood **AOO** would involve the fallacy of *illicit major*, in the fourth figure, it would lead to the fallacy of the *undistributed middle* It is valid only in the second figure, the mood being technically called *Baroco*.

16. *If a valid syllogism contains* **O** *as premise, its middle term must occupy the same position in both premises*

If one premise be **O**, the other must be **A** and the conclusion must be **O**. The **O** conclusion will distribute the major term, which must, therefore, be distributed in the major premise Now, if the major premise be **A**, the major term will occupy the position of subject, and the middle term that of predicate, the middle term being undistributed in the major premise must be distributed in the minor premise, which is **O**. In other words, the middle term will occupy the position of predicate in the minor premise as well as in the major. Again, if the major premise be **O**, the major term will stand as predicate of the major premise and the middle term as subject of the major premise. As the middle term is undistributed in the major premise, it must be distributed in the minor premise which is an **A** proposition; so the middle term must stand as subject of the minor premise. Thus here

also, the middle term occupies the same position, viz., that of subject in both premises.

17. *Show that double distribution of the middle term is impossible in Figs. I and II.*

In Fig. I the middle term occupies the position of subject in the major premise and that of predicate in the minor. So, if the middle term be distributed in both the premises, the major premise must be universal, and the minor premise must be negative. But as already proved, a negative minor premise in Fig. I would involve the fallacy of *illicit major* (Fig. I—special rule 2). Again, in Fig. II the middle term is predicate in both the premises; so, if the middle term be twice distributed, both the premises must be negative, which is impossible (Rule V).

18. *Given the major term distributed in the major premise without being distributed in the conclusion, determine the mood and figure.*

The major term is undistributed in the conclusion, where it stands as the predicate. The predicate of the conclusion being undistributed, the conclusion must be affirmative, and consequently both the premises must be affirmative. As the major premise is affirmative, the major term, to be distributed in it, must occupy the position of subject. Thus we see that the major premise is universal and affirmative—the major term standing as the subject, and the minor premise is also affirmative. These conditions are possible only in Fig. IV, for in Figs. I and III, the middle term, and not the major, stands as the subject of the major premise, and in Fig. II, one of the premises must be negative. Now, in Fig. IV, if the major premise be universal affirmative, the minor premise also must be universal, for otherwise there will arise the fallacy of *undistributed middle*. The minor term is not distributed in the minor premise, being the predicate of an affirmative proposition; so it must also remain undistributed in the conclusion, where it stands as the subject. In other words, the conclusion is particular as well as affirmative. Thus we see that the mood is in the fourth figure, and it is **AAI** (*Bramantip*).

19. *If the middle term be twice distributed, what moods and figures are possible? How would you justify your answer?*

In Figs. I and II, the middle term cannot be twice distributed (*vide* problem 17). In Fig. III, the middle term occupies the position of subject in both the premises; so, if the middle term be distributed twice, both the premises must be universal. Thus the possible combinations will be **AA**, **AE**, **EE** and **EA**. Of these, **EE** yields no conclusion (General Rule V), and **AE** also leads to no conclusion in the third figure (§ 11). **AA** and **EA** yield **I** and **O** conclusions respectively, and the moods are *Darapti* and *Felapton* (§ 11). In Fig. IV, the middle term is predicate in the major premise and subject in

the minor. So, if the middle term be distributed in both the premises, the major premise must be negative, *i.e.*, E or O, and minor must be universal, *i.e.*, A or E. Hence the possible combinations will be EA, EE, OA, and OE; of these, EE and OE are invalid by General Rule V, and OA also gives no conclusion in the fourth figure (§ 12). EA yields O conclusion, and the mood is *Fesapo* (§ 12) Thus we see that double distribution of the middle term is possible only in Figs. III and IV, and in the moods *Darapti*, *Felapton* of Fig. III and *Fesapo* of Fig. IV.

§ 22. How to test Syllogistic Arguments. In testing an argument, we have first to change the constituent propositions into their logical forms If any of the premises or the conclusion be suppressed, it is to be supplied. Next, we have to ascertain the conclusion, which will enable us to know the major and minor terms (being respectively the predicate and subject of the conclusion) and thus to distinguish between the major and minor premises. Next, we are to arrange the premises and the conclusion in the form of a syllogism and to determine the figure and the mood of the argument Lastly, we have to see whether the argument violates any of the general or special rules of the Syllogism If it does, it is fallacious; if not, it is valid

Question · Test the following arguments :—

(*a*) The Moon goes round the Earth; the Earth round the Sun; and therefore, the Moon goes round the Sun.

(*b*) Socrates was wise, and wise men alone are happy; therefore, Socrates was happy.

(*c*) None but students are allowed concession, he is a student; therefore, he is allowed concession.

(*d*) James cannot be rich, for he is not fashionable and only the rich are fashionable.

(*e*) I am egoist! I know I am not one. Envy is an egoist; and on whatever ways I have strayed, you have never found me on the path of envy.

(*f*) Bees are useful little insects; they gather honey; therefore, the little insects that gather honey are useful creatures

(*g*) Skilful labour is highly paid; the work of the metaphysician is skilful labour: therefore, it is highly paid.

(*h*) He who says that you are a sentient being speaks truly; but he who says that you are a monkey says that you are a sentient being; therefore, he who says that you are a monkey speaks truly.

Answer :

(*a*) When reduced to logical form, the argument stands thus :—

The Earth is what goes round the Sun,
The Moon is what goes round the Earth;
∴ The Moon is what goes round the Sun.

The argument is fallacious, for there is no middle term: 'the Earth' and 'what goes round the Earth' are not the same either in form or in meaning. Thus the argument involves the *Fallacy of Four Terms*.

(*b*) When reduced to logical form, the argument stands thus :—

All happy men are wise,
Socrates was wise;
∴ Socrates was happy.

Here the middle term 'wise' is undistributed in both the premises, for affirmative propositions do not distribute their predicates. Thus the argument involves the *Fallacy of Undistributed Middle*.

(*c*) When reduced to logical form, the argument stands thus :—

All who are allowed concession are students,
He is a student;
∴ He is allowed concession.

The argument involves the *Fallacy of Undistributed Middle* as in (*b*) above.

(*d*) When reduced to logical form, the argument stands thus :—

All fashionable people are rich,
James is not fashionable;
∴ James is not rich.

Here the major term 'rich' has been distributed in the conclusion, being the predicate of a negative proposition, while it has not been distributed in the major premise, being the predicate of an affirmative proposition. Hence the argument involves the *Fallacy of Illicit Major*.

(*e*) The argument is simply this :—

Envious persons are egoists,
I am not an envious person;
∴ I am not egoist.

This argument involves the *Fallacy of Illicit Major*, as the major term 'egoist' has been distributed in the conclusion, without being previously distributed in the major premise, where it stands as predicate of an affirmative proposition.

(*f*) When reduced to logical form, the argument stands thus :—

Bees are useful creatures,
Bees are little insects that gather honey;
∴ The little insects that gather honey are useful.

Here the minor term 'little insects that gather honey' is distributed in the conclusion, being the subject of a universal proposition, while it is not distributed in the minor premise, where it is predicate of an affirmative proposition. Hence the argument involves the *Fallacy of Illicit Minor*.

(*g*) Skilful labour is highly paid,
The work of the metaphysician is skilful labour;
∴ The work of the metaphysician is highly paid.

Here the middle term 'skilful labour' is used ambiguously. In the major premise it means 'skilful manual labour', while in the minor premise it means 'skilful intellectual labour.' Hence the argument involves the *Fallacy of Ambiguous Middle or of Four Terms.*

(h) When reduced to logical form, the argument stands thus :—

> He who says that you are a sentient being is one who speaks truly,
> He who says that you are a monkey is one who says that you are a sentient being;
> He who says that you are a monkey is one who speaks truly

Here the middle term is used in two different senses. In the major premise it means 'he who *explicitly* says that you are a sentient being,' while in the minor premise it means 'one who simply *implies* or *supposes* that you are a sentient being' Moreover, 'sentient being' in the first premise means 'sentient *rational* being,' while in the second premise it means 'sentient *non-rational* being.' Hence the argument involves the *Fallacy of Ambiguous Middle or of Four Terms*

§ 23. Exercises.

1 Define a Syllogism and state its characteristics Distinguish between Syllogism, Immediate Inference and Induction Illustrate your answers

2. Explain the structure of the Syllogism Show the importance of the middle term in mediate inference

3 Clearly explain what you mean by the 'hypothetical character of the syllogism' Show that false premises of a Syllogism may lead to a true conclusion Can a false conclusion be derived from true premises ?

4. What is the *Dictum de omni et nullo?* What is its importance in syllogistic reasoning ? How does it affect the relative value of the Figures of the Syllogism ?

5 Discuss and compare the various renderings of Aristotle's *Dictum* What is Mill's objection to the *Dictum*? What does he propose to substitute ? What expression and interpretation do you think ought to be given to it ?

6 What different views have been held as to the *Dictum de omni et nullo?* With what qualifications may the *Dictum* be adopted as the basis of the Syllogism ?

7. 'Whatever is affirmed or denied of a class, is affirmed or denied of every member of that class.' Is this a verbal or a real proposition ? Give reasons for your answer.

8. What do you mean by a Figure? How many Figures are there? Indicate the peculiarities and uses of the different Figures.

9. Define Mood. How many possible Moods are there? Distinguish between *possible* and *legitimate* Moods.

10. Show the position of the terms in each of the four Figures. Discuss the propriety of accepting the Fourth Figure as an independent Figure.

11. In what different ways can the valid Moods be determined? What are Subaltern Moods and why are they so called?

12. Show that though the four kinds of propositions, *viz*., **A, E, I, O** may be combined into sixty-four different groups of three propositions each, only eleven of these groups can be valid syllogisms in any Figure.

13. State the General Rules of the Syllogism, and show the reason of each rule.

14. Give the two premises, determine the possible Moods and Figures. Strike your pen through the illegitimate Moods; and of the legitimate Moods, show which are valid in each Figure.

15. Show how the Syllogistic rules may be deduced from the *Dictum de omni et nullo*.

16. Is it absolutely true, two particular premises never admit of a conclusion? If not, state what sort of conclusion may be drawn, and under what circumstances, and give examples.

17. What precaution would you take in applying the rule '*From two negative premises nothing can be inferred*?'

18. State and prove the General Rules that relate to the distribution of terms.

19. Explain and illustrate the Fallacies of Four Terms, Undistributed Middle, Illicit Major and Illicit Minor.

20. Prove the following :–

(*a*) If either premise be negative or particular, the conclusion must also be the same. Is the converse of either of these rules true?

(*b*) If both the premises be particular or negative, no conclusion follows.

(*c*) Whenever the minor premise is negative, the major must be universal.

21. State and prove the Special Rules of the different Figures.

22. Explain—"Logicians have thought that each figure was best suited for certain special purposes." Which Figure is most convenient (*a*) for overthrowing an adversary's conclusion; (*b*) for establishing a negative conclusion; (*c*) for proving a universal conclusion?

23. Explain and Illustrate :—Perfect, Imperfect, Subaltern and Indirect Moods; Fundamental, Strengthened and Weakened Syllogisms.

24. What are the Subaltern Moods in the several Figures? In what Moods are they respectively included?

25. Wherefore is **OAO** invalid in Fig. I, **AEI** in Fig. III, and **AII** in Fig. IV? Which of the following Moods are valid, and in which Figure? State the grounds of exclusion for each you reject:—**AAO, AII, EAO, EII, IAI, IEO, OAO, OEI.**

26. What can be determined respecting a syllogism under each of the conditions:

(a) When only one term is distributed, and that only once;
(b) When only one term is distributed, and that twice;
(c) When two terms only are distributed, each only once;
(d) When two terms only are distributed, each twice.

27. What is the least, as well as the greatest, number of terms that can be distributed in the premises of a syllogism?

28. Two propositions are given having a common term. If they are **I** and **A**, show that they justify either no conclusion or two conclusions; but if **I** and **E**, they always lead to a single conclusion.

29. Show that **A** proposition can be proved only in the First Figure. Justify "**A** is the most difficult proposition to establish and the easiest to disprove."

30. Two premises differ in quantity and quality. If the one is major, the syllogism is valid in every Figure, but if the other, the syllogism is valid in none. Find the premises.

31. Show in what Figures and Moods the following premises give valid conclusions: (a) **EI**; (b) All planets are heavenly bodies, No planets are self-luminous.

32. Draw the conclusions, if any, which follow from the following combinations of premises:—(a) **AE**; (b) **IA**; (c) All birds are oviparous, All birds cannot fly; (d) None but animals are sentient beings, All planets are insentient beings; (e) No branch of science can be made absolutely perfect, Yet all branches of science are worthy of diligent culture; (f) None but gentlemen are members of the club, All members of the club are invited to compete.

33. What are Pure Conditional Syllogisms? How do you test them? Give concrete examples and reduce them to the Categorical forms.

34. What is Reduction? Explain the two methods of Reduction. Why is it thought necessary? Frame concrete examples of *Camestres*, *Camenes*, *Darapti*, *Disamis*, *Dimaris*, *Baroco*, *Bocardo* and *Bramantip*, and reduce them both *directly* and *indirectly*.

35. What Mood can alone be valid in all Figures? Show that **IEO** violates the special rules of all the Figures, and **AEO** is superfluous in any Figure.

36. What are the different Methods of testing Syllogisms? Explain and illustrate the Method of testing Syllogisms by Diagrams.

37. Mention several important forms of the Non-Syllogistic Mediate Inference. Is the following an instance of syllogistic reasoning or not?—A is equal to B, B is equal to C; therefore, A is equal to C. Give reasons for your answer. Is the 'Syllogism of Singulars' a Deductive inference?

38. Give concrete examples of **EAO, EAE, OAO** and **AEE** in the Figures in which they are valid, and reduce them to the First Figure.

39. Two premises and a conclusion are given. Show how to determine (*a*) the major, the middle and the minor terms; (*b*) the order of the premises; (*c*) the Figure; (*d*) the Mood; (*e*) the validity of the syllogism. Apply your method to the following example:—All stars twinkle, but planets do not twinkle; therefore, planets are not stars.

40. Prove the following by the general syllogistic rules:—

(1) If the minor premise be negative, the middle term is but once distributed.

(2) In the Fourth Figure neither of the premises can be particular negative, nor the conclusion universal affirmative.

(3) When the major term is predicate in its premise, the minor premise must be affirmative; also when the minor term is predicate in its premise, the conclusion cannot be universal affirmative.

(4) If the middle term be twice distributed in useful Moods, the syllogism has universal premises and particular conclusion.

(5) In a negative Mood, the major premise cannot be particular affirmative.

(6) In a syllogism with the minor premise universal negative, the conclusion (unless weakened) must also be the same.

(7) In the Fourth Figure, the conclusion may be either affirmative or negative; if negative, either universal or particular. In the same Figure, the conclusion can never be both affirmative and universal.

(8) The conclusion cannot be universal affirmative, when the minor term is predicate in the minor premise and the major term subject in the major.

(9) In the First Figure the conclusion must have the quality of the major and the quantity of the minor premise.

(10) The middle term cannot be distributed twice when a premise is particular.

(11) In the Second Figure the conclusion must be negative and have the quality of the minor premise; and in the Third Figure, the conclusion must be particular and have the quality of the major premise

(12) An O proposition can be the major premise only in the Third Figure, and the minor premise only in the Second.

(13) There must be at least one more term distributed in the premises than in the conclusion.

(14) The number of distributed terms in the premises cannot exceed those in the conclusion by more than two

(15) The number of undistributed terms in the premises cannot exceed those in the conclusion by more than one

(16) A negative minor premise necessitates a universal major, and a particular major premise precludes a negative minor

41 What should be the quantity of the conclusion when the middle term is twice distributed? In what Moods and Figures is such an argument possible?

42 What do you know about the minor premise when the middle term is subject in the major?

43 Given the minor term as predicate in the minor premise, and the major term subject in the major, show that the conclusion cannot be universal affirmative

44 Test the following arguments —

(1) Every hen comes out of an egg, every egg comes out of a hen therefore, every egg comes out of an egg.

(2) Only industrious students pass their examinations, John failed in the examination, therefore, he is not an industrious student

(3) None but the wise are good, and none but the good are happy, therefore, none but the wise are happy

(4) You are not what I am, but I am a man; therefore, you cannot be a man

(5) None but the honest are honoured; but James is not honoured therefore, he is not honest

(6) How can you say he is not a careful examiner, when he is severe in examining his papers, as careful examiners are known to be?

(7) Learned men sometimes become mad; but as he is not learned, there is no danger of his sanity.

(8) The bat is a bird, since the bat can fly as all birds do.

(9) All nations which are self-governed are prosperous; India is not self-governed; therefore, it is not prosperous.

(10) He must have the plague, for he has fever, and fever is one of the symptoms of plague.

(11) Every book is not useful; and all books are not interesting: therefore, some useful things are not interesting.

(12) India comprehends Bengal; Bengal does not comprehend Bombay; therefore, India does not comprehend Bombay.

(13) All men are not Europeans; but there are no Englishmen who are not Europeans; therefore, there are no Englishmen who are men.

(14) Cloven feet being found universally in horned animals, we may conclude that this fossil animal, since it appears to have cloven feet, was horned.

(15) I shall be admitted because I have passed in the first division, and only first division candidates will be admitted.

(16) Solon was really competent to rule, for we know he was wise, and it is the wise only who are fitted to rule.

(17) Every man has a right to inculcate his own opinions; therefore a magistrate is justified in using his power to propagate his own religious views.

(18) Most plants decompose carbonic acid, but fungi do not. Fungi, therefore, are not plants.

(19) No kind of spirituous liquor ought to be drunk in excess. But water is not a spirituous liquor. Therefore, water ought to be drunk in excess.

(20) He who is most hungry eats most; he who eats least is most hungry: therefore, he who eats least eats most

(21) Every country which possesses abundance of gold money is prosperous. India does not possess much gold money. It cannot, therefore, be a prosperous country.

(22) All material things gravitate, air gravitates; air is, therefore, material.

(23) All quadrupeds are animals, a bird is not a quadruped; therefore, a bird is not an animal.

(24) All poets are imaginative; some philosophers are poets: therefore, some philosophers are not imaginative.

(25) None but the Hindus worship Siva, all Bengalees are Hindus; therefore, all Bengalees worship Siva.

(26) What is related in the Talmud is unworthy of credit, miraculous stories are related in the Talmud; therefore, miraculous stories are unworthy of credit.

(27) Everyone desires happiness, virtue is happiness; therefore, everyone desires virtue.

(28) He is, to be sure, a highly educated man, but all highly educated men are not qualified to be inspectors.

(29) Some who are truly wise are not learned; but the virtuous alone are truly wise: the learned, therefore, are not always virtuous.

(30) None but Hindus are Brahmos; this man is not a Brahmo; therefore, he is not a Hindu.

(31) Every book is liable to error, every book is a human production; therefore, all human productions are liable to error.

(32) None but the good are really to be envied; all truly wise men are good: therefore, all truly wise men are to be envied.

(33) All criminal actions should be legally punished; prosecutions for theft are criminal actions: therefore, these should be punished by law.

(34) If I had read as much as my neighbours, I should have been as ignorant

(35) Rational beings are accountable for their actions; brutes not being rational are, therefore, exempt from responsibility.

(36) This man is a scoundrel, for he is very much afraid, and "ill-doers are ill-dreaders."

(37) Education removes or prevents error; the use of a good dictionary removes or prevents error: therefore, the use of a good dictionary is education.

(38) All, who think this man to be innocent think that he should not be punished; you think that he should not be punished · therefore, you think that he is innocent.

(39) This bright flower is from the east, for flowers of eastern countries have bright colours.

(40) Beggars, who have no property, can receive no injustice, because injustice is nothing but violation of property.

(41) It follows that all men desire to end their existence; for all men desire to be happy, and happiness is the end of human life

(42) Branches of Education are good which inform the mind with moral ideals; Mathematics is a branch of Education which does not inform the mind with moral ideals. Therefore, Mathematics is not good.

(43) Pious men only are fit to be ministers of religion; some ignorant men are pious: therefore, ministers of religion may be ignorant men.

(44) All winged creatures must be bipeds, for all birds are winged and they are bipeds.

(45) Studious men often break down their health; but as he is not studious, there is little possibility of his doing so.

(46) He who calls you an animal speaks truly; he who calls you an ass calls you an animal: therefore, he who calls you an ass speaks truly.

(47) The end of a thing is its perfection; death is the end of life: therefore, death is the perfection of life.

(48) The bamboo is a grass, because all grasses have parallel-veined leaves and so has the bamboo.

(49) None but express trains stop at this station; and as the last train did not stop, it could not have been the express train.

(50) James is wise, for he is careful, as wise men always are.

(51) The honest alone are trustworthy; and John being honest, he may be trusted.

(52) Everything is allowed by law which is morally right; indulgence in pleasure is allowed by law: therefore, indulgence in pleasure is morally right.

(53) It is impossible that thought can be a function of matter, because all functions of matter are modes of motion, which thought is not.

(54) The express train alone does not stop at this station, and as the last train did not stop, it must have been the express train.

(55) Some bacteria must be animals, because they require organic food, as all animals do.

(56) This syllogism must be valid; for, like valid syllogisms, it has three terms.

(57) The conclusion of this syllogism cannot be particular, as both of its premises are universal, and syllogisms with a particular premise give a particular conclusion.

(58) The conclusion of this syllogism cannot be valid, because in this syllogism the middle term has been distributed twice, whereas we see that syllogisms in which the middle term is distributed only once are valid.

(59) How can you deny that John is deeply versed in the subject? You see he criticizes all topics pertaining to it, and evidently such critics alone can be said to have a thorough insight into the subject.

(60) Dr. Johnson remarked that "A man who sold a pen-knife was not necessarily an ironmonger." Against what logical fallacy was this remark directed?

(61) Only the best graduates of the University are nominated for executive service under Government; but Jones has not been so nominated: therefore, he is not one of the best graduates of the University.

(62) Sankaracharyya was a great religious reformer and teacher; and as he was a bachelor, we may safely conclude that any one living a single life may become a great reformer and teacher.

(63) John must be thoroughly honest, since he is very loud in denouncing evil, and only those who so denounce are known to be honest.

(64) A boy looking at a white powder says: surely, this is sugar.

(65) No creature is immortal and no mortal being is perfect.

(66) Warm countries alone produce wine; but Abyssinia is a warm country; therefore, Abyssinia produces wine.

(67) Since the laws allow all that is innocent, and avarice is allowed, it must be innocent.

(68) Animals cannot live without atmosphere; there is no atmosphere on the moon: therefore, there can be no animal life on the moon.

(69) Men are sinners, saints are men: therefore, saints are sinners.

(70) Some one like me has come, none but Orestes is like me: therefore, Orestes has come.

(71) My hand touches the pen, the pen touches the paper: therefore, my hand touches the paper.

(72) A is larger than B, B is larger than C; therefore, *à fortiori* A is larger than C.

(73) No pauper has a vote, but James is not a pauper; therefore, he has a vote.

(74) None can deny that Faraday was a true man of science; for he spent his life in searching after truth, which is the object of search with all men of science.

(75) This candidate cannot expect to pass, for he has not studied his subject sufficiently.

(76) How can any one maintain that pain is an evil, who admits that remorse involves pain and yet may sometimes be a real good?

(77) Plato is not Socrates, Socrates is a man: therefore, Plato is not a man.

(78) God created man, man created sin; therefore, God created sin.

(79) I knew he was a Bohemian, for he was a good musician, and Bohemians are always good musicians.

CHAPTER XVIII

Mixed Syllogisms

§ 1. **Hypothetical-categorical Syllogisms.** As already mentioned (Ch. XVII § 3), a Hypothetical-categorical Syllogism is a form of mixed syllogism consisting of a hypothetical major premise, a categorical minor premise, and a categorical conclusion. The major premise may be any of the four forms of the hypothetical proposition, *viz.*, 'If A is B, C is D,' 'If A is B, C is not D,' 'If A is not B, C is D', and 'If A is not B, C is not D.' With any of these as major, we may combine four possible forms of the minor premise which may affirm or deny either the antecedent or the consequent of the hypothetical major. Thus we may have four possible varieties of the Hypothetical-categorical syllogism in connection with each of the four possible forms of the hypothetical major premise. All these varieties, however, are not valid. There are *two rules* which guide us in determining the validity of such syllogisms. These are :—

A Hypothetical-categorical syllogism consists of a hypothetical major premise, a categorical minor premise, and a categorical conclusion.

Two rules of inference.

(1) *If we affirm the antecedent, we may affirm the consequent, but not conversely,* i.e., *we cannot affirm the antecedent on affirming the consequent.*

(2) *If we deny the consequent, we may deny the antecedent, but not conversely,* i.e., *we cannot deny the consequent on denying the antecedent.*

Two fallacies arise from the violation of these two rules. The violation of the first rule is called the **Fallacy of affirming the consequent,** *i.e.*, the fallacy of affirming the antecedent on affirming the conse-

Fallacies arising from the violation of the two rules—Fal-

lacies of Affirming the Consequent and Denying the Antecedent.

quent. The violation of the second rule is called the **Fallacy of Denying the Antecedent**, *i.e.*, the fallacy of denying the consequent on denying the antecedent. Thus, taking the hypothetical proposition 'If A is B, C is D' as the major premise, we may have the following four forms of the Hypothetical-categorical syllogism :—

(1) If A is B, C is D; A is B : . C is D	(2) If A is B, C is D; C is not D : A is not B
(3) If A is B, C is D, C is D A is B.	(4) If A is B, C is D; A is not B : C is not D.

Of these, forms (1) and (2) are valid ; form (3) commits the *Fallacy of Affirming the Consequent*, and form (4) commits the *Fallacy of Denying the Antecedent* No valid conclusion may ever follow from affirming the consequent or denying the antecedent.

Reason of the two rules

The proof of the above two rules is implied in the very nature of a hypothetical proposition. It has already been pointed out (Ch. XI, § 3) that in a hypothetical proposition, the antecedent corresponds to the ground or cause, and the consequent to the effect or what necessarily follows from it. In the hypothetical proposition 'If A is B, C is D,' 'A being B' is a condition of 'C being D ;' but it is not to be supposed that the given antecedent is the only condition of the given consequent. 'If a man is stabbed, he dies,' but stabbing cannot be said to be the only condition of a man's death, for his death may be due to other causes as well. For this reason, from the affirmation of the antecedent, *viz.*, of being stabbed, we may infer a man's death, for the cause being present, the effect must be present ; but from the

affirmation of the consequent, *viz.*, his death, we cannot infer that he has been stabbed, for his death may be due to various other causes, such as disease, old age, starvation, etc. Similarly, from the denial of the consequent, *viz.*, his death, we may infer that he has not been stabbed, for the effect being absent, the cause must be absent ; but from the denial of the antecedent, *viz.*, from the fact that he has not been stabbed, we cannot infer that he is not dead, for his death may be produced by many other causes.

Note. In interpreting the rules, it is to be remembered that to affirm the antecedent or consequent does not mean to change a negative antecedent or consequent into the affirmative form, but to take it in its given quality, whether it be affirmative or negative. Also, to deny the antecedent or consequent means to change the given quality, *i.e.*, to make it affirmative when it is given negative, and to make it negative when it is given affirmative. Thus, if we are to affirm the antecedent of the proposition 'If A is not B, C is D', we are to take it in the form 'A is not B;' and if we are to deny the consequent of the proposition 'If A is B, C is not D,' we are to make it 'C is D'

It is also to be noted that the above two rules do not apply when the antecedent is the only and sole-sufficing condition of the consequent. In such cases, the affirmation of the consequent, or the denial of the antecedent, yields a valid conclusion. Thus the arguments—

'If a triangle is equilateral, it is also equiangular,
This triangle is not equilateral;
∴ This trangle is not equiangular :' and

'If he marries, he becomes a husband,
He becomes a husband;
∴ He marries'—are quite right

Two 'Moods' of Hypothetical-categorical Syllogisms.

Corresponding to the two rules given above, there are two Moods of Hypothetical-categorical syllogisms. If the minor premise affirms the antecedent of the major premise, the syllogism is called **Modus Ponens** or **Constructive** Syllogism. If the minor premise denies the consequent of the major, the syllogism is called **Modus Tollens** or **Destructive** Syllogism. Let us now illustrate the different forms

of these two Moods by means of symbolic and concrete examples :—

I. *Modus Ponens* (*Constructive*).

Illustrations.

(1) If A is B, C is D;	If the sun rises, there is light;
A is B :	The sun rises :
∴ C is D.	∴ There is light.
(2) If A is B, C is not D;	If you are moderate, you do not suffer;
A is B :	You are moderate :
∴ C is not D	∴ You do not suffer.
(3) If A is not B, C is D;	If the weather is not cold, you go out;
A is not B :	The weather is not cold :
∴ C is D.	∴ You go out.
(4) If A is not B, C is not D;	If the man is not honest, he is not to be trusted;
A is not B :	The man is not honest :
∴ C is not D.	∴ He is not to be trusted.

II. *Modus Tollens* (*Destructive*).

(1) If A is B, C is D;	If the sun rises, there is light;
C is not D .	There is no light :
∴ A is not B.	∴ The sun does not rise.
(2) If A is B, C is not D;	If you are moderate, you do not suffer;
C is D .	You suffer :
∴ A is not B	You are not moderate.
(3) If A is not B, C is D,	If the weather is not cold, You go out;
C is not D :	You do not go out :
∴ A is B.	∴ The weather is cold.
(4) If A is not B, C is not D,	If the man is not honest, he is not to be trusted;
C is D :	The man is trusted :
∴ A is B.	∴ He is honest.

Illustrations of the violation of the two rules.

To these examples of valid Hypothetical-categorical syllogisms we may also add some examples of the violation of the two rules :—

(1) If A is B, C is D;	If there is rain, the ground is wet;
A is not B :	There is no rain :
∴ C is not D.	∴ The ground is not wet.

This argument involves the *Fallacy of Denying the Antecedent.*

(2) If A is not B, C is D;	If he is not ill, he attends;
C is D:	He attends:
∴ A is not B.	∴ He is not ill.

This argument involves the *Fallacy of Affirming the Consequent.*

(3) If A is B, C is not D;	If you are vaccinated, you do not get an attack of small-pox;
A is not B:	You are not vaccinated:
∴ C is D.	∴ You get an attack of small-pox.

This argument involves the *Fallacy of Denying the Antecedent.*

Reduction of Hypothetical-categorical Syllogisms to Pure Categorical Syllogisms.

As hypothetical propositions are capable of being reduced to categorical forms, it is apparent that Hypothetical-categorical syllogisms may also be reduced to Pure Categorical syllogisms. When thus reduced, *Modus Ponens* takes the form of *Barbara* or *Celarent* in Fig. I, and *Modus Tollens* that of *Camestres* or *Cesare* in Fig. II. Thus example (1) under *Modus Ponens* may be reduced to the corresponding categorical form as shown below:—

All cases of A being B are cases of C being D;
This is a case of A being B:
∴ This is a case of C being D.
The case of the rising of the sun is a case of light;
This is the case of the rising of the sun:
∴ This is a case of light.

This is *Barabara* in Fig. I.

We may similarly reduce example (2) under *Modus Tollens* thus:—

No case of A being B is a case of C being D;
This is a case of C being D:
∴ This is not a case of A being B.
No case of being moderate is a case of suffering;
Yours is a case of suffering:
∴ Yours is not a case of being moderate.

This is *Cesare* in Fig. II.

It may easily be understood that if an argument in the Hypothetical-categorical form is fallacious, it must also be fallacious when reduced to its corresponding categorical form. Thus the fallacy of *Denying the Antecedent* is equivalent to the fallacy of *Illicit Process of the Major*, and the fallacy of *Affirming the Consequent* is equivalent to the fallacy of *Undistributed Middle*. If the fallacious arguments (1) and (2) given above are reduced to their corresponding categorical forms, we may easily detect the fallacies involved in them:—

(1) All cases of rain are cases of ground being wet;
This is not a case of rain
∴ This is not a case of ground being wet.
It involves the fallacy of *Illicit Major*.

(2) All cases of his not being ill are cases of his attending;
This is a case of his attending
∴ This is a case of his not being ill.
It involves the fallacy of *Undistributed Middle*.

A Disjunctive-categorical Syllogism consists of a disjunctive major premise, a categorical minor premise and a categorical or disjunctive conclusion.

§ 5. **Disjunctive-categorical Syllogisms.** A Disjunctive-categorical Syllogism is a form of mixed syllogism consisting of a disjunctive major premise, a categorical minor premise, and a categorical or disjunctive conclusion. The following are examples of such syllogisms:—

(1)	(2)
A is either B or C;	A is either B or C or D;
A is not B	A is not B;
∴ A is C.	∴ A is either C or D.

We have already seen (Ch. XII, § 3) that when the alternatives of a disjunctive proposition are mutually exclusive, the truth of the one implies the falsity of the rest, and *vice versa*; but when the alternatives are not mutually exclusive, the falsity of the one implies the truth of the rest, but not *vice versa*. Hence the rule of determining the validity of disjunctive-categorical Syllogisms is: *To deny one*

Rule of inference.

alternative is to affirm the rest, but from affirming

any alternative nothing follows unless the two alternatives are mutually exclusive.

According to the above rule, we get two Moods, *viz.*, **Modus Tollendo Ponens** (*i.e.*, the Mood which by denying affirms) and **Modus Ponendo Tollens** (*i.e.*, the Mood which by affirming denies), when the alternatives are mutually exclusive; while we get only one Mood, *viz.*, **Modus Tollendo Ponens,** when the alternatives are not mutually exclusive. Thus:— *'Moods' of Disjunctive-categorical Syllogisms.*

Modus Tollendo Ponens

(1) He is either alive or dead;
He is not alive:
∴ He is dead.

(2) He is either alive or dead;
He is not dead:
∴ He is alive

Modus Ponendo Tollens

(3) He is either alive or dead;
He is alive:
∴ He is not dead.

(4) He is either alive or dead;
He is dead:
∴ He is not alive.

Here all the four forms lead to valid conclusions, inasmuch as the alternatives 'alive' and 'dead' are mutually exclusive. Again:—

Modus Tollendo Ponens

(1) He is either a fool or a knave;
He is not a fool:
∴ He is a knave.

(2) He is either a fool or a knave;
He is not a knave:
∴ He is a fool.

Modus Ponendo Tollens

(3) He is either a fool or a knave;
He is a fool:
∴ He is not a knave.

(4) He is either a fool or a knave;
He is a knave:
∴ He is not a fool.

Here the forms (1) and (2) in the *Modus Tollendo Ponens* yield valid conclusions, but the forms (3) and (4) in the *Modus Ponendo Tollens* are invalid, inasmuch as the alternatives 'fool' and 'knave' are not mutually exclusive, for a person may be both a *Modus Ponendo Tollens* should be treated as a fallacy in Deductive Logic.

fool and a knave. As, without material knowledge, we can never be sure whether the alternatives are mutually exclusive or not, it may, as a general rule, be laid down that *Modus Ponendo Tollens* should be regarded as a fallacy in Deductive Logic.

As we have seen before (Ch. XVI, § 7) that a disjunctive proposition may be reduced to a hypothetical proposition, and a hypothetical proposition, again, may be reduced to a categorical one, it is easy to reduce a mixed Disjunctive syllogism into its corresponding mixed Hypothetical syllogism or into a pure syllogism, either categorical or hypothetical.

The Dilemma is a mixed syllogism consisting of a compound hypothetical major premise, a disjunctive minor premise, and a categorical or disjunctive conclusion.

§ 5. Hypothetical-disjunctive Syllogisms: The Dilemma. The Dilemma is a form of mixed syllogism consisting of a compound hypothetical major premise, a disjunctive minor premise, and a categorical or disjunctive conclusion. Thus the major premise is a compound hypothetical proposition (*i.e.*, it is a combination of two distinct hypothetical propositions) with more than one antecedent, or more than one consequent, or more than one of both. The minor premise is a disjunctive proposition, either alternatively affirming the antecedents, or alternatively denying the consequents, of the major premise. The conclusion is categorical, if there be only one antecedent or only one consequent in the hypothetical major; and it is disjunctive, if there be more than one of both. Moreover, the conclusion is equally unpleasant to the opponent, whichever of the alternatives in the disjunctive minor may be accepted as true. Sometimes the name 'Dilemma' is restricted to mean that form of Hypothetical-disjunctive Syllogism, which consists of two hypotheticals and two alternatives. If there are three hypotheticals and three alternatives, the argument is called a

Trilemma; if four, a *Tetralemma*; if more than four, a *Polylemma*. These more complex forms, however, are regulated by the same rules as those of the Dilemma. For this reason, the name *Dilemma* is sometimes used generically for all forms of Hypothetical-disjunctive Syllogisms.

Different forms of the Dilemma.

Having thus understood the meaning of the dilemmatic argument, let us now explain and illustrate its different **Forms.** In the first place, a dilemma may be either Simple or Complex. If the compound hypothetical major premise consists of one antecedent and two consequents, or one consequent and two antecedents, or, in other words, if the two hypothetical propositions forming the compound hypothetical major have a common antecedent or a common consequent, thus leading to a categorical conclusion, the dilemma is called **Simple.** If, however, the compound hypothetical major consists of two separate antecedents and two separate consequents, thus leading to a disjunctive conclusion, the dilemma is called **Complex.** In the second place, a dilemma may be either Constructive or Destructive. If the disjunctive minor premise affirms the antecedents of the major, the dilemma is called **Constructive.** If it denies the consequents of the major, the dilemma is called **Destructive.** Thus we have altogether four forms of the dilemma, *viz.*, (1) Simple Constructive, (2) Simple Destructive, (3) Complex Constructive, and (4) Complex Destructive.

1. Simple Constructive Form.

(1) **Simple Constructive Form.** Here the major premise has two antecedents and one consequent. The minor premise disjunctively affirms the antece-

dents of the Major, leading to a categorical conclusion. Thus:—

Examples.

(i) If A is B, C is D; and if E is F, C is D;
(or, as it is sometimes put, if A is B or E is F, then C is D;)
Either A is B or E is F:
∴ C is D.

(ii) If you are destined to pass, you need not read; and if you are destined to fail, you need not read;
But you are destined either to pass or to fail:
Therefore, you need not read.
(This is known as the *Ignava Ratio* or the Lazy Argument.)

(iii) If a man is unmarried, he is unhappy because he has no wife to take care of him; and if he is married, he is unhappy because he has to take care of his wife;
But he is either married or unmarried:
Therefore (in either case) he is unhappy.

2. Simple Destructive Form.

(2) **Simple Destructive Form.** Here the major premise has one antecedent and two consequents. The minor premise disjunctively denies the consequents, leading to a categorical conclusion. Thus:—

Examples.

(i) If A is B, C is D; and if A is B, E is F;
(or, as it may also be put, If A is B, then C is D, and E is F.)
Either C is not D or E is not F.
∴ A is not B

(ii) If the ship spring a leak, she will settle down; and if the ship spring a leak, she will capsize;
Either she will not settle down or she will not capsize
Therefore, she does not spring a leak.

(iii) If I am to make myself rich, I must earn money and be economic,
But either I cannot earn money or cannot be economic
Therefore, I cannot be rich.

(iv) If a body moves, it must move either in the place where it is, or in the place where it is not;
But it can move neither in the place where it is, nor in the place where it is not:
Therefore, it cannot move at all (*i.e.*, motion is impossible).
(This is Zeno's famous dilemma to prove that motion is impossible.)

3. Complex Constructive Form.

(3) **Complex Constructive Form.** Here the major premise has two antecedents and two consequents. The minor premise disjunctively affirms the

antecedents, leading to a disjunctive conclusion. Thus:—

(i) If A is B, C is D; and if E is F, G is H; — Examples.
Either A is B, or E is F:
∴ Either C is D, or G is H.

(ii) If Æschines joined in the public rejoicings, he is inconsistent; if he did not, he is unpatriotic;
But he either joined, or not:
Therefore, he is either inconsistent or unpatriotic.

(This example is found in the oration of Demosthenes *On the Crown*.)

(iii) If these books contain the same doctrines as the Koran, they are unnecessary; and if they are at variance with the Koran, they are wicked;
But they must either contain the same doctrines as the Koran or be at variance with it:
Therefore, these books are either unnecessary or wicked.

(Caliph Omar thus justified himself in burning the famous Alexandrian library.)

(iv) If you act justly, men will hate you; and if you act unjustly, the gods will hate you;
But you must act either justly or unjustly:
Therefore (by entering public life) you will be hated.

(The Athenian mother thus tried to dissuade her son from entering public life.)

(4) **Complex Destructive Form.** Here the major premise has two antecedents and two consequents. The minor premise disjunctively denies the consequents, leading to a disjunctive conclusion. Thus:— — 4. Complex Destructive Form.

(i) If A is B, C is D; and if E is F, G is H; — Examples.
Either C is not D, or G is not H:
∴ Either A is not B, or E is not F.

(ii) If he were clever, he would see his mistake, and if he were candid, he would acknowledge it;
Either he does not see his mistake or he does not acknowledge it:
Therefore, either he is not clever or he is not candid. (Stock)

(iii) If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest;
But he does it, either in jest or earnest:
Therefore, he is either not wise, or not good. (Whately)

Note. The true dilemma is defined by Whately as "a conditional syllogism with several antecedents in the major and a disjunctive minor." This definition excludes the — The different views

of the dilemma.

simple destructive form which has only one antecedent in the major. Mansel, Jevons and others are at one with Whately in respect of this view. These Logicians, as Coffey puts it, reject the simple destructive form on the ground that "the same conclusion may be reached by substituting for the denial contained in the alternative or disjunctive minor ('either C is not D or E is not F'), the more complete denial contained in the *remotive* minor 'neither C is D or E is F.' This would give us two distinct mixed hypothetical syllogisms (in the *Modus Tollens*) :—

(a) If A is B, C is D;	(b) If A is B, E is F;
But C is not D :	But E is not F :
∴ A is not B.	∴ A is not B."

(*Science of Logic*, I, p. 368.)

Following Welton, the same writer next proceeds to show reasons why four, instead of three, forms should be recognized. "This is true," he says, "but it is no reason why we should not recognize both ways of reaching the conclusion as distinct. Moreover, the same is true of the simple constructive form : we can get the same conclusion by conjunctively (copulatively) affirming the antecedents of the major, as by alternatively, affirming them, *i.e.*, by the *Modus Ponens* of the mixed hypothetical syllogism. Hence, to be consistent, those logicians should reject the simple constructive, as well as the simple destructive, dilemma. Besides, these are mutually reducible; hence, they must stand or fall together" (*ibid.*, p. 375). This simple view, recognizing four distinct forms of the dilemma, is accepted by most modern Logicians such as Keynes, Welton, Coffey, Fowler, and others. For the different views of the dilemma held by Ueberweg, Thomson, Hamilton and others, *vide* Welton, *Manual of Logic*, pp. 381-4; Dr. Ray, *Deductive Logic*, pp. 286-9; Coffey, *Science of Logic*, I, pp. 374-5.

Rules or Conditions of a valid dilemma.

What, then, are the **Rules** or **Conditions of a valid dilemma?** In the first place, the formal correctness of a dilemma depends on that of its constituent hypothetical-categorical syllogisms. Thus, let us take example (*i*) under the complex constructive dilemma given above, and resolve it into two hypothetical-categorical syllogisms :—

If A is B, C is D;	If E is F, G is H;
A is B :	E is F :
∴ C is D,	∴ G is H.

Now, either A is B, or E is F;
∴ Either C is D, or G is H.

Now, as these two constituent syllogisms are formally correct, inasmuch as their conclusions affirm the

consequents on affirming the antecedents, the dilemma itself must also be regarded as formally correct. Similarly, we may resolve any of the examples of the dilemma given above in order to ensure its formal correctness. Thus it appears that the dilemma is formally incorrect, if its minor premise disjunctively affirms the consequents or denies the antecedents of the major premise, and thereby the conclusion respectively affirms the antecedents or denies the consequents. Hence we may enumerate the *formal conditions* of the dilemma as follows:— Formal conditions of the dilemma

(1) The major premise must contain two distinct hypothetical propositions.

(2) The minor premise must be a disjunctive proposition, and must either affirm the antecedents or deny the consequents of the major premise.

(3) The conclusion must be the same (in simple form) or similar (in complex form), and must always be unfavourable or perplexing to the opponent, whichever of the alternatives in the minor premise be accepted as true.

In the second place, the material validity of a dilemma depends on the material truth of its premises. Thus the first material condition is that the minor premise must be materially true. In other words, the alternatives mentioned in the disjunctive minor premise must be exhaustive, for, if any possible alternative be excluded, the premise must be regarded as materially false. But it is very difficult to fulfil this condition, for the alternatives mentioned in the minor premise can seldom exhaust all the possible cases unless they happen to be negatives or contradictories. For this reason, Jevons remarks that "dilemmatic arguments are more often fallacious

than not." Thus, in example (*ii*) under simple constructive form given above, the possible alternative 'you are destined to pass if you read' has altogether been ignored. The second material condition is that the major premise must be materially true. This conditions is fulfilled if the consequents really and necessarily follow from the antecedents. In other words, the connection between the antecedents and consequents must be real and not imaginary or false. Thus, in example (*iii*) under simple contructive form, we wrongly assume that a married man is unhappy because he has a wife to take care of, and that an unmarried man is also unhappy because he has no wife to take care of him.

Material conditions of the dilemma

Hence we may sum up the *material conditions* of the dilemma as follows:—

(1) The alternatives mentioned in the disjunctive minor premise must be exhaustive.

(2) There must be real connection between the antecedents and consequents of the major premise.

The alternatives assumed in the hypothetical major are called the horns of the dilemma

The dilemma has been characterized as a 'rhetorical device', 'a controversial form of argument,' a 'favourite weapon of orators and wits.' The alternatives contained in a dilemmatic argument are known as the *horns of a dilemma*, and the opponent, against whom it is directed, is said to be *impaled on its horns*, inasmuch as he finds himself forced to choose between the alternatives, any one of which leads to an equally disagreeable and unfavourable conclusion. The opponent, on the contrary, has to devise means of refuting the dilemma, which, as we have seen above, is more often fallacious than valid.

Refutation of a faulty dilemma

The next question, therefore, is: **How to refute a faulty dilemma?** Having tested the formal correct-

ness of the dilemma by the rules of the hypothetical-categorical syllogism (as above), we may next attempt to *refute* it with reference to its material invalidity in three different ways. Thus:—

(1) We may *take it by one of its horns, i.e.*, we may show that the connection between the antecedent and consequent of one of the hypothetical propositions forming the major premise is not real, but only fictitious. Thus we may refute the above example (*ii*) under the complex constructive form by pointing out that there is no inevitable and necessary connection between 'Æschines' not joining public rejoicings' and 'his being unpatriotic.' As a true patriot, he might have believed that the time was not then ripe for those rejoicings. 1. By taking it by one of its horns.

(2) We may *take it by both the horns, i.e.*, we may show that in both the hypothetical propositions forming the major premise, there is no real connection between the antecedents and consequents. Thus we may refute the above example (*iii*) under the complex constructive form by showing that the first hypothetical proposition of the major premise wrongly assumes that what is not in the Koran is unnecessary, and the second hypothetical wrongly assumes that what is not in conformity with the Koran is wicked. Again, in example (*iv*) under the simple destructive form, the major premise wrongly assumes that there is no change of place in motion. In fact, when a body moves, it moves from the place where it is to the place where it is not. 2. By taking it by both the horns.

(3) We may *escape between the horns, i.e.*, we may show that the alternatives mentioned in the minor premise are not exhaustive. Thus we may refute the above example (*ii*) under the simple con- 3. By escaping between the horns.

structive form by showing that the alternatives do not exhaust all possibilities ; one may, for example, be destined to pass, if he reads ; and his idleness may lead to his failure. Similarly, in example (*iii*), under the complex destructive form, the possible alternative of "speaking irreverently of Scripture quite in a criticizing spirit" has been ignored.

Rebutting a dilemma.

Rebutting a dilemma. We may, in certain cases, attempt to meet a faulty dilemma by *rebutting* it, *i.e.*, by advancing a counter-dilemma which proves an opposite conclusion. The opposing dilemma is formed by *transposing, and changing the quality of*, the consequents of the original dilemma. Thus the above example (*iv*) under the complex constructive form may be rebutted thus: –

If I act justly, the gods will love me ; if I act unjustly, men will love me ;

But I must act either justly or unjustly :

Therefore (by entering public life) I shall be loved.

Similarly, the above example (*iii*) under the complex constructive form may be rebutted thus :—

If these books contain the same doctrines as the Koran, they are not wicked ; and if they are at variance with it, they are not unnecessary ;

But they must contain the same doctrines as the Koran or be at variance with it :

Therefore, these books are either not wicked or not unnecessary.

It should be noted here that simple dilemmas cannot be rebutted, because there being only one antecedent or only one consequent, transposition is impossible. Moreover, complex destructive dilemmas

cannot be rebutted, because such an attempt will lead to the fallacy of *Affirming the Consequent*. Thus we see that only complex constructive Dilemmas can be rebutted as above.

It may further be observed that the "rebutting" a dilemma, though it may silence the opponent, should not be regarded as a form of refutation; since, in the majority of such cases, both dilemmas are equally fallacious and lead to two conclusions which are only *apparently* opposite. For example, in the above instance of "rebutting" the dilemma on public life, the two conclusions are not really incompatible, for, according to the given premises, a public man must always be both hated and loved. Hence the "rebutting" is only an apparent refutation.

Classical examples of rebutted dilemmas. The Litigiosus.

Note. Let us close this section by citing two classical examples of rebutted dilemmas. "Most famous is the *Litigiosus*. Protagoras agreed to train Euathlus as a lawyer, one-half the fee to be paid at once, and the other half when Euathlus won his first case. As Euathlus, engaged in no suit, Protagoras sued him, and confronted him with this dilemma: 'Most foolish youngman, if you lose this suit you must pay me by order of the court, and if you gain it you must pay me by our contract.' To which Euathlus retorted: 'Most sapient master, I shall not pay you; for if I lose this suit I am free from payment by our contract, and if I gain it, I am exonerated by the judgment of the court.' Of this difficulty several solutions have been offered. The most reasonable seems to be this. As Euathlus had until then won no case, the condition of the bargain was not fulfilled, and the judges should have decided in his favour. It was then open to Protagoras to bring a fresh suit, when the judgment must have gone against Euathlus."

The Crocodilus.

"Somewhat similar is the *Crocodilus*: A crocodile had seized a child, but promised the mother that if she told him truly whether or not he was going to give it back, he would restore it. Fearing that if she said he was going to give it back, he would prove her wrong by devouring it, she answered, 'You will not give it back,' and argued: 'Now you must give it back—on the score of our agreement if my answer is true, and to prevent its becoming true if it is false.' But the crocodile answered: 'I cannot give it back, for if I did your answer would become false, and thus I should break our agreement; and even could your answer be correct I could not give it back, as that would

make it false.' On this Lotze says: 'There is no way out of this dilemma; as a matter of fact, however, both parties rest their cases on unthinkable grounds; for the answer really given can as little be true or untrue independently of the actual result as could the answer she might have given, an answer which only differs from this in being more fortunate.' For, had she said 'you will give it back,' then its restoration would both have made her answer true and have fulfilled the agreement" (Welton, *Manual of Logic*, I, pp. 385-6).

Is the reasoning in the mixed syllogism Mediate or Immediate?

§ 4. Is the Reasoning in the Mixed Syllogism Mediate or Immediate? Kant, Hamilton, Bain and others argue that the reasoning in the mixed syllogism is to be regarded as immediate, and not as mediate. Thus Hamilton represents the hypothetical-categorical syllogism—"If A is B, C is D,' 'A is B,' 'C is D' in the following form:—

If A is B, C is D; (Premise)
∴ A being B, C is D. (Conclusion)

Again, he represents the disjunctive-categorical syllogism—'A is either B or C,' 'A is not B,' 'A is C'—in the following form:

A is either B or C, (Premise)
∴ A not being B, A is C. (Conclusion)

Similarly, the dilemma is represented as a form of immediate inference, inasmuch as it is only a combination of two hypothetical-categorical syllogisms.

Bain, according to whom immediate inference is no inference at all, goes a step further and maintains that in mixed syllogism there is no real inference, because 'the conclusion is implied in what has already been stated.' Thus he writes: "In the Conditional Proposition—If A is B, C is D, the equivalent is—A being assumed to be B, it follows that C is D. There is no inference in this case. Accepting 'A is B,' we accept 'C is D;' this is another expression for the same fact. 'If the weather continues fine, we shall go to the country,' is transformable into the equivalent form 'The weather continues fine, and so we shall go to the country.' Any person affirming the one, does not, in affirming the other, declare a new fact, but the same fact. No new matter is introduced into the assertion, it is a pure instance of the Law of Consistency." To this objection raised by Bain, Keynes rightly replies: "But is not this the case in all formal mediate inference? It cannot be maintained that the categorical syllogism is more than a pure instance of the law of consistency; or that the conclusion in such a syllogism is not implied in what has already been stated." To the above objection that the mixed syllogism is a form of immediate inference on the ground that the conclusion follows from one premise only, it has been rightly urged that the major and minor premises in the mixed syllogism are quite distinct

propositions, and that the conclusion follows, not from one premise taken singly, but from two premises taken together. Thus Coffey observes: "In the mixed hypothetical syllogism we infer from two propositions, neither of which is inferable from the other, and both of which are necessary to reach the conclusion. From the proposition 'If A is B, C is D' we can infer *immediately* 'If C is not D, A is not B;' but we cannot infer categorically that 'A is not B' unless we are also given, as in the mixed hypothetical syllogism, the premise that 'C is not D'" (*Science of Logic*, I, p. 361). Kant's objection that in the mixed hypothetical syllogism there are only two terms and there is no middle term has justly been met by Dr. Ray and others who hold that here there are really three terms as in a pure syllogism. If a mixed hypothetical syllogism be translated into a pure categorical one, we find that there are three terms in it. Thus if we change the mixed hypothetical syllogism—'If A is B, C is D;' 'A is B;' ∴ 'C is D'—into the corresponding categorical form, we have—

The case of A being B is a case of C being D,
This is a case of A being B;
∴ This is a case of C being D.

Thus it is apparent that "in the example taken above, the consequent as a many-worded term, is the major term, the antecedent as a many-worded term, is the middle term, and 'this case' or 'the case in question' *understood*, is the minor term" (Ray). Moreover, it may be observed that a valid mixed syllogism may be resolved into a valid categorical syllogism, 'and the latter is admittedly a mediate inference;' and we have already seen how the fallacies of mixed hypothetical syllogisms, *viz.*, those of *'affirming the consequent'* and *'denying the antecedent'* are respectively equivalent to the fallacies of the corresponding categorical syllogisms, *viz.*, those of *undistributed middle* and *illicit major*. Thus pure categorical syllogisms and mixed syllogisms must stand or fall together. Hence we conclude that mixed syllogisms have every claim to be regarded as a process of mediate inference. For a fuller treatment of the subject, students are advised to consult Bain, *Deduction*, pp. 116-9; Hamilton, *Logic*, pp. 378-85; Joseph, *Introduction to Logic*, p. 316; Keynes, *Formal Logic*, pp. 354-7; Ray, *Deductive Logic*, pp. 289-96; Minto, *Logic*, pp. 217-8.

§ 5. Typical examples worked out.

Question: Test the following arguments:—

(1) If men are wise, they would be happy; but they are not wise: therefore, they would not be happy.

(2) If Cæsar was a tyrant, he deserved to die, Cæsar was not a tyrant; therefore, he did not deserve to die.

(3) Logic is indeed worthy of being cultivated, if Aristotle is to be regarded as infallible, but he is not; Logic, therefore, is unworthy of being cultivated.

(4) If a country is prosperous, the people will be loyal, the people of this country are loyal; and, therefore, it must be prosperous.

(5) Logic is either a Science or an Art, it is a Science; therefore, it is not an Art.

(6) If you pass your B.A. Examination, you can study for the B.L.; but you have not passed your B.A. Examination: therefore, you cannot study for the B.L.

(7) An emperor is admirable either for his military skill or for his political wisdom. Akbar was admirable for his political wisdom. Therefore, he cannot have been remarkable as a military leader.

(8) Whether a man is poor through his own fault or through pure misfortune, he should be helped, through forgiveness in the former case, and through pity in the latter.

(9) If the studies prescribed by the Calcutta University tend to advance a man in life, they are useful; if they tend to increase national and private wealth, they are useful; but the prescribed course of studies has neither the one tendency nor the other; therefore, they are useless.

Answer: (1) This is an example of hypothetical-categorical syllogism. It is invalid, because it involves the *Fallacy of Denying the Antecedent*.

(2) This also involves the same fallacy as (1).

(3) When reduced to logical form, the argument stands thus:—

> If Aristotle is to be regarded as infallible, Logic is indeed worthy of being cultivated;
> But Aristotle is not infallible
> ∴ Logic is indeed unworthy of being cultivated.

This is an instance of mixed hypothetical syllogism. It involves the *Fallacy of Denying the Antecedent*.

(4) This is an instance of mixed hypothetical syllogism. It involves the *Fallacy of Affirming the Consequent*.

(5) This is an instance of mixed disjunctive syllogism. Being a case of *Modus Ponendo Tollens*, it is invalid, for the alternatives 'Science' and 'Art' are not mutually exclusive, and so Logic may be both a Science and an Art.

(6) This is an instance of mixed hypothetical syllogism. It seems to involve the fallacy of *Denying the Antecedent*, but it is valid, for here the antecedent is the only condition of the consequent.

(7) This is an instance of mixed disjunctive syllogism. Being a case of *Modus Ponendo Tollens*, it is invalid, for the alternatives are not mutually exclusive, and so Akbar may be admirable both for his military skill and for his political wisdom.

(8) When reduced to logical form, it stands thus:—

If a man is poor through his own fault, he should be helped through forgiveness; if he is poor through pure misfortune, he should be helped through pity;

But a man is poor either through his own fault or through pure misfortune :

Therefore (in either case) he should be helped.

This is an instance of simple constructive dilemma. It is formally valid, but the alternatives mentioned in the minor premise are not exhaustive, for a man may be poor partly through his own fault and partly through pure misfortune. Hence it is materially invalid.

(9) According to Keynes, Welton, Fowler and other modern Logicians, this is an instance of simple destructive dilemma. According to Whately, Mansel, Jevons and others, this is a mixed hypothetical syllogism in the *Modus Tollens*, and not a dilemma, because the minor premise is not a disjunctive proposition but a remotive proposition. According to both views, however, the argument is formally invalid, for the constituent syllogisms involve the *Fallacy of Denying the Antecedent*, as will appear from the following analysis :—

If A is B (If studies.. advance a man in life), or
E is F (If studies... national and private wealth),
C is D (they are useful) ;
Neither A is B nor E is F :
∴ C is not D (they are not useful, *i.e.*, they are useless).

This yields two distinct hypothetical-categorical syllogisms :—

If A is B, C is D;	If E is F, C is D;
A is not B :	E is not F :
∴ C is not D.	∴ C is not D.

Both these syllogisms commit the *Fallacy of Denying the Antecedent*.

§ 6. Exercises.

1. Explain and illustrate Hypothetical-categorical syllogisms. What are the rules of inference? Are there any exceptions to these rules?

2. Prove the rules of inference applicable to Hypothetical-categorical syllogisms. What fallacies arise from their violation? To what categorical fallacies do they correspond? Illustrate your answers.

3. Explain and illustrate the *Modus Ponens* and *Modus Tollens* of Hypothetical-categorical syllogisms.

4. Explain the nature of Disjunctive-categorical syllogisms. Exhibit their different forms, giving their rules, and the reason for them.

5. Take the proposition 'If S is P, Q is R' as major premise. Supply the possible categorical minors; and show which of them lead to valid conclusions.

6. Take the proposition 'S is either P or Q' as a major premise. Supply the different possible minor premises, and show which of them lead to valid conclusions, giving examples.

7. Discuss the question when in Deductive Logic we ought to regard the alternatives in a disjunctive proposition as mutually exclusive or not. Show how the analysis of the Disjunctive Syllogism depends on the answer given to the question.

8. Explain and illustrate the nature of Dilemmatic argument. What are its various forms?

9. State and explain the formal and material conditions of a Valid Dilemma.

10. What are the different views as to the various forms of the Dilemma? Which view would you accept and why?

11. Explain and illustrate the following statement:—
'Dilemmatic arguments are more often fallacious than not.'

12. Discuss the value of the Dilemma as an instrument of reasoning. What is meant by the expressions—'the horns of the dilemma' and 'placed on the horns of the dilemma'?

13. What are the different ways of refuting a faulty Dilemma? Fully explain in this connection what you mean by 'rebutting a Dilemma.' Take a concrete example of the Dilemma and rebut it.

14. Show that valid Mixed syllogisms may be resolved into valid categorical forms, and that the invalid ones in the first case may also be tested by the special canons of Categorical syllogisms.

15. Are mixed syllogisms forms of Immediate Inference? Discuss.

16. Examine the following arguments:—

(1) If a man takes poison he will die; but he does not take poison; therefore, he will not die.

(2) John is either intelligent or hardworking, but he is not intelligent; therefore, he is hardworking.

(3) If virtue is involuntary, vice is also involuntary; vice is voluntary; therefore, virtue is also voluntary.

(4) If the triangle is equilateral, its sides and its angles are equal; but neither its sides nor its angles are equal; therefore, it is not equilateral.

(5) The boy failed in the examination either through illness or through misfortune. But it was not illness. Therefore, it was through misfortune.

(6) If virtue were knowledge, it would be capable of being taught; but where are the teachers of virtue?

(7) If I were in your place, I should punish this man severely, but I am not.

(8) John is wise, if he is learned. As, however, he is wise, he must be learned.

(9) Gibbon was not very talented, for a successful author must be either very industrious or very talented, and Gibbon was very industrious.

(10) Where there is no law there is no injustice.

(11) The more abundant a thing becomes, the more will it sink in value, and silver is becoming more abundant every day.

(12) No one should be punished, if he is innocent; this man should not be punished: therefore, he is innocent.

(13) If the sun shines, it will be a brilliant day; if it is not foggy or cloudy, the sun will shine; therefore, if it is not foggy or cloudy, it will be a brilliant day.

(14) If a substance has inertia, it has gravity; if it does not resist, it has no inertia; therefore, if a substance does not resist, it has no gravity.

(15) An organized body is either an animal or a plant; this substance is neither; therefore, it is not an organized being.

(16) All existences are either mental or material; nothing is neither mental nor material: therefore, nothing is not in existence

(17) If the temperature rises, the barometer will fall; if the barometer falls, the weather will not be fine; therefore, if the temperature rises, the weather will not be fine.

(18) If a solid is heated, it becomes a liquid; if a liquid is heated, it becomes a gas; therefore, if a solid is heated, it becomes a gas.

(19) If men have free-will, they are responsible for their actions, but men have no free-will. therefore, they are not responsible for their actions.

(20) Man, apart from society, would be either a god or a beast; but man is neither.

(21) If I am fated to be drowned now, there is no use in my struggling; if not, there is no need of it. But either I am fated to be drowned now or I am not; so that it is either useless or needless for me to struggle.

(22) Protective laws should be abolished; for they are injurious, if they produce scarcity, and they are useless if they do not.

(23) A land army is unnecessary to an inland country; for, if its navy holds the sea, there is nothing for the army to do; and if its navy is driven from the sea, there is nothing that its army can do (to save the country from starvation).

(24) I do not see how Mr. Rhodes can escape censure. If he knew of Dr. Jameson's raid, he was guilty of complicity; if he did not, of negligence; and he either knew it or he did not.

(25) Electricity is either a form of matter or a mode of motion; mind is neither a form of matter nor a mode of motion; therefore, mind is not electricity.

(26) If I tell the truth, I shall offend the people; and if I tell a lie, I shall offend my conscience. Either I must tell the truth or tell a lie. Therefore, either I shall offend the people or offend my conscience.

(27) If they stay abroad, the wife will die; while the husband's lung will not stand the English climate; it is to be feared, therefore, that one must fall a victim.

(28) If schoolmasters can claim exemption from poor rates, it must be either by statute or by common law; but neither does the statute exempt them nor does the common law apply; therefore, they can claim no such exemption.

(29) If students are idle, examinations are unavailing; and if they are industrious, examinations are superfluous; students are either idle or industrious, therefore, examinations are either unavailing or superfluous.

(30) If emigrants are useless they are a burden to the colonies; if they are useful they are a loss to the mother country, but they are either useless or useful; therefore, emigration is either a burden to the colonies or a loss to the mother country.

(31) If I am to regain health, I must give up work and take a sea trip, but I cannot do both; therefore, I cannot regain my health.

(32) If you work hard, you will get a prize. Therefore, you must have worked hard, for you have got a prize.

(33) The standard of the Calcutta University must be low, since the percentage of success at its examinations is comparatively high, and it is a well known fact that the percentage of success is high when the standard is low.

(34) If the charge is false, the author of it is either ignorant or malicious; but the charge is true; therefore he is neither.

(35) Compulsory attendance at College lectures is absurd. For, if lectures are valuable, the boys will attend them without any compulsion, if they are not valuable, they need not be attended.

(36) Moral exhortations are useless; for good men do not need them, and bad men will pay no heed to them.

CHAPTER XIX

Abridged and Conjoined Syllogisms

§1. **Enthymemes.** The word 'Enthymeme' (from Gr. *en*, in, and *thymos*, mind) implies that some part of an argument, instead of being explicitly stated in language, has been kept in the mind and is to be understood. Thus, an Enthymeme has been defined as an abridged syllogism, *i.e.*, a syllogism of which some part (either a premise or the conclusion) has been suppressed or omitted. An Enthymeme is often called a form of **irregular syllogism,** because here the premises and the conclusion are not stated in the strict logical order. Thus we usually say 'John must die,' for 'he is a man;' here the major premise 'All men are mortal' has been suppressed. In common speech, ordinary conversation and even in literature, we often argue in this way by suppressing the most obvious element of an argument. Hence such a form of reasoning is sometimes characterized as a natural mode of reasoning. It shall, however, be remembered that to test the validity of an enthymematic argument, we must have it expanded into its proper logical form.

An Enthymeme is a syllogism abridged by the omission of some of its constituent propositions

Three orders or forms of enthymemes have been ordinarily recognized by Logicians, *viz.*,—

Four orders of Enthymemes

1. An *Enthymeme of the First Order*:—Here the major premise is suppressed; *e.g.*, 'Camels are herbivorous, for they are ruminants.' Evidently here the major premise 'All ruminants are herbivorous' has been suppressed.

2. *An Enthymeme of the Second Order*.—Here the minor premise is suppressed; *e.g.*, 'Air has weight, since all material bodies have weight.' Evidently here the minor premise 'Air is a material body' has been suppressed.

3. *An Enthymeme of the Third Order*:—Here the conclusion is suppressed, *e.g.*, 'John is a man, and no men are infallible.' Evidently here the conclusion 'John is not infallible' has been suppressed.

To these three orders, we may also add—

4. *An Enthymeme of the Fourth Order*:—Here only a premise or only the conclusion is stated, and the remaining parts are suppressed. For example, if we say 'John is not perfect,' we have expressed only the conclusion and we have to supply the major and minor premises, *viz.*, 'No man is perfect' and 'John is a man.' Again, seeing a particular poet imaginative, we may simply say 'All poets are imaginative,' which, however, may be expanded thus:—'All poets are imaginative,' 'This man is a poet,' therefore, 'This man is imaginative.'

An Enthymeme, according to Aristotle, is an incomplete syllogism, 'a syllogism from probabilities or signs.'

Note. Aristotle used 'enthymeme' in a different sense. He defines it as 'a syllogism from probabilities or signs.' In other words, according to Aristotle, the enthymeme was an *incomplete syllogism*, where the conclusion, instead of being conclusive, was only probable. This might happen when the major premise does not state a necessary or universal fact, but only a probable event. The weaker premise is often suppressed, and then the argument, though only probable, looks as if it were certain. Thus the argument "'This patient is fever-stricken, for he is thirsty' assumes that 'All fever-stricken patients are thirsty,' this is an argument in the Second Figure, but it is not a valid argument. Thirst is a sign or symptom of fever, but not a conclusive sign, because it is indicative of other ailments also. Yet the argument has a certain probability" (Minto, *Logic*, pp. 206-7). Hence we may say with Creighton that "the term 'enthymeme' seems to have been used by Aristotle for an argument from signs or from likelihood, without complete proof. Form this sense of logical incom-

pleteness, the name has come to be applied in modern times to an argument in which some part is omitted" (*Introductory Logic*, p 133).

Trains of Syllogistic Reasoning. A Train of Syllogistic Reasoning is a combination of two or more syllogisms so linked together as to prove a single conclusion. Such a train is also known as a **Polysyllogism.** In order to illustrate this, let us combine syllogisms in the following two forms —

Two or more syllogisms, so linked together as to justify a final conclusion, are said to form a Train of Syllogistic Reasoning

First Form	*Second Form*
(1) All A is B (minor premise), All B is C (major premise), All A is C (conclusion)	(1) All A is B (conclusion), All D is B (major premise), and All A is D (minor premise)
(2) All A is C (minor premise), All C is D (major premise), All A is D (conclusion)	(2) All A is D (conclusion), All B is D (major premise), and All A is B (minor premise)
(3) All A is D (major premise), All D is E (major premise), All A is E (conclusion)	(3) All D is E (conclusion), All C is E (major premise), and All D is C (minor premise)

In order to understand the distinction between the two forms given above, we are first to know the meanings of the terms 'Prosyllogism' and 'Episyllogism' When two syllogisms are related to each other in such a way that the conclusion of one forms a premise of the other, the former is called a **Prosyllogism** in relation to the latter, and the latter is called an **Episyllogism** in relation to the former In other words, a syllogism, whose conclusion forms a premise of another syllogism, is called in relation to that other a Prosyllogism. And a syllogism, which has for either of its premises the conclusion of another, is called in relation to that other an Episyllogism. Thus in the example of the *First Form* given above, syllogism (1) is called a prosyllogism in

Meanings of 'Prosyllogism' and 'Episyllogism'

relation to syllogism (2), and syllogism (2) is called a prosyllogism in relation to syllogism (3). Again, syllogism (2) is an episyllogism in relation to syllogism (1), and syllogism (3) is an episyllogism in relation to syllogism (2). Similarly, in the example of the *Second Form*, syllogism (1) is an episyllogism in relation to syllogisms (2) and (3), which latter are prosyllogisms in relation to the former. It is also evident that the two terms 'prosyllogism' and 'episyllogism' are only relative. The same syllogism may be a prosyllogism in relation to one, and an episyllogism in relation to another. In the example of the *First Form*, syllogism (2) is an episyllogism in relation to syllogism (1), while it is a prosyllogism in relation to syllogism (3).

Two forms of the train of syllogistic reasoning—Progressive or Episyllogistic train and Regressive or Prosyllogistic train.

Now, there are **two forms** of the train of syllogistic reasoning, *viz.*, Progressive, Episyllogistic or Synthetic train and Regressive, Prosyllogistic or Analytic train. In a **Progressive train**, the conclusion of each prior syllogism forms a premise of the next, until we arrive at the final conclusion, which thus occurs last and is determined by all the preceding syllogisms. It is called *Progressive*, because here the reasoning progresses, *i.e.*, advances, from premises to conclusion. It is also called *Episyllogistic*, because here the reasoning proceeds from a prosyllogism to an episyllogism. It is further called Synthetic, because here we have a synthesis or combination of two or more syllogisms, all of which taken together justify the final conclusion. The example of the *First Form* given above illustrates this train.

In a **Regressive train**, on the other hand, the conclusion is stated first, and the premises are next advanced for proving it; these premises, again, form

conclusions of, and are thus proved by, other succeeding syllogisms in like manner. Thus all the syllogisms of the train tend to support the first conclusion. It is called *Regressive*, because here we go backward from the conclusion to the premises supporting it. It is also called *Prosyllogistic*, because here the reasoning proceeds from an episyllogism to a prosyllogism. It is further called *Analytic*, because it consists in finding out by gradual analysis the reasons or premises which together prove the conclusion stated at the outset. The *Second Form* given above illustrates this train.

Trains of reasoning are generally in the first figure and in the mood *Barbara*. But they may be in any other mood and in any other figure as well. If one of the syllogisms in the train belongs to any other mood than *Barbara* and to any other figure than the first figure, the whole train is regarded as belonging to that other mood and figure. Thus, if one syllogism in the train is in *Cesare* and others are in *Barbara*, then that whole train is to be regarded as being in *Cesare*. It is also to be remembered that the train is taken to be valid if all the constituent syllogisms are valid, but if anyone of these is invalid, the train itself is taken to be invalid.

Abridged forms of Trains of Syllogistic Reasoning.

So far we have dealt with trains of syllogistic reasoning, wherein all the premises and conclusions are fully expressed. But a train of syllogisms may also be stated in an incomplete or irregular form. In other words, a train may be abridged by the omission of some of its parts. Corresponding to the above two trains, *viz.*, Progressive and Regressive, we have two abridged trains, *viz.*, **Sorites** and **Epicheirema**.

We shall consider these two abridged forms of trains of syllogistic reasoning in the following sections.

A Sorites is an episyllogistic train of reasoning, abridged by the omission of the conclusions of its constituent prosyllogisms.

§ 8. Sorites. A Sorites, as said above, is an abridged form of Progressive Train of reasoning in which all the conclusions, except the last one, and the corresponding premises, are suppressed. In other words, it is an episyllogistic train of reasoning, abridged by the omission of the conclusions of its constituent prosyllogisms.

Two forms of Sorites—the Aristotelian and the Goclenian

There are *two forms* of sorites, *viz.*, the Aristotelian and the Goclenian. If the suppressed conclusion of each prosyllogism in a progressive train of reasoning forms the *minor* premise of the next episyllogism, the sorites is called **Aristotelian**;* and if it forms the *major* premise, the sorites is called **Goclenian**.* Examples of these two kinds, symbolic and concrete, are given below :—

Illustrations.

Aristotelian Sorites	*Goclenian Sorites*
All A is B (minor),	All D is E (major),
All B is C (major),	All C is D (minor),
All C is D (major),	All B is C (minor),
All D is E (major),	All A is B (minor),
∴ All A is E	∴ All A is E.
John is an Englishman,	All men are mortal,
All Englishmen are Europeans,	All Europeans are men,
All Europeans are men,	All Englishmen are Europeans,
All men are mortal,	John is an Englishman,
∴ John is mortal.	∴ John is mortal.

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* *Note.* "The name Sorites is not employed by Aristotle in the modern sense, though he alludes to such chains of arguments. It appears to have been first used in this way by the Stoics, from whom it was adopted by Cicero, though its common acceptance was much later. The Goclenian Sorites received its name from Goclenius, who first discussed it in his *Isagoge in Organum Aristotelis* (1598)."—Welton, *Logic*, I, p. 309.

The symbolic illustrations, when fully expressed, stand thus :—

Analysis of the illustrations.

Aristotelian Form	*Goclenian Form*
(1) All A is B (minor), All B is C (major); ∴ All A is C (conclusion) :	(1) All D is E (major), All C is D (minor); ∴ All C is E (conclusion) :
(2) All A is C (minor), All C is D (major); ∴ All A is D (conclusion) :	(2) All C is E (major), All B is C (minor); ∴ All B is E (conclusion) :
(3) All A is D (minor), All D is E (major); ∴ All A is E (Final conclusion).	(3) All B is E (major), All A is B (minor); ∴ All A is E (Final conclusion).

In the same way, the concrete illustrations may also be analysed into their constituent syllogisms.

From a comparison of the two different kinds of sorites given above, it is evident that in both of them the conclusion as well as the premises are the same. But the two series differ in the order of arrangement of the premises. Thus the first premise of the Aristotelian sorites is the last of the Goclenian, the second of the Aristotelian sorites is the last but one of the Goclenian, and so forth.

Points of Difference between the two kinds of Sorites.

From what has been said above, it is now easy to characterize the two kinds of sorites more fully and to indicate the points of difference between them in the following manner :—

(1) In the Aristotelian form, the argument proceeds from terms of narrower extent to those of wider extent ; while in the Goclenian, the argument proceeds from terms of wider extent to those of narrower extent.

(2) In the Aristotelian form, the conclusion of each prosyllogism becomes the *minor* premise of the next episyllogism ; while in the Goclenian, the

conclusion of each prosyllogism becomes the *major* premise of the next episyllogism.

(3) In the Aristotelian form, the first proposition is a minor premise, and the other expressed propositions are major; while in the Goclenian, the first proposition is a major premise, and the other expressed propositions are minor.

(4) In the Aristotelian form, the propositions are so arranged that the predicate of one proposition becomes the subject of the next (giving it the appearance of Fig. IV; though, in reality, the constituent syllogisms are all in Fig. I); while in the Goclenian, the propositions are so arranged that the subject of one premise becomes the predicate of the next (and this, we know, is the order of Fig. I; and the constituent syllogisms are also in Fig. I).

(5) In the final conclusion of the Aristotelian form, the last predicate is proved of the first subject; while in the final conclusion of the Goclenian form, the first predicate is proved of the last subject.

Regular and Irregular Sorites.

Regular and Irregular Sorites. A sorites is called *regular* when all the constituent syllogisms are in the first figure, and the middle terms maintain a fixed relative position, so that if the middle term be predicate of one premise, it will always be subject of the next, or if it be subject of one premise, it will always be predicate of the next. The above two forms are examples of regular sorites. On the other hand, a sorites is called *irregular* when the constituent syllogisms are in different figures. Sorites are generally in the first figure, though according to Keynes and others, sorites in the second and third

figures are also possible.* As examples of irregular sorites, we may take the following:—

(1)	(2)
Some A is not B,	All A is B,
All C is B,	All B is C,
All D is C,	No D is C,
All E is D,	All E is D,
∴ Some A is not E.	∴ No A is E.

In example (1), all the constituent syllogisms are in Fig. II and in the mood *Baroco*. In example (2), the first syllogism is in Fig. I and in the mood *Barbara*, while the second and third syllogisms are in Fig. II and in the moods *Cesare* and *Camestres* respectively.

Rules of Regular Sorites. The following two rules are followed by all valid regular sorites:— Rules of Regular Sorites

(1) Only one premise can be particular, *viz.*, the first in the Aristotelian form and the last in the Goclenian.

(2) Only one premise can be negative, *viz.*, the last in the Aristotelian form and the first in the Goclenian.

Proof:—(1) If more than one premise be particular, then evidently there would arise the fallacy of *two particular premises*. One particular premise in combination with a universal one would yield a particular conclusion, which will next be used as a premise. Now, if this particular premise happens to be combined with another particular premise, there would arise the fallacy of two particular premises. Proof of the rules.

In the Aristotelian sorites, only the first premise is a minor premise, all others are major. Now, in

* On the possibility of a Sorites in a figure other than the first—*Vide* Hamilton, *Lectures on Logic* II, p. 403; Keynes, *Formal Logic*, p. 373.

the first figure, the major premise must be universal; so, if any premise other than the first be particular, there would be a particular major, and this would lead to the fallacy of *undistributed middle*. Again, in the Goclenian sorites, the conclusion of a preceding syllogism forms the major premise of the next syllogism; so, if any premise other than the last be particular, there would be a particular conclusion, which will next be used as a major premise. But a particular major in the first figure would give rise to the fallacy of *undistributed middle*.

(2) If more than one premise be negative, then evidently there would arise the fallacy of *two negative premises*. One negative premise in combination with an affirmative one would yield a negative conclusion, which will next be used as a premise. Now, if this negative premise happens to be combined with another negative premise, there would arise the fallacy of two *negative premises*.

In the Aristotelian sorites, the conclusion of a preceding syllogism forms the minor premise of the next syllogism; so, if any premise other than the last be negative, there would be a negative conclusion, which will next be used as a minor premise. But a negative minor in the first figure would give rise to the fallacy of *illicit major*. Again, in the Goclenian sorites, only the first premise is a major premise, all others are minor. Now, in the first figure the minor premise must be affirmative; so, if any premise other than the first be negative, there would be a negative minor, and this would lead to the fallacy of *illicit major*.

An Epicheirema is a pro-

54. Epicheiremas. An Epicheirema, as already said, is an abridged form of Regressive Train

of reasoning in which one of the premises of each prosyllogism is suppressed. In other words, it may be described as a syllogism of which one or both premises are supported by a reason or reasons. It thus appears that in an epicheirema, we first state the conclusion, and next the two premises proving it ; and then add a reason or reasons for one or both of the premises (in the form of enthymemes). When one premise only is supported by a reason, the epicheirema is called **Single**; when both are thus supported, it is called **Double.** Again, when the reasons themselves are also supported by further reasons, the epicheirema is called **Complex,** as opposed to **Simple,** where the reasons for the premises are not supported by further reasons. Let us illustrate these different forms below :—

syllogistic train of reasoning abridged by the omission of one of the premises of its prosyllogisms. An Epicheirema may be (1) Single or (2) Double; and each of these, again, may be either (*a*) Simple or (*b*) Complex.

Illustrations.

1. Single Simple

All A is B,	
∵ All C is B,	∵ All D is B.
and All A is C.	

2. Double Simple

All A is B,	
∵ All C is B,	∵ All D is B.
and All A is C,	∵ All E is C

3. Single Complex

All A is B,		
∵ All C is B,	∵ All M is B,	∵ All X is B.
and All A is C.		

4. Double Complex

All A is B,		
∵ All C is B,	∵ All M is B,	∵ All X is B.
and All A is C,	∵ All P is C,	∵ All Q is C.

We may easily expand these different forms of epicheirema into the corresponding prosyllogistic

A symbolic illustration

expanded into a fully expressed train.

trains. Thus the above argument in the Double Complex form, when expanded, stands thus:—

(1) All A is B,
∵ All C is B,
and All A is C.

(2) All C is B,
∵ All M is B,
and All C is M (This was suppressed).

(3) All M is B,
∵ All X is B,
and All M is X (This was suppressed).

(4) All A is C,
∵ All P is C,
and All A is P (This was suppressed).

(5) All P is C,
∵ All Q is C,
and All P is Q (This was suppressed).

A concrete illustration expanded into a fully expressed train.

We may similarly frame concrete examples of the different forms. A concrete example of the Double Complex form is given below:—

Socrates is mortal,
∵ All men are mortal, ∵ All animals are mortal, ∵ All finite creatures are mortal.
and Socrates is a man, ∵ All philosophers are men, ∵ All lovers of truth are men.

This argument, when fully expressed, stands thus:—

(1) Socrates is mortal,
∵ All men are mortal,
and Socrates is a man

(2) All men are mortal,
∵ All animals are mortal,
and All men are animals (This was suppressed).

(3) All animals are mortal,
∵ All finite creatures are mortal,
and All animals are finite creatures (This was suppressed).

(4) Socrates is a man,
∵ All philosophers are men,
and Socrates is a philosopher (This was suppressed).

(5) All philosophers are men,
∵ All lovers of truth are men,
and All philosophers are lovers of truth (This was suppressed).

It is to be noted here that in the examples given above, all the prosyllogisms are enthymemes of the second order, for the conclusions and major premises are given, while the minor premises are suppressed. Similarly, examples of epicheirema composed of

enthymemes of the first order may also be framed, as in the following argument:—

All A is E,
∴ All A is D, ∵ All A is B.
and All D is E, ∵ All D is C.

§ 5. Classification of Trains of Syllogistic Reasoning. The different forms of Trains of Syllogistic Reasoning in their relation to each other may be exhibited in a tabular form thus:— Tabular view of Trains of Syllogistic Reasoning.

Trains of Syllogistic Reasoning
- Progressive, Episyllogistic or Synthetic
  - Fully expressed
    - Aristotelian
  - Condensed (Sorites)
    - Goclenian
- Regressive, Prosyllogistic, or Analytic
  - Fully expressed
    - Single
      - Simple
      - Complex
  - Condensed (Epicheirema)
    - Double
      - Simple
      - Complex

§ 6. Directions for working out Exercises. 1. In order to test the validity of an enthymematic argument, we have at first to supply the suppressed premise of the enthymeme. The argument is next put in its proper logical form and then tested by the syllogistic rules. Thus, the important question here is:—How are we to supply the suppressed premise of an enthymeme and to determine the figure and mood of an enthymematic argument?

(1) *To find out the order of an enthymeme*—We know that the subject of the conclusion is the minor term; and the predicate, the major term. So, if the conclusion and a premise be given, we can easily determine which premise is given. If the given premise contains the subject of the conclusion, it is the minor premise; and if it contains the predicate, it is the major premise. Thus we come to know which premise is suppressed. If the major be suppressed, the enthymeme is of the first order; if the minor, it is of the second order.

(2) *To supply the suppressed premise*—That term which occurs in the given premise, but which does not occur in the conclusion, is the middle term. So, in order to find out the suppressed premise, we are to combine the middle term with that term of the conclusion, which does not occur in the given premise. If the predicate of the conclusion be thus combined with the middle term, it is the major premise; and if the subject of the conclusion be thus combined, it is the minor premise.

(3) *To determine the figure and mood of an enthymematic argument*.—If the given premise and the conclusion have the same subject, the enthymeme is either in the first or in the second figure; if they have the same predicate, the argument is either in the first or in the third figure; if the predicate of the conclusion is subject of the given premise, the enthymeme is either in the second or in the fourth figure; if the subject of the conclusion is predicate of the given premise, the argument belongs either to the third or to the fourth figure When the enthymeme is of the third order, we can at once determine its figure and mood by the position of the middle term.

(4) *To find out premises for a given conclusion*—As we know, the subject and predicate of the conclusion are respectively the minor and major terms of the required syllogism, the only important thing is to find out a suitable middle term. We are next to frame the required syllogism with reference to the general syllogistic rules.

(5) *To draw the conclusion from two given premises* When the conclusion is suppressed, we do not know which is the major term and which the minor, and consequently it is not sure which one is the major premise and which one is the minor premise Here we can arrange the premises in any order to form a valid mood in any of the four figures. Any pair of premises should not be hastily rejected, because it does not readily yield a conclusion. By applying the processes of immediate inference to the given premises we may arrive at results leading to a valid conclusion. This has already been explained in the case of two negative premises (Ch XVII, § 8, Rule V, *Note*).

II. In order to test the validity of a train of reasoning, we have to examine separately each constituent syllogism. If any of the constituent syllogisms turn out to be incorrect, the whole train is to be regarded as invalid.

III. In order to test the validity of a Sorites or an Epicheirema, we have first to state it fully in the expanded form and next to examine the validity of all the constituent syllogisms. If all the constituent syllogisms are found to be correct, the argument itself is valid; if any of these is found to be incorrect, the argument is wrong.

Examples.

Question: Test the following arguments :—

(1) The people of England are wealthy, because they are industrious.

(2) James shall be admitted, for only first division candidates are admitted

(3) No men are immortal, and no mortal beings are perfect.

(4) He is too weak to walk.

(5) The Americans are a happy people; since they are allowed by everyone to be courageous, and those who are courageous are free, and those who are free are happy.

(6) All men are fallible, for they are finite. All Popes are men, for they are born of women.

(7) No church institutions are useful, for they teach religious matters, which are not business matters; and business matters are useful, being profitable.

Answer:

(1) It is an enthymeme of the First Order. The major premise 'All wealthy people are industrious' has been suppressed. It may be put in the logical form thus :—

All wealthy people are industrious,
The people of England are industrious;
∴ The people of England are wealthy.

The argument is invalid, as it involves the *Fallacy of Undistributed Middle*.

(2) It is an enthymeme of the Second Order. The minor premise 'James is a first division candidate' has been suppressed. It may be put in the logical form thus :—

All who are admitted are first division candidates,
James is a first division candidate;
∴ James shall be admitted.

This is invalid, as the argument involves the *Fallacy of Undistributed Middle*.

(3) Obviously it is an enthymeme of the Third Order, for the conclusion has been suppressed; we shall not, however, hastily reject the premises, because both of them are negative. Let us see if any conclusion may be drawn from them.

No men are immortal.......................................(*a*)
No mortal beings are perfect...............................(*b*)

Obverting (*a*) we get—all men are mortal.......................(*c*)
Taking (*b*) as the major premise and (*c*) as the minor, we get the following syllogism :—

No mortal beings are perfect,
All men are mortal;
∴ No men are perfect.

This argument violates no syllogistic rule and is in the mood *Celarent*. It is, therefore, valid.

(4) This is an enthymeme of the First Order. Here the major premise 'No too weak persons can walk' has been suppressed. When put in the logical form, it stands thus :—

No too weak persons are those who can walk,
He is a too weak person;
∴ He is not one who can walk (*i.e.*, He cannot walk).

The argument is in the valid mood *Celarent*. Hence it is valid.

(5) This is an Aristotelian Sorites. It may be arranged thus :—

All Americans are courageous..................(minor),
All who are courageous are free ....(major),
All who are free are happy......................(major),
∴ All Americans are happy ....(conclusion).

When the argument is fully expressed, we have the following two syllogisms :—

(*a*) All Americans are courageous..................(minor),
All who are courageous are free. ....(major),
∴ All Americans are free... ..(This was suppressed).
(*b*) All Americans are free ....(minor),
All who are free are happy..............................(major),
∴ All Americans are happy. ....(Final conclusion).

Both the constituent syllogisms are in the valid mood *Barbara*. Hence the argument is valid.

(6) This is a double Epicheirema. It may be arranged thus :—

All Popes are fallible ....(Final conclusion),
∵ All men are fallible, ∵ They are finite.
and All Popes are men, ∵ They are born of women.

The argument, when fully expressed, stands thus : —

(1) All Popes are fallible ....(Final conclusion),
∵ All men are fallible.......................................(major),
and All Popes are men...(minor),
(2) All men are fallible. ....(conclusion),
∵ All finite beings are fallible.. . ...(major. This was suppressed),
and All men are finite..(minor).
(3) All Popes are men.......................................(conclusion),
∵ All born of women are men.... ....(major. This was suppressed),
and All Popes are born of women........................(minor).

All the constituent syllogisms are in the valid mood *Barbara*. Hence the argument is valid.

(7) This is an Aristotelian Sorites. It may be arranged thus :—

All church institutions teach religious matters...(minor),
No religious matters are business matters.........(major),
All business matters are profitable.....................(major),
All profitable matters are useful ....(major),
∴ No church institutions are useful.................(conclusion).

The argument, however, is invalid, for in the first two premises we have really *four terms*, inasmuch as 'that which teaches religious matters' and 'religious matters' are not the same. Moreover, in the Aristotelian Sorites, only the last premise can be negative, but here the second premise is negative; consequently the term 'useful' has been distributed in the conclusion without being distributed in the premise, thus leading to the *Fallacy of Illicit Major*.

§ 7. **Exercises.**

1. What is an Enthymeme? What do you mean by the Order of an Enthymeme? What are the different Orders? Explain and illustrate the method of supplying the suppressed premise of an Enthymeme.

2. What is meant by a Train of Syllogistic Reasoning? Distinguish between a Prosyllogism and an Episyllogism, and between a Prosyllogistic Train and an Episyllogistic Train.

3. Why is a Prosyllogistic Train called Regressive or Analytical, and an Episyllogistic Train Progressive or Synthetical?

4. What is a Sorites? Exhibit the different kinds of Sorites, and give a concrete example of each.

5. Clearly distinguish between the Aristotelian and the Goclenian Sorites.

6. What do you mean by the Regular Sorites? State and prove the rules of the Regular Sorites.

7. What is an Epicheirema? Explain the different forms of Epicheirema, illustrating your answer by examples.

8. Examine the following arguments :—

(1) The night is cloudy; therefore, the dew will not be excessive.

(2) Cæsar deserved death, because he was a tyrant.

(3) The present universe began in time, and will in time come to an end.

(4) John is not happy, for he is envious.

(5) The Kritias cannot be a work of Plato; it is of small literary value.

(6) Death is nothing terrible; if it were so, it would have appeared so to Socrates.

(7) The sky being cloudy, the day is sultry.

(8) No imperfect beings are happy, and no unhappy beings are contented.

(9) This news is too good to be true.

(10) James is not a Christian; for the Christians alone hold such views.

(11) These merit our respect, for they are honest men.

(12) He is too good to be worldly successful.

(13) No dishonest men are virtuous; and the covetous are not honest.

(14) The world cannot be eternal, because it is not perfect.

(15) None but physicians came to the meeting. There were, therefore, no nurses there.

(16) Whatever has no parts does not perish by the dissolution of its parts. Therefore, the soul of man is imperishable.

(17) He speaks so frankly that I do believe him.

(18) Do not trust him, he has deceived me more than once.

(19) You must not believe him, because he has never been known to speak the truth.

(20) This proposition is too good to be practicable.

(21) All law is an abridgment of liberty, and consequently, of happiness.

(22) He was too impulsive a man not to have committed many errors.

(23) No metal is a compound substance, and gold is not a non-metal.

(24) None but animals are sentient beings, and all plants are insentient beings.

(25) Only material bodies gravitate; and light does not gravitate.

(26) There have been astronomers who were mere mathematicians; and there have been mathematicians who were mere calculating machines.

(27) All life, Hobbes argues, is activity; all activity arises from want; and therefore from sufferings.

(28) John is evidently pious, for he regularly says his prayers thrice a day.

(29) The last speaker is opposed to the motion, but every sensible man wishes it to pass.

(30) All novelty is injury, for it defaces the present state of things.

(31) All men are mortal; and all men are rational.

(32) All mortals are created; and no immortal beings are men.

(33) No fixed stars are planets; and all planets are round bodies.

(34) What fault has he committed that he should be punished?

(35) Darwin long ago taught us that the clover crop is dependent on the number of maiden ladies in the district. For the ladies keep cats, and the cats destroy field-mice, which prey on the bees, which, in their turn, are all-important agents in the fertilization of the clover flowers.

(36) The complete development of a being is its highest good. But complete development can be attained only through independence, and independence is possible only to members of a state. Therefore, man is naturally a political animal.

(37) The child of Themistocles governed her mother; she governed her husband; he Athens; and Athens Greece; the child of Themistocles, therefore, governed Greece.

(38) The prisoner should be executed, for it has been proved that he administered arsenic to the person who died; and as arsenic is poison, it is clear that he is guilty of murder.

(39) Athletic games are duties; for whatever is necessary to health is a duty and exercise is necessary to health, and these games are exercise.

(40) Gold is a metal; all metals are elements; all elements are material bodies; therefore, gold is a material body.

(41) I cannot assist you, because I do not think it right to encourage beggars.

(42) Bucephalus is a horse; a horse is a quadruped; a quadruped is an animal; an animal is a substance: therefore, Bucephalus is a substance.

(43) Sentient beings seek happiness; all finite beings are sentient; all men are finite beings; Caius is a man: therefore, he seeks happiness.

(44) All Malays are cruel, because all savages are; all the aboriginal inhabitants of Singapore are Malays, because all the natives of that part of Asia are; therefore, all the natives of Singapore are cruel.

(45) Whatever promotes happiness is good; whatever perfects the soul promotes happiness; therefore whatever perfects the soul is good; misfortune which happens to the good, serves either to discipline or to improve the soul: hence misfortune which befalls the good is good.

(46) If a gas is heated, its temperature rises; if its temperature rises, its elastic force increases; if its elastic force increases, the pressure on the walls of the containing vessel increases; therefore, if a gas is heated, the pressure on the walls of the containing vessel increases.

(47) All men are mortal, for they are animals; Socrates is a man, for rational bipeds are men; therefore, Socrates is mortal.

(48) The cost of labour depends on the efficiency of labour; the rate of profits depends on the cost of labour; the investment of capital depends on the rates of profits; wages depend on the investment of capital; therefore, wages depend on the efficiency of labour.

(49) God cannot be both good and omnipotent: if He is good, He cannot be omnipotent, since He is powerless against evil; and if He is omnipotent, He cannot be good, for He connives at the presence of evil.

(50) We cannot allow that AB has died from natural causes as he has evidently been poisoned, for he is known to have taken a large quantity of this substance which has been proved by the usual tests to be arsenic.

(51) Mercy but murders, pardoning those that kill.

(52) None but the contented are happy, none but the virtuous are contented, none but the wise are virtuous; therefore, none but the wise are happy.

9. Construct a valid Sorites consisting of five propositions and having 'Some A is not B' as its first premise. Point out the mood and figure of each of the distinct syllogisms into which the Sorites may be resolved.

CHAPTER XX

Functions and Value of the Syllogism

§ 1. **Validity and Use of the Syllogism as a Mode of Reasoning.** We have already described the characteristics of the syllogism and have seen that the syllogistic inference consists in deducing particular cases from universal truths (Ch. XVII, § 1). We have also examined the soundness of its underlying principle, *viz.*, the *Dictum de omni et nullo*, and have seen that this *Dictum*, rightly interpreted and truly understood, safeguards the syllogistic process of reasoning as a legitimate method of extending our knowledge from the known to the unknown, which is involved in all real inference (Ch. XVII, § 5). We shall now examine the truth of the objections urged against this mode of inference by John Stuart Mill, the most determined opponent of the syllogism.

The syllogism, as the type of Deductive reasoning, consists in passing from general to particular truths.

The validity of the syllogistic reasoning depends on its sufficiency to extend our knowledge from *given* to *new* truths.

First, Mill holds that the syllogism does not represent the true nature of the reasoning process, that it is not the normal mode in which we actually argue, and that, properly speaking, it is the reverse of the true way of reasoning. All reasoning, according to Mill, is *from particulars to particulars.* He represents the true type of reasoning thus:—

J. S. Mill's objections against the syllogism. (1) The syllogism does not represent the normal mode of reasoning, because, all reasoning is *from particulars to particulars,*

Individual men A, B, C, etc, are mortal; ∴ All men are mortal.	Induction, establishing a general proposition.
All men are mortal; Socrates is a man: ∴ Socrates is mortal.	Syllogism, interpreting the general proposition and testing it by applying it to a new case.

and not from generals to particulars.

Thus Mill says, "All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulæ for making more. Certain individuals have a given attribute ; an individual or individuals resemble the former in certain other attributes ; therefore they resemble them also in the given attribute" (*System of Logic*, Bk. II, Ch. III, § § 4 and 7). According to Mill, therefore, the conclusion of a syllogism is in every case drawn from the same set of particular facts from which the general proposition (the major premise) is itself derived. The general proposition is simply a summing up of the particular facts that we have actually observed. It thus appears that whenever we are using the general proposition to arrive at some particular facts, we are simply proceeding from one set of particular facts to other particular facts. And the only legitimate function, which Mill assigns to the syllogism, is to *interpret* the general proposition and to *test* it by applying it to particular cases.

(2) The syllogism, as a distinct kind of reasoning, involves the fallacy of *Petitio Principii*,

Secondly, Mill brings the charge of the fallacy of *Petitio Principii* against the syllogism. Logical inference, as we know, consists in passing from a given truth or truths to some new truth. When, however, in our reasoning, we assume in the premise what is to be proved in the conclusion, *i.e.*, when the conclusion asserts no new truth but is presupposed in the premise, we are said to commit the fallacy, technically known as *Petitio Principii* or *Begging the Question*. Thus Mill argues that the syllogism, considered as an independent process of reasoning, involves this fallacy ; for, when we say—'All men are mortal,' 'Socrates is a man ;' therefore, 'Socrates is mortal'—the conclusion 'Socrates is mortal' is presupposed in the more general (major)

premise 'All men are mortal.' In fact, there are two possible ways of arriving at the universal (major) premise 'All men are mortal.' Either we have examined all the individual cases of human mortality including the case of Socrates, or we have examined only some cases excluding others, of which the case of Socrates is one. If we have not examined all cases of human mortality, *i.e.*, if we have not got sufficient and full evidence for the major premise with which we start, we cannot use it as a basis for proving further facts. If we do so, we simply take for granted the very thing that we have to prove. If the major premise unwarrantably contains the case of Socrates, then it assumes the very thing that we have to prove. This is what Mill means when he says that the syllogism, as a distinct kind of inference, involves the fallacy of *Petitio Principii*. For the conclusion reveals no new truth; we only assume the conclusion in the major premise and next make a show of proving it by the syllogism, as if it were a new truth. If, on the other hand, we have examined all cases of human mortality including that of Socrates, *i.e.*, if we have got full evidence for our universal (major) premise, then the particular fact embodied in the conclusion must have already been examined and so need not be separately proved, and hence the syllogism turns out to be *useless*. This has sometimes been expressed in the form of the following dilemma:—

because the conclusion is already assumed in the major premise.

"If *all* the facts of the major premise of any syllogism have been examined, the syllogism is needless; and if *some* of them have not been examined, it is a *petitio principii*. But either all have been examined, or some have not. Therefore, the syllogism is either useless or fallacious."

Mill's contention that the syllogism is either useless or fallacious.

Mill's objections criticized.

In answer to the first objection that the syllogism is not the usual process of reasoning and that all reasoning is from particulars to particulars, it has been truly remarked that Mill makes a confusion between the province of Psychology and that of Logic. "It is true that the syllogism is not the process by which we usually reason. But it is equally true that our usual reasonings will not be valid, and therefore not deserve the name, unless they are capable of being reduced to the syllogistic form. Mill seems to make a confusion between the business of Psychology and that of Logic. It is not the business of the latter to give an account of the various processes by which people reason correctly or incorrectly, but to give an account of the processes by which they *ought* to reason, and *must* reason if they wish to reason correctly. The former is the business of the Psychology of Reasoning, while the latter is the business of the Logic of Reasoning. Mill confuses these two, and makes both the business of Logic. Recognizing the distinction here drawn, it may be said that the syllogism is the type of all valid reasoning; for no reasoning will be valid, as Mill also allows, unless it can be thrown into the form of a syllogism" (Ray, *Deductive Logic*, p. 255).

(1) Mill makes a confusion between the business of Psychology and that of Logic.

(2) What Mill calls inference from particulars to particulars is really inference from universals to particulars.

We may further observe that Mill's objection is based on a misunderstanding of the true nature of inference. He maintains that all reasoning is from particulars to particulars, but he cannot be right in this. We can never reason from particulars to particulars except through the medium of a universal. Thus, whenever we pass from particulars to particulars, we do so on the ground of essential identity between them; and this essential identity is vaguely

assumed in a general proposition implicit in thought. Hence we find that what Mill calls simply inference from particulars to particulars is at bottom inference from universals to particulars (*vide* Ch. V, § 4, and Ch. XXXI, § 5).

Next, as to the charge of *Petitio Principii*, we may remark that the general proposition used as a premise in the syllogism is to be understood in the *connotative* sense. All that it implies is that from what we have observed to be true in a number of individual cases, it has been found that a certain universal and necessary connection subsists between the predicate taken attributively (mortality) and the group of attributes implied by the subject (humanity). It is on the strength of this connection that we are warranted in using it as a basis for drawing further conclusions. Even if we interpret it *denotatively*, we are not involved in the fallacy of *Petitio Principii*. For the general proposition does not mean that we have examined *all* cases, neither does it mean that we have had no sufficient evidence for getting at it, but it means that from what we have observed, we have had reason enough to find some essential connection between the subject and the predicate. (In our subsequent treatment of Induction we shall see that Mill himself admits that we can arrive at universal propositions on actual observation of a few instances, and that such universal propositions are expressions of general and necessary connections of attributes.) Thus, at first, the general proposition is *implicit* to us in its full signification. It becomes *explicit* only through its being used as a premise for arriving at some particular truths. C. Read also expresses the same truth by saying that the syllogism 'begs the question formally, but not materially.' If,

(3) The syllogism does not appear to involve the fallacy of *Petitio Principii*, if we understand the universal premise in the connotative sense.

The true meaning of the universal major premise of a syllogism.

C. Read's interpretation of the syllogism. Syllogism begs the

question formally, but not materially.

in order to decide whether 'Socrates is mortal,' we are told that 'All men are mortal,' laying stress upon the 'all,' as if all had been examined, though in fact Socrates has not been, then the question as to Socrates is begged. But the question is not begged if we interpret the argument thus: 'Mortality' being connected with 'humanity' is connected with Socrates, because he is 'human'—for here the argument rests on material evidence, *i.e.*, on actual observation (of a few cases) leading to the establishment of a necessary connection between 'humanity' and 'mortality.'

Martineau's interpretation of the same. The *Petitio Principii* is relative to the degree of knowledge. The conclusion is not present in thought when the major premise is stated.

Martineau's defence of the syllogism is substantially the same. He remarks that the *petitio principii* is 'entirely relative to the state and range of the individual understanding.' What is a *petitio principii* to the teacher may not be so to the learner. "To the omniscient mind all reasoning must involve a *petitio principii*.' To us what is a *petitio principii* at one time was not so at another. Hence the syllogism does not involve the fallacy of *petitio principii*, because the conclusion is not known when the universal (major) premise is asserted (*vide* Ray, *Deductive Logic*, pp. 255-60). In fact, Mill overlooks the distinction between logical implication and actual knowledge, between what is only *implicitly* understood and what is *explicitly* known. Indeed, the universal premise logically implies and thus *implicitly* contains the conclusion, but one may not *explicitly* know it to be so contained. Hence to such a person the syllogism is not a *petitio principii*. All the theorems and problems of Euclid are implicitly contained in the definitions and axioms, but one cannot be said to have known them all simply because one has known the definitions and axioms.

Distinction between logical implication and actual knowledge, between *implicit* and *explicit* knowledge.

As a further answer to the same charge, it has also been remarked that Mill's objection makes the minor premise superfluous and unnecessary. We have already had occasion to point out the important function of the minor premise in a syllogism (Ch. XVII, § 5). In fact, the conclusion does not follow from the major premise alone (as Mill seems to suppose), nor from the minor alone, but from the two premises taken jointly. A general proposition is applicable to a new case, only when the latter is shown, by means of a minor premise, to have the essential characteristics of the subject of the general proposition.

(4) Mill's objection makes the minor premise superfluous.

Lastly, we may bring the following counter-charge against Mill. Mill cannot, without inconsistency, raise the objection of the fallacy of *Petitio Principii* against the syllogism, for he accepts the connotative interpretation of propositions (*vide* Ch. XIV, § 2), according to which view, the question of the knowledge of the conclusion will not arise at all in interpreting the meaning of the universal premise. According to the connotative interpretation, the major premise 'All men are mortal' does not mean that all men possess the quality of mortality, but that humanity is a mark of mortality. Following his connotative interpretation of the *Dictum*—'the mark of a mark is a mark of the thing itself' (*vide* Ch. XVII, § 5)—the syllogistic argument may be expressed thus:—

(5) To be consistent, Mill cannot raise the objection of *Petitio* against the syllogism.

Humanity is a mark of mortality,
Socrates is a mark of humanity:
∴ Socrates is a mark of mortality.

From what has been said above, it is clear that the syllogism is not a *useless* process of inference, because both the form of the argument and the con-

Thus the syllogism is not useless.

Various uses of the syllogism.

clusion obtained by it may represent some new features not known previously. Even Mill is of opinion that the syllogistic form of reasoning is not altogether useless, because :—

(*a*) The major premise is a convenient way of remembering the result of our previous observation and experiment. In short, it is a memorandum of past experience.

(*b*) It is also a recommendation for drawing other inferences in the light of previous experience.

(*c*) The syllogistic reasoning is a good method of testing and verifying the major premise by applying and extending it to new cases.

(*d*) Above all, the most important function of the syllogism is to test our previous Induction. If we cannot throw an Induction into the form of a syllogism we must suspect that there is something wrong in our previous Induction.

This conclusively establishes the validity and use of the syllogistic inference.

§ 2. **Exercises.**

1. In what does Logical Inference consist? Is the Syllogism a form of Logical Inference?

2. State and examine Mill's objection to the Syllogism as mode of argument

3. Does Aristotle's *Dictum de omni et nullo* involve the fallacy of *Petitio Principii*? Fully discuss (*vide* Ch. XVII, § 5).

4. What is the real significance of the universal major premise of a syllogism?

5. "The charge against the Syllogism that it involves the *petitio principii* is founded on a misunderstanding": explain the nature of, and the supposed warrant for, this charge, and the misunderstanding on which it is founded.

6. 'All inference is from particulars to particulars.' Test this statement.

7. Give a clear statement of the use and importance of the Syllogism. What, according to Mill, are its uses?

8. 'The Syllogism begs the question formally, but not materially.' Explain what you mean by the statement.

9. Reproduce the connotative restatements of Aristotle's *Dictum* as propounded by Mill and C. Read, in order to free it from the charge of *petitio principii* (Ch. XVII, § 5).

10. Discuss critically the question of the function and value of the Syllogism.

11 What is Mill's view of the major premise of a syllogism? And how does this view affect his estimate of the value of the Syllogistic theory?